2

A Sort of Sorts

Analyzing and Sorting Graphs

MATERIALS

Scissors

Lesson Overview

Students begin this lesson by cutting out 17 different graphs. They sort the graphs into different groups based on their own rationale, compare their groupings with their classmates, and discuss the reasoning behind their choices. Next, four different groups of graphs are given, and students analyze the groupings and explain possible rationales behind the choices made. Students explore different representations of relations. Students need to keep their graphs as they will be used in lessons that follow.

Algebra 1

Linear Functions, Equations, and Inequalities

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:

(C) graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems.

Quadratic Functions and Equations

(7) The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to:

(A) graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including *x*-intercept, *y*-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry.

Exponential Functions and Equations

(9) The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:

(D) graph exponential functions that model growth and decay and identify key features, including y-intercept and asymptote, in mathematical and real-world problems.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G

Essential Ideas

- A relationship between two quantities can be graphed on the coordinate plane.
- Graphical behaviors can reveal important information about a relationship.
- A graph of a relationship can have a minimum or maximum or no minimum or maximum. A graph can pass through one or more quadrants. A graph can exhibit vertical or horizontal symmetry. A graph can be increasing, decreasing, neither increasing nor decreasing, or both increasing and decreasing.

Lesson Structure and Pacing: 1 Day

Engage

Getting Started: Let's Sort Some Graphs

Students cut out 17 graphs and sort the graphs into different categories based on their own rationale. They then compare their categorizations with their classmates' choices and explain their reasoning. The emphasis is on the variety of ways to correctly categorize these graphs.

Develop

Activity 2.1: Identifying Graphical Behaviors

Four different scenarios that show groups of graphs are given, and students explain the rationale behind the groups and the errors in the reasoning behind a grouping. Rationales for groups include graphs being discrete, having vertical symmetry, existing in only two quadrants, and not being a function.

Demonstrate

Talk the Talk: Compare and Contrast

Students use the graphs they cut out and sorted at the beginning of the lesson to create a list of all the different types graphical behaviors.

Facilitation Notes

In this activity, students cut out 17 graphs and sort the graphs into different categories based on their own rationale. They then compare their categorizations with their classmates' choices and explain their reasoning. The emphasis is on the variety of ways to correctly categorize these graphs.

Ask a student to read the introduction aloud and discuss the activity as a class. Provide scissors and the time necessary to cut out each of the 17 graph cards.

Have students work with a partner or in a group to complete the activity. Student responses will be shared in the Talk the Talk at the end of the lesson.

As students work, look for

- Conflicts and reasoning about the best way to group the graphs.
- Creative strategies, such as Venn diagrams, to deal with more than one graphical characteristic at a time.

Differentiation strategy

To scaffold support, reduce the number of graphs that they must sort.

Questions to ask

- How many different categories of graphs do you have?
- How did you decide which graphs to include in each category?
- Do any of your categories contain a single graph?
- Did you and your group members disagree about any particular categorization?
- Were there any graphs that didn't fit into any category?

Misconception

Students may not realize the significance of the arrowheads included on various graphs in terms of continuation. In these situations, you could suggest that they visualize these graphs beyond the viewable window.

Summary

Graphs of relationships have a variety of characteristics.

DEVELOP

Activity 2.1

Identifying Graphical Behaviors

Facilitation Notes

In this activity, four different scenarios that show groups of graphs are given, and students explain the rationale behind the groups and the errors in the reasoning behind a grouping. Rationales for categorizations include graphs being discrete, having vertical symmetry, existing in only two quadrants, and not being a function.

At this point, students are not required to use the terms *discrete* and *continuous*. These terms will be defined in the next lesson.

Have students work with a partner or in a group to complete this activity. Share responses as a class.

Questions to ask

- How are Matthew's graphs different than other graphs you may have seen?
- Which of Matthew's graphs appear to be linear? Non-linear?
- What kind of scenarios can you think of for each of Matthew's graphs?
- Is the *y*-axis the axis of symmetry in all of Ashley's graphs? Explain.
- What graphs have a horizontal axis of symmetry? What does that mean?
- Are Duane's graphs only what they appear to be, or is only part of each graph visible? Explain.
- How are Judy's graphs different from other graphs you have seen?
- For each *x*-value on one of Judy's graphs, how many *y*-values are there?
- Will graphs that have a horizontal axis of symmetry always represent non-functions? Why or why not?

Misconception

Students may think the term *axis of symmetry* implies that the axis of symmetry must be the *x*-axis or *y*-axis. Use graphs *Q* and *S* to disprove this claim. If helpful, use the term *line of symmetry* rather than the term *axis of symmetry*.

Differentiation strategies

- To scaffold support for Question 4, remind students of the Vertical Line Test and have them label points that have the same *x*-value but different y-values. Revisit graphs of functions to emphasize the difference.
- To extend the activity, ask students to sketch other graphs that could belong in the groups.

Summary

Graphs of relationships can exhibit symmetry and can represent functions and non-functions.

Talk the Talk: Compare and Contrast

DEMONSTRATE

Facilitation Notes

In this activity, students use the graphs they cut out and sorted at the beginning of the lesson to create a list of different graphical behaviors. Remind students to keep their graph cutouts. They will need them for the next two lessons.

Have students work with a partner or in a group to complete this activity. Share responses as a class.

Questions to ask

- How many different categories of graphs do you have?
- How did you decide which graph to include in each category?
- Do any of your categories contain a single graph?
- Did you and your group members disagree about any particular categorization?
- Are there any graphs that don't fit into any category?
- Would you prefer to change your groupings after seeing other groupings? Why or why not?
- Is there a way you could creatively group the graphs so that they may identify with more than one graphical characteristic at a time?
- What mathematical term could you use to identify that characteristic?

Summary

Graphs of relationships have a variety of characteristics.

NOTES

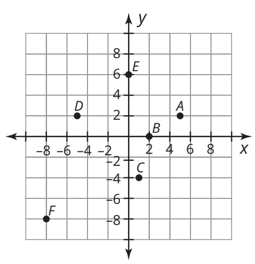
2

A Sort of Sorts

Analyzing and Sorting Graphs

Warm Up

1. Write the coordinates of each point and name the quadrant or axis where the point is located.



Learning Goals

- Review and analyze graphs and graphical behavior.
- Determine similarities and differences among various graphs.
- Sort graphs and give reasons for the similarities and differences between the groups of graphs.

You have used graphs to analyze the relationship between independent and dependent quantities. Do the graphs of certain types of relationships share any characteristics?

Warm Up Answers

Point A: (5,2) Quadrant I

Point B: (2,0) on the *x*-axis

Point C: (1,−4) Quadrant IV

Point D: (−5,2) Quadrant II

Point E: (0,6) on the *y*-axis

Point F: (−8,−8) Quadrant III

Answers

1. Answers will vary.

GETTING STARTED

Let's Sort Some Graphs

Mathematics is the science of patterns and relationships. Looking for patterns and sorting patterns into different groups based on similarities and differences can provide valuable insights. In this lesson, you will analyze many different graphs and sort them into various groups.

1. Cut out the 17 graphs at the end of the lesson. Then analyze and sort the graphs into at least 2 different groups. You may group the graphs in any way you feel is appropriate.

Record the following information for each of your groups.

- **Name each group of graphs.**
- **List the letters of the graphs in each group.**
- **Provide a rationale for why you created each group.**

Keep your graphs, you will need them in the next lesson.

ELL Tip

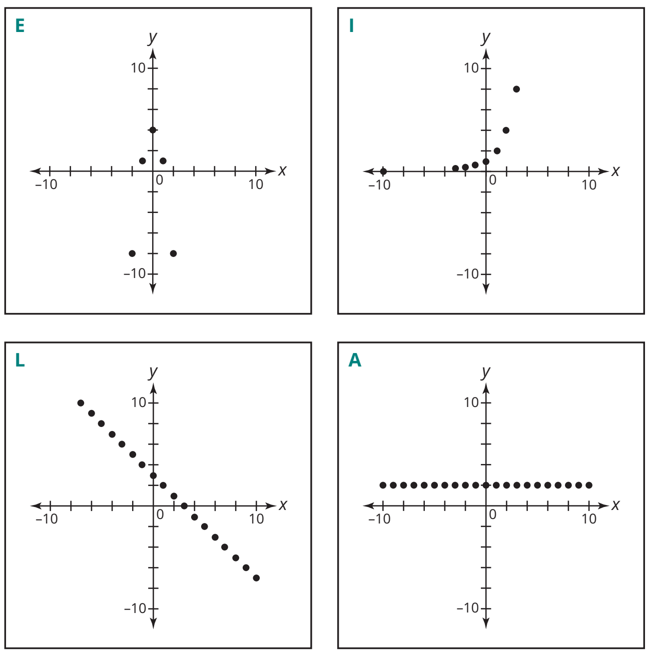
Allow students time to identify vocabulary and create basic descriptions for each group of graphs to prepare for classroom discussions. As you ask groups members for their grouping decisions, encourage students to orally contribute with the help of their prepared descriptions.

ACTIVITY **2.1**

Identifying Graphical Behaviors

In this activity, consider the different ways the graphs are grouped.

1. Matthew grouped these graphs together. Why do you think Matthew put these graphs in the same group?

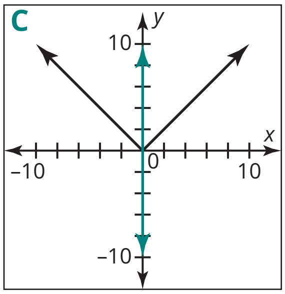


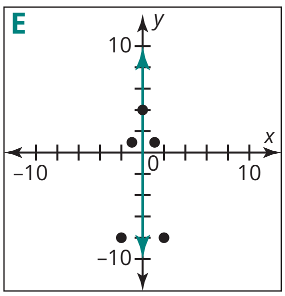
Answers

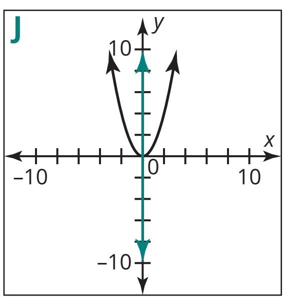
1. Sample answer.
   The graphs have points that are not connected.

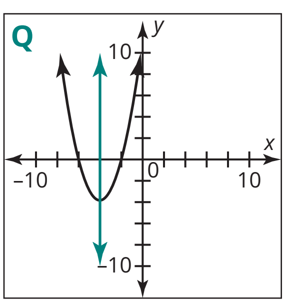
Answers

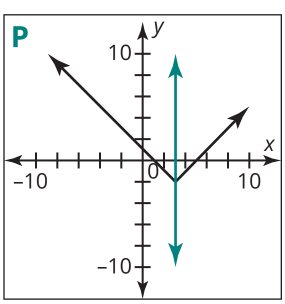
2a. Each of the graphs can be divided in half by drawing a vertical line.







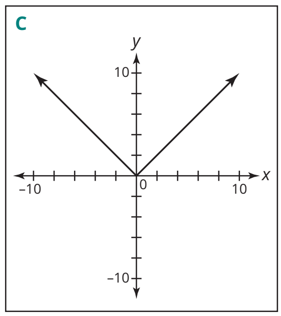


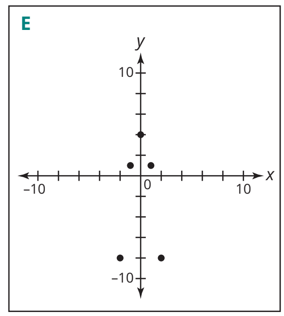


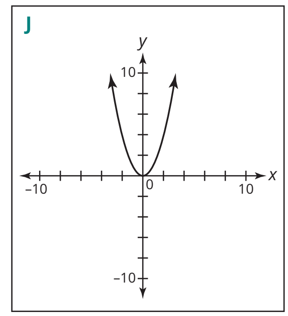
2b. Graphs B, D, H, N, A

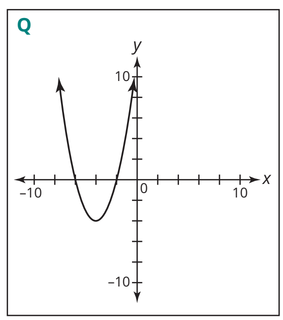
2. Consider Ashley's correct grouping.

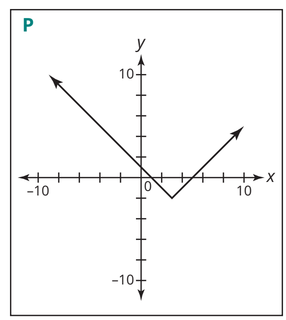
> Ashley
>
> I grouped these graphs together because they all have a vertical axis of symmetry. If I draw a vertical line through the middle of the graph, the image is the same on both sides.











a. Show why Ashley's reasoning is correct.

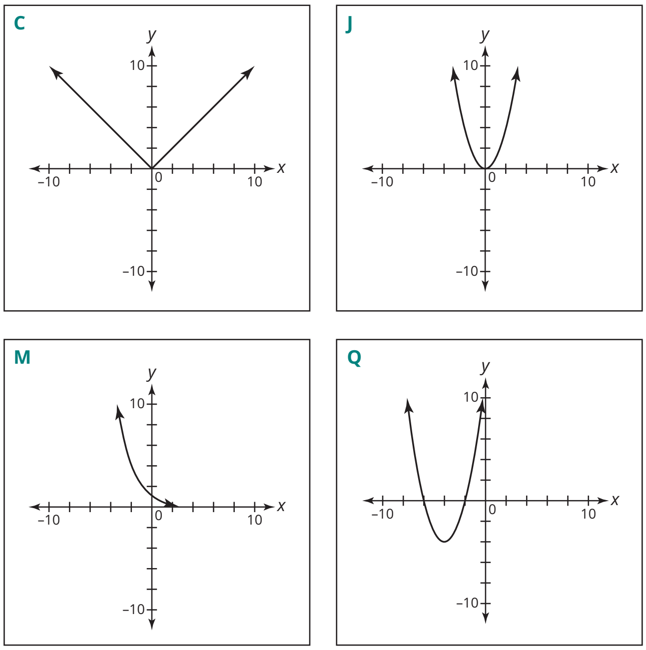
b. If possible, identify other graphs that have a vertical axis of symmetry.

ELL Tip

Have students define each term as needed with an illustration: *symmetry, horizontal, vertical, linear*. Provide connections with familiar terms, such as horizon/horizontal and line/linear.

3. Consider Duane's incorrect grouping.

> Duane
> I grouped these graphs together because each graph goes through only two quadrants.



a. Explain why Duane's reasoning is not correct.

b. If possible, identify other graphs that go through only two quadrants.

Answers

3a. Even though it is not visible, Graph Q continues into the first quadrant. Therefore, the graph goes through three quadrants. Each of the other graphs C, J, and M satisfy Duane's reasoning.

3b. Graphs A, F, H, I, K, O

Answers

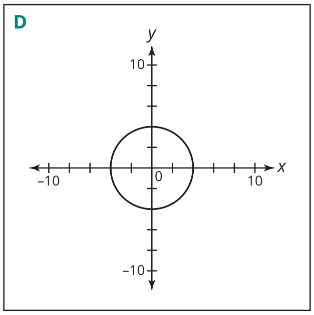
4a. Sample answer.

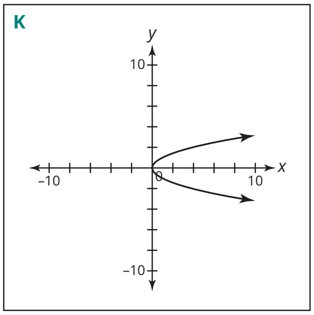
In each graph, for at least one value of x, there is more than one value of y. Each of these graphs has a horizontal axis of symmetry.

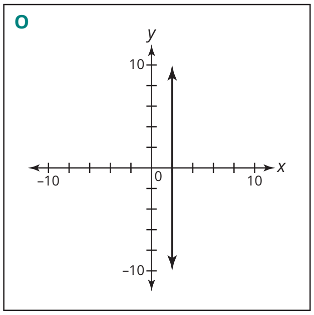
4b. Sample answer.

The graphs are not functions.

4. Judy grouped these graphs together, but did not provide any rationale.







a. What do you notice about the graphs?

b. What rationale could Judy have provided?

6 • TOPIC 1: Quantities and Relationships

TALK the TALK

Compare and Contrast

1. Compare your groups with your classmates' groups. Create a list of the different graphical behaviors you noticed.

NOTES

Ask yourself:

Are any of the graphical behaviors shared among your groups? Or, are they unique to each group?

Answers

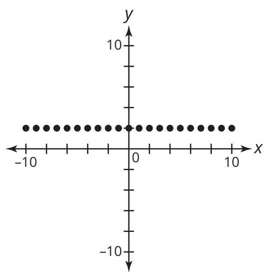
Sample answers.

Possible graphical behaviors:

- always increasing from left to right
- always decreasing from left to right
- the graph both increases and decreases
- straight lines
- smooth curves
- discrete data values
- the graph has a maximum value
- the graph has a minimum value
- the graph is a function
- the graph is not a function
- the graph goes through the origin
- the graph forms a U shape
- the graph forms a V shape

Graph Cards

A



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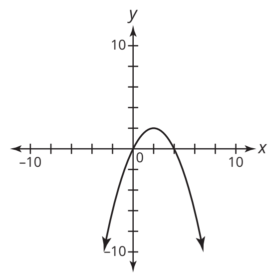

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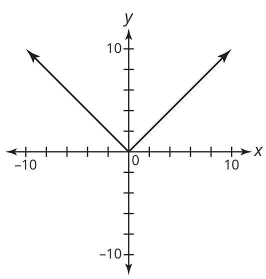

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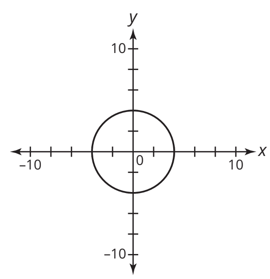

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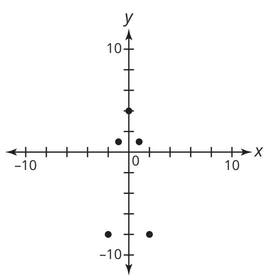

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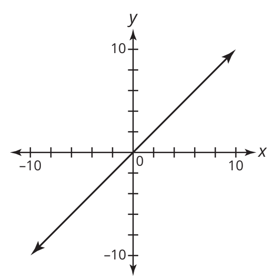

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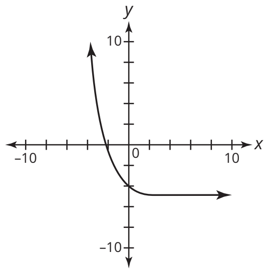

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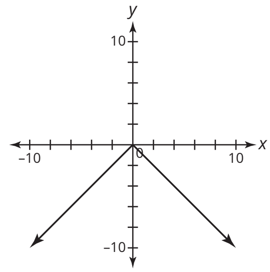

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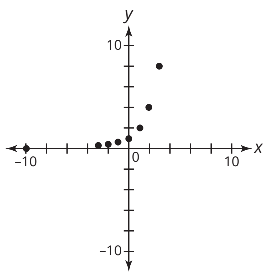

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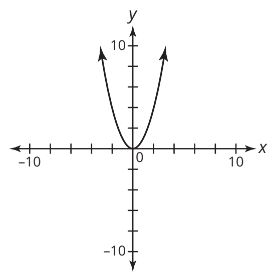

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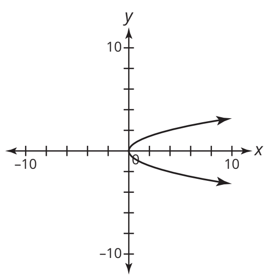

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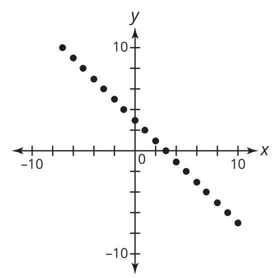

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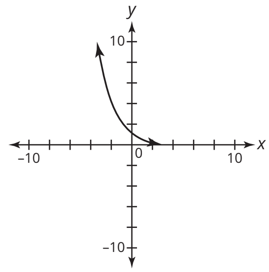
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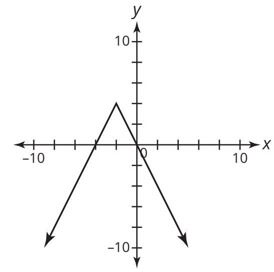
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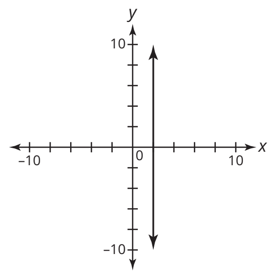
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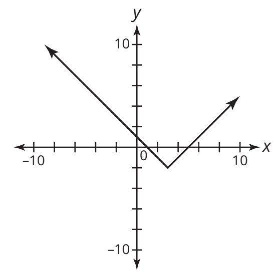
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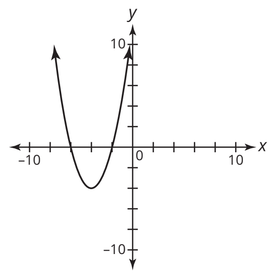
Q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