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A Picture Is Worth a Thousand Words

Understanding Quantities and Their Relationships

MATERIALS

Glue sticks
Scissors

Lesson Overview

Students are presented with various scenarios and identify the independent and dependent quantities for each. They then match a graph to the appropriate scenario, label the axes using the independent and dependent quantities, and create the scale for the axes. Students make basic observations about the similarities and differences in the graphs. They then look more deeply at pairs of scenarios along with their graphs to focus on characteristics of the graphs, such as intercepts, increasing and decreasing intervals, and maximum and minimum points. The lesson concludes with students creating their own scenario and a sketch of a graph to model the scenario.

Algebra 1

Linear Functions, Equations, and Inequalities

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations.

The student is expected to:

(C) graph linear functions on the coordinate plane and identify key features, including x -intercept, y -intercept, zeros, and slope, in mathematical and real-world problems.

Quadratic Functions and Equations

(7) The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to:

(A) graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including x -intercept, y -intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry.

Exponential Functions and Equations

(9) The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the

reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:

(D) graph exponential functions that model growth and decay and identify key features, including y-intercept and asymptote, in mathematical and real-world problems.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G

Essential Ideas

- There are two quantities that change in problem situations.
- When one quantity is determined by another, it is said to be the dependent quantity. The quantity that the dependent quantity is determined from is called the independent quantity.
- The independent quantity is used to label the x-axis. The dependent quantity is used to label the y-axis.
- The domain includes the values that make sense for the independent quantity. The range includes the values that make sense for the dependent quantity.
- Graphs can be used to model problem situations.

Lesson Structure and Pacing: 1 Day

Day 1

Engage

Getting Started: What Comes First?

Students read descriptions of relationships between two quantities and identify which is independent and which is dependent.

Develop

Activity 1.1: Connecting Scenarios and Their Graphs

Students are presented with six different scenarios. For each scenario, they identify the independent and dependent quantities and match a graph. Students then scale the axes and determine the domain and range for each scenario.

Day 2

Activity 1.2: Comparing and Contrasting Graphs

Students make basic observations about the similarities and differences in the graphs from the previous activity. They then look more deeply at pairs of scenarios along with their graphs to focus on key characteristics, such as intercepts, increasing and decreasing intervals, and maximum and minimum points.

Demonstrate

Talk the Talk: A Writer and a Mathematician

Students create a scenario based upon a possible trip to school. They then sketch a graph to model their scenario. Students share their work with classmates and note similarities and differences.

Facilitation Notes

In this activity, students read descriptions of relationships between two quantities and identify which is the independent and which is the dependent.

Ask a student to read the introduction before Question 1 aloud. Review the definitions of dependent quantity and independent quantity as a class.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

As students work, look for

Strategies and phrases they use to determine which quantity depends on the other.

Questions to ask

- Which quantity forces the other quantity to change?
- Which quantity depends on the other quantity?
- What information is determined first, the number of movie tickets purchased or the total cost of the tickets?
- Does the number of movie tickets purchased depend of the cost of the tickets, or does the cost of the tickets depend on the number of movie tickets purchased?
- What information is determined first: the number of eggs used or the number of cakes baked?
- Does the number of students in attendance at school determine the number of lunches served or vice versa?
- Does the time driven determine the number of miles to a destination, or does the number of miles to a destination determine the time driven?
- Does the number of minutes the swimming pool is being filled depend on the number of gallons of water, or does the number of gallons of water depend on the number of minutes the swimming pool is being filled with water?

Differentiation strategy

To scaffold support when identifying which variable is dependent or independent, provide arbitrary numbers relevant to the situation. For example, ask students “If one movie ticket costs \$7.50, what is the cost of two movie tickets? Three movie tickets? Four movie tickets?” Use a follow up question similar to “Which value depends on the other, or which value is used to determine the other value?”

Misconception

Students may confuse the independent variable with the dependent variable. For example, they could think the number of movie tickets is determined by the total cost of the tickets (if the cost of three tickets is \$22.50, then each ticket must have been \$7.50). Just because the value of one variable can be determined using the value of a second variable, this does not signify dependence or independence.

Summary

There are two quantities that change in problem situations. When one quantity is determined by another, it is said to be the dependent quantity. The quantity that the dependent quantity is determined from is called the independent quantity.

DEVELOP

Activity 1.1 Connecting Scenarios and Their Graphs



Facilitation Notes

In this activity, students are presented with eight different scenarios. For each scenario, they identify the independent and dependent quantities and match a graph. Students then scale the axes and determine the domain and range for each scenario.

Ask a student to read the introduction before Question 1 aloud. As a class, discuss the directions to this task because it has several parts and includes cutting out and gluing graphs next to their scenario descriptions.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

As students work, look for

- Characteristics of the graphs that students use to connect them to the scenarios.
- Clues students use in the scenarios to determine the scale.

Differentiation strategy

As an alternative grouping method, use the jigsaw strategy for scaling the axes for each scenario. This strategy is meant to save time while providing a brief recall of scaling, but the sharing part is necessary so that students can use the information to determine the domain and range for each problem.

Misconception

Students confuse the concepts of domain and range. Ask students to explain the difference between the terms *domain* and *range*.

Questions to ask for *Music Club*

- How is the number of songs measured?
- How is the cost measured?
- What is the cost for zero songs? One song? Five songs?
- Does the number of songs determine the cost, or does the cost determine the number of songs?
- Can this scenario be described using a rate of change?
- Is this an increasing or decreasing function?
- What is the meaning of the point located on the x-axis?

Questions to ask for *Something's Fishy*

- How is time measured?
- How is water measured?
- How many gallons of water are drained from the aquarium in one minute? Two minutes? Three minutes?
- Does the amount of time determine the number of gallons of water emptied, or does the gallons of water emptied determine the amount of time?
- What is the rate of change in this situation? Is it a constant rate of change?
- What does a constant rate of change look like on a graph?
- Is this an increasing or decreasing function?

Questions to ask for *Smart Phone, but Is It a Smart Deal?*

- How is time measured?
- How is interest measured?
- How much interest will you pay if you borrow the money for one week? Two weeks? Three weeks? Four weeks? Five weeks?
- Does the amount of time the money was borrowed determine the amount of interest paid, or does the amount of interest paid determine the amount of time the money was borrowed?
- What is the rate of change in this situation? Is it a constant rate of change?
- Is this an increasing or decreasing function?
- What does this increasing rate of change look like on a graph?

Questions to ask for *It's Magic*

- How is the length of the pieces of rope measured?
- What is the length of the rope after the first cut? Second cut? Third cut? Fourth cut?
- Does the number of cuts in the rope determine the length of each piece of rope, or does the length of each piece of rope determine the number of cuts in the rope?

- What is the rate of change in this situation? Is it a constant rate of change?
- Is this an increasing or decreasing function?
- What does this decreasing rate of change look like on a graph?

Questions to ask for *Baton Twirling*

- How is time measured?
- How is the height of the baton measured?
- What is the height of the baton after 0.5 seconds? 1 second? 1.5 seconds? 2 seconds?
- Does the amount of time elapsed determine the height of the baton, or does the height of the baton determine the amount of time elapsed?
- Can this scenario be described using a rate of change?
- Is this an increasing or decreasing function?
- Can a function have intervals that increase and decrease?
- Does the function contain a minimum value or a maximum value? What is that value?
- What does a function that is both increasing and decreasing look like on a graph?
- If Jill wants to twirl around more times, what impact will it have on the maximum height of the baton?

Questions to ask for *Jelly Bean Challenge*

- What information does Mr. Wright record?
- Does Mr. Wright determine the number of jelly beans guessed, or the number of jelly beans each guess is off by?
- Does the number of jelly beans guessed determine the number of jelly beans the guess is off by, or does the number of jelly beans the guess is off by determine the number of jelly beans guessed?
- What is the meaning of the point located on the x-axis?

Questions to ask for all six scenarios

- What units of measure are used?
- Why did you decide to use this graph to describe this scenario?
- What words in the scenario helped you to decide the appropriate graph?
- Could more than one graph model this scenario? Why or why not?
- Is there any scenario that cannot be modeled using one of the graphs?
- How did you decide the label for the x-axis of the graph?
- How did you decide the label for the y-axis of the graph?
- Is the independent quantity located on the x-axis or the y-axis? Does it make a difference? Explain.
- Is the dependent quantity located on the x-axis or the y-axis? Does it make a difference? Explain.

Summary

Graphs can be used to model scenarios. Knowing the independent and dependent variables, as well as the domain and range, is helpful in making connections between the scenario and its graph.

Activity 1.2 Comparing and Contrasting Graphs



Facilitation Notes

In this activity, students make basic observations about the similarities and differences in the graphs from the previous activity. They then look more deeply at pairs of scenarios along with their graphs to focus on key characteristics, such as intercepts, increasing and decreasing intervals, and maximum and minimum points.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

As students work, look for

- Mathematical terms used to describe similarities and differences in the graphs.
- Instances where students would benefit from an increased mathematical vocabulary to describe graphical characteristics.

Questions to ask

- Is the independent quantity always located on the same axis?
Which axis?
- Is the dependent quantity always located on the same axis?
Which axis?
- Which graphs contain straight lines? Curves?
- Which graphs could be described as increasing from left to right?
- Which graphs could be described as decreasing from left to right?
- Could any graphs be considered both increasing and decreasing?
- Is it possible for a graph to be both increasing and decreasing?
- Can the curves on the graph be described as smooth curves? Are all curves considered smooth curves?
- Which graphs have a maximum value? A minimum value?
- Do both the *Something's Fishy* and *It's Magic* graphs decrease?
- Why is the *Something's Fishy* graph a straight line, while the *It's Magic* graph is a smooth curve?
- Do both the *Baton Twirling* and *Jelly Bean Challenge* graphs increase and decrease?

- Why is the *Baton Twirling* graph a smooth curve, while the *Jelly Bean Challenge* graph includes straight lines?

Summary

Key characteristics of graphs, such as intercepts, increasing and decreasing intervals, and maximum and minimum points are used to interpret scenarios and differentiate graphs.

DEMONSTRATE

Talk the Talk: A Writer and a Mathematician

Facilitation Notes

In this activity, students create a scenario based upon a possible trip to school. They then sketch a graph to model their scenario. They share their work with classmates and note similarities and differences.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- What are your independent and dependent variables?
- What units of measure were used to describe your variables?
- What do the values on the x -axis represent with respect to the problem situation?
- What do the values on the y -axis represent with respect to the problem situation?
- What x - and y -values make sense in this problem situation?
- Can your graph be described as increasing or decreasing?
- Is your graph curved or linear in nature?
- Does your graph contain any horizontal line segments? If so, what does this represent in the scenario?
- How many different pieces are on your graph?
- Does your graph contain any parallel line segments? What does this imply with respect to the scenario?
- If your graph contains a line segment having a negative slope, what would this imply with respect to the scenario?
- What point on your graph represents your home?
- What point on your graph indicates that you arrived at school?

Differentiation strategies

- To support students who struggle when setting up their graph, ask them questions about the significance of points on the x -axis, on the y -axis, and at the origin.

- To extend the activity, have students create a scenario and trade papers with a partner. Then have the partner draw a graph for the scenario. Have partners discuss if the graph drawn was the intent of the writer of the scenario.

Misconception

Students can confuse the domain and range of the problem situation with the domain and range of the algebraic function representing the situation. It is important to emphasize that the graph of a function may contain points that do not make sense with respect to the problem situation.

Summary

A graph is an efficient way to model and interpret a scenario.

NOTES

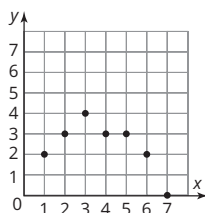
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A Picture Is Worth a Thousand Words

Understanding Quantities and Their Relationships

Warm Up

Emma bought a new video game. The graph shown describes the number of hours Emma spent playing the game over a period of 7 days.



1. Label the axes.
2. What does the highest point on the graph represent with respect to the scenario? The lowest point?

Learning Goals

- Understand quantities and their relationships with each other.
- Identify the independent and dependent quantities for a scenario.
- Match a graph with an appropriate scenario.
- Use a reasonable scale for a graph modeling a scenario.
- Identify key characteristics of graphs.
- Describe similarities and differences between pairs of graphs and scenarios.

Key Terms

- dependent quantity
- independent quantity

You have analyzed graphs of relationships and identified important features such as intercepts and slopes. How can the key characteristics of a graph tell a story?

Warm Up Answers

1. x-axis: Time (days)
y-axis: Time Playing Game (hours)
2. The highest point, (3, 4), represents that Emma played video games for 4 hours on the 3rd day. The lowest point, (7, 0), represents that Emma did not play video games on the 7th day.

Answers

- the number of movie tickets purchased and the total cost
 - the number of eggs used and the number of cakes baked
 - the number of students in attendance at school and the number of lunches served
 - the number of hours driven and the number of miles to a vacation destination
 - the number of minutes a swimming pool is filled with water and the number of gallons of water in the swimming pool
2. The independent quantity is the one that is necessary to know first. It effects the dependent quantity.

GETTING STARTED

What Comes First?

Have you ever planned a party? You may have purchased ice, gone grocery shopping, selected music, made food, or even cleaned in preparation. Many times, these tasks depend on another task being done first. For instance, you wouldn't make food before grocery shopping, now would you?

Consider the two quantities that are changing in each relationship.

- the number of movie tickets purchased and the total cost
- the number of eggs used and the number of cakes baked
- the number of students in attendance at school and the number of lunches served
- the number of hours driven and the number of miles to a vacation destination
- the number of minutes a swimming pool is filled with water and the number of gallons of water in the swimming pool

1. Circle the independent quantity and underline the dependent quantity in each relationship.

2. Describe how you can determine which quantity is independent and which quantity is dependent in any problem situation.

When one quantity is determined by another in a problem situation, it is said to be the **dependent quantity**. The quantity it is determined from is called the **independent quantity**.

ELL Tip

The terms *independent* and *dependent* are cognates in many languages and may be easily identified by English Learners. Review the verb *depend* with students and discuss how it is related to *independent* and *dependent*. Give them the following sentence frame to practice sentence structure and independent/dependent identification. “

_____ depend(s) on _____ to _____.

_____ is dependent and _____ is independent.”

ACTIVITY

1.1

Connecting Scenarios and Their Graphs



While a person can describe the monthly cost to operate a business, or talk about a marathon pace a runner ran to break a world record, graphs on a coordinate plane enable people to see the data. Graphs relay information about data in a visual way.

You can use lines or smooth curves to represent relationships between points on a graph. In some problem situations, all the points on the line will make sense. In other problem situations, not all the points will make sense. So, when you model a relationship with a line or a curve, it is up to you to consider the situation and interpret the meaning of the data values shown.

This activity includes six scenarios and six graphs that are located at the end of the lesson.

1. **Read each scenario. Determine the independent and dependent quantities. Then match each scenario to its corresponding graph. Glue the graph next to the scenario. For each graph, label the x- and y-axis with the appropriate quantity and a reasonable scale, and then interpret the meaning of the origin.**

Music Club

Jermaine loves music. He can lip sync almost any song at a moment's notice. He joined Songs When I Want Them, an online music store. By becoming a member, Jermaine can purchase just about any song he wants. Jermaine pays \$1 per song.

- **independent quantity:**

- **dependent quantity:**

Think

about:

Be sure to include the appropriate units of measure for each quantity.

Answers**Music Club**

Graph E

Independent Quantity:
number of songs

x-axis: Number of Songs;
interval of 1

Dependent Quantity:
cost (dollars)

y-axis: Cost (dollars)
interval of 1

Origin: (0 songs, 0 dollar
cost)

ELL Tip

Students who understand the concept of independent and dependent quantities may be unable to show their knowledge if they lack the linguistic or cultural understanding required to identify the variables used in the activity. Ensure students have a clear understanding of all variables before they begin.

Answers

Something's Fishy

Graph A

Independent Quantity:
time (minutes)

x-axis: Time (minutes);
interval of 1

Dependent Quantity: water
(gallons)

y-axis: Water (gallons);
interval of 20

Origin: (0 minutes, 0 gallons
of water)

Smart Phone, but Is It a Smart Deal?

Graph B

Independent Quantity:
time (weeks)

x-axis: Time (weeks);
interval of 1

Dependent Quantity:
interest (dollars)

y-axis: Interest (dollars);
interval of 10

Origin: (0 weeks, 0 dollars
of interest)

Ask

• yourself:

What strategies
will you use to match
each graph with one
of the eight scenarios?

Something's Fishy

Candice is a building manager for the Crowley Enterprise office building. One of her responsibilities is cleaning the office building's 200-gallon aquarium. For cleaning, she must remove the fish from the aquarium and drain the water. The water drains at a constant rate of 10 gallons per minute.

• independent quantity:

• dependent quantity:

Smart Phone, but Is It a Smart Deal?

You have your eye on an upgraded smart phone. However, you currently do not have the money to purchase it. Your cousin will provide the funding, as long as you pay him back with interest. He tells you that you only need to pay \$1 in interest initially, and then the interest will double each week after that. You consider his offer and wonder if this *really* is a good deal.

• independent quantity:

• dependent quantity:

It's Magic

The Amazing Aloysius is practicing one of his tricks. As part of this trick, he cuts a rope into many pieces and then magically puts the pieces of rope back together. He begins the trick with a 20-foot rope and then cuts it in half. He then takes one of the halves and cuts that piece in half. He repeats this process until he is left with a piece so small he can no longer cut it.

- **independent quantity:**

- **dependent quantity:**

Baton Twirling

Jill is a drum major for the Altadena High School marching band. For the finale of the halftime performance, Jill tosses her baton in the air so that it reaches a maximum height of 22 feet. This gives her 2 seconds to twirl around twice and catch the baton when it comes back down.

- **independent quantity:**

- **dependent quantity:**

Answers

It's Magic

Graph D

Independent Quantity:
number of cuts

x-axis: Number of Cuts;
interval of 1

Dependent Quantity:
length of each piece of
rope (feet)

y-axis: Length of Each Piece
of Rope (feet);
interval of 2

Origin: (0 cuts, 0 feet of
rope)

Baton Twirling

Graph F

Independent Quantity:
time (seconds)

x-axis: Time (seconds);
interval of 0.25

Dependent Quantity:
height of baton (feet)

y-axis: Height of Baton
(feet): interval of 3

Origin: (0 seconds, height
of 0 feet)

Answers

Jelly Bean Challenge

Graph C

Answers may vary for intervals, sample answers provided.

Independent Quantity:
number of jelly beans guessed

x-axis: Possible Number of Jelly Beans;
possible interval of 100

Dependent Quantity:
number of jelly beans the guess is off by

y-axis: Number of Jelly Beans the Guess is Off by
possible interval of 100 (same a x-axis scale)

Origin: (0 jelly beans, 0 jelly beans the guess is off by)

Jelly Bean Challenge

Mr. Wright judges the annual Jelly Bean Challenge at the summer fair. Every year, he encourages the citizens in his town to guess the number of jelly beans in a jar. He records all the possible guesses and the number of jelly beans that each guess was off by.

- **independent quantity:**

- **dependent quantity:**

ACTIVITY

1.2

Comparing and Contrasting
Graphs

Now that you have matched a graph with the appropriate problem situation, let's go back and examine all the graphs.

1. What similarities do you notice in the graphs?

2. What differences do you notice in the graphs?

3. How did you label the independent and dependent quantities in each graph?

4. Analyze each graph from left to right. Describe any graphical characteristics you notice.

Think

about:

Look closely when analyzing the graphs. What do you see?

Answers

1. Sample answer.
 - The independent quantity is graphed on the x -axis while the dependent quantity is graphed on the y -axis.
 - All the graphs are continuous.
2. Sample answer.
 - Some graphs contain straight lines, while some contain curves.
 - Some graphs seem to move up as they go from left to right, some move down from left to right.
 - Some graphs are made of pieces that go up, go down, or stay constant from left to right.
3. I labeled the independent quantity on the x -axis and the dependent quantity on the y -axis in each graph.
4. Sample answer.
 - Some graphs only increase.
 - Some graphs only decrease.
 - Some graphs both increase and decrease.
 - Some graphs have a minimum or maximum value.
 - Some graphs increase or decrease at a constant rate.

ELL Tip

Provide additional vocabulary support by giving students illustrated examples of the following terms: *increase*, *decrease*, *straight*, *curved*, *maximum*, *minimum*. Using one of the graphs in the assignment, point to a portion of the graph and call on students to describe it with one of the illustrated terms.

Answers

5a. Sample answer.

- Both graphs increase from left to right.
- The graph of the *Smart Phone, but is it a Smart Deal?* situation is a smooth curve, but the graph of the *Music Club* situation is a straight line.

5b. Sample answer.

- Both graphs decrease from left to right.
- The graph of the *Something's Fishy* situation is a straight line, but the graph of the *It's Magic* situation is a smooth curve.

5c. Sample answer.

- The graphs have either a minimum or a maximum value, Both graphs increase and decrease.
- The graph of the *Baton Twirling* situation is a smooth curve, but the graph of the *Jelly Bean Challenge* situation is made up of two straight lines.



What do the points on each graph represent?

5. Compare the graphs for each pair of scenarios given and describe any similarities and differences you notice.

a. *Smart Phone, but Is It a Smart Deal?* and *Music Club*

b. *Something's Fishy* and *It's Magic*

c. *Baton Twirling* and *Jelly Bean Challenge*



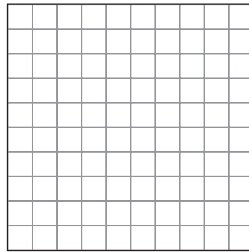
TALK the TALK

A Writer and a Mathematician

1. Write a scenario and sketch a graph to describe a possible trip to school.

Scenario

Graph



2. Describe the meaning of the points, or smooth curve, represented by your graph.
3. Compare your scenario and sketch with your classmates' scenarios and sketches. What similarities do you notice? What differences do you notice?

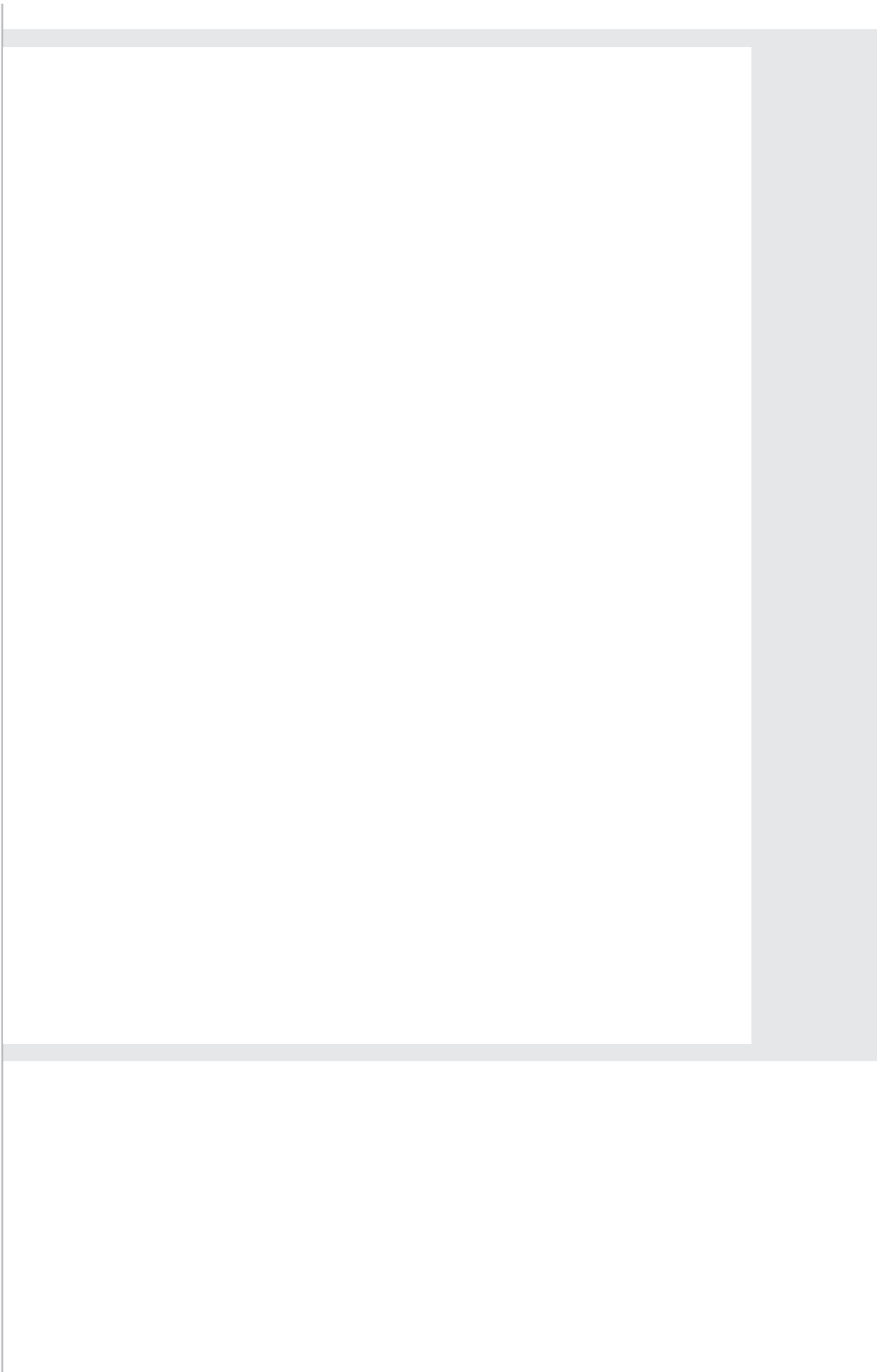
NOTES

Answers

1. Answers will vary.
2. Sample answer.
Each point on the graph represents possible times and the corresponding distances.
3. Answers will vary.

ELL Tip

Ask students what they think of when they hear the word *respect*. After asking “What do the values on the x -axis represent with respect to the problem situation,” ask whether the word *respect* has the same meaning in that sentence. Based on context, ask students to determine the meaning of the phrase *with respect to*.



Graph Cutouts

