3

F of X

Recognizing Functions and Function Families

Warm Up

Identify the domain and range of the relation described by each set of ordered pairs. Write an equation using the variables *x* and *y* that could map the domain to the range.

1. $\{(-3, -6), (-2, -4), (-1, -2), (0, 0), (1, 2)\}$
2. $\{(-3, 9), (-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$
3. $\{(-3, 3), (-2, 2), (-1, 1), (0, 0), (1, -1), (2, -2)\}$

Learning Goals

- Define a function as a relation that assigns each element of the domain to exactly one element of the range.
- Write equations using function notation.
- Recognize multiple representations of functions.
- Determine and recognize characteristics of functions.
- Determine and recognize characteristics of function families.

Key Terms

- relation
- domain
- range
- function
- function notation
- Vertical Line Test
- discrete graph
- continuous graph
- increasing function
- decreasing function
- constant function
- function family
- linear functions
- exponential functions
- absolute minimum
- absolute maximum
- quadratic functions
- linear absolute value functions
- *x*-intercept
- *y*-intercept

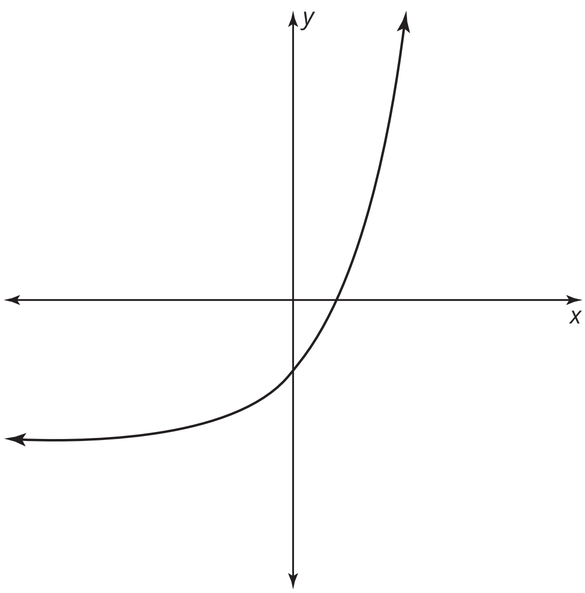
You have sorted graphs by their graphical behaviors. How can you describe the common characteristics of the graphs of the functions?

GETTING STARTED

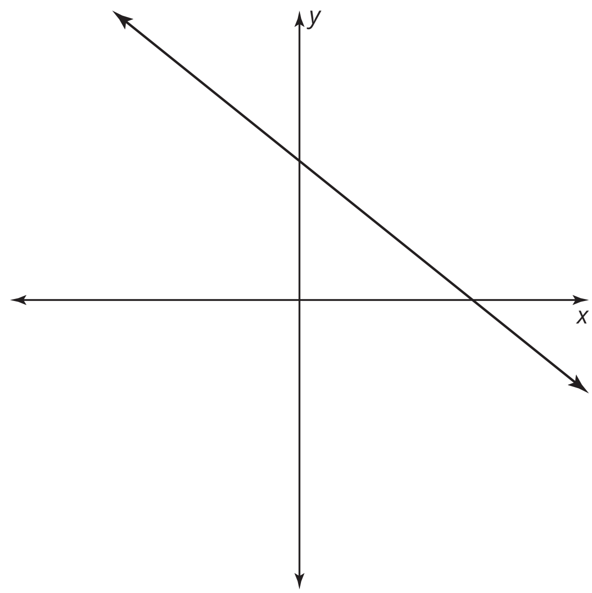
Odd One Out

1. Which of the graphs shown does not belong with the others? Explain your reasoning.

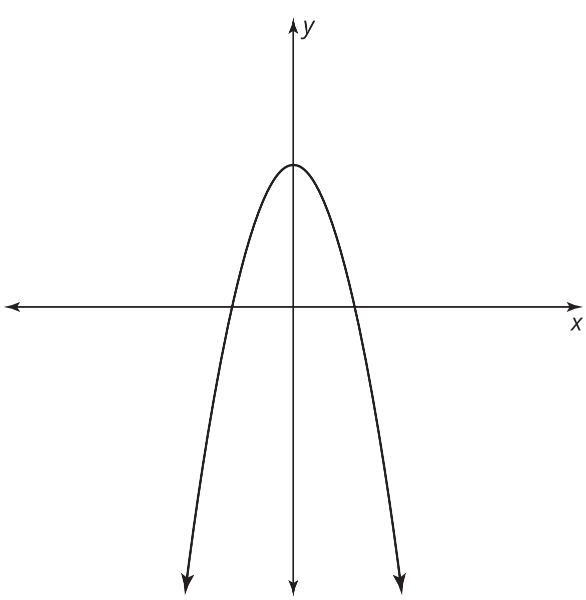
Graph A



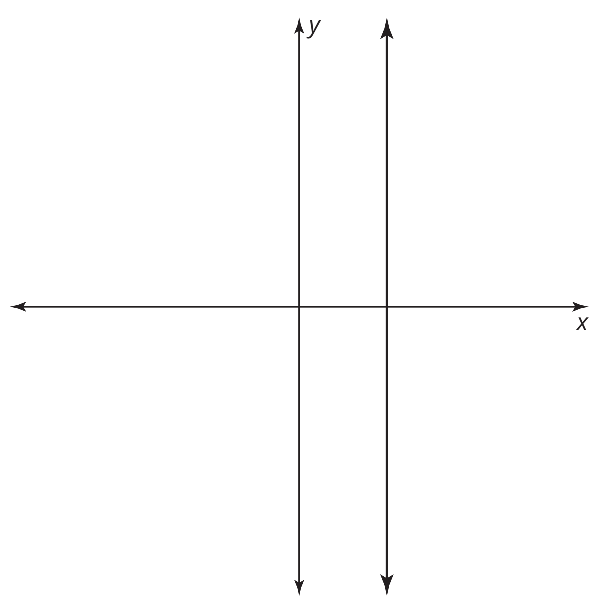
Graph B



Graph C



Graph D



ACTIVITY 3.1 Functions and Non-Functions

A relation can be represented in the following ways.

Ordered Pairs

{(−2, 2), (0, 2), (3, −4), (3, 5)}

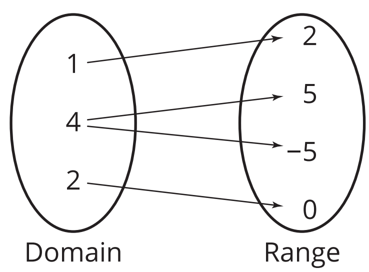
Equation

$y = \frac{2}{3}x - 1$

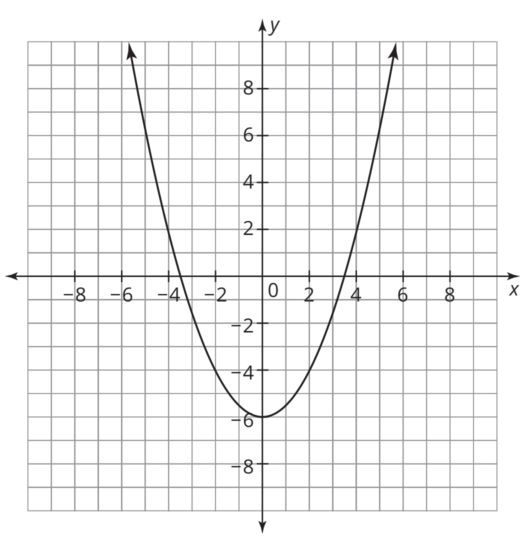
Verbal

The relation between students in your school and each student's birthday.

Mapping



Graph



Table

Domain	Range
−1	1
2	0
5	−5
6	−5
7	−8

A **relation** is the mapping between a set of input values called the **domain** and a set of output values called the **range**.

Think about:

So all functions are relations, but only some relations are functions.

A **function** is a relation that assigns to each element of the domain exactly one element of the range. Functions can be represented in a number of ways.

1. **Analyze the relation represented as a table. Is the relation a function? Explain your reasoning.**

2. **Analyze the relation represented as a mapping. Is the relation a function? Explain your reasoning.**

3. **Analyze the relation represented verbally. Is the relation a function? Explain your reasoning.**

You can write an equation representing a function using *function notation.* Let's look at the relationship between an equation and function notation.

Consider this scenario. U.S. Shirts charges \$8 per shirt plus a one-time charge of \$15 to set up a T-shirt design.

The equation $y = 8x + 15$ can be written to model this situation. The independent variable x represents the number of shirts ordered, and the dependent variable y represents the total cost of the order, in dollars.

This is a function because for each number of shirts ordered (independent value) there is exactly one total cost (dependent value) associated with it. Because this relationship is a function, you can write $y = 8x + 15$ in function notation.

$$f(x) = 8x + 15$$

name of function

independent variable

Function notation is a way of representing functions algebraically. The function notation $f(x)$ is read as "f of x" and indicates that x is the independent variable.

The cost, defined by f, is a function of x, defined as the number of shirts ordered.

If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x.

You can write a function in a number of different ways. You could write the T-shirt cost function as $C(s) = 8s + 15$, where the cost, defined as C, is a function of s, the number of shirts ordered.

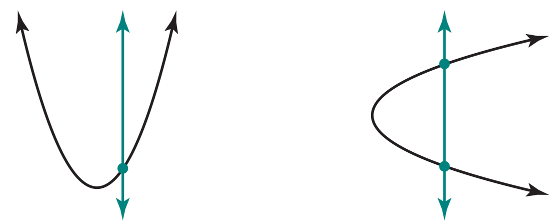
4. Consider the U.S. Shirts function, $C(s) = 8s + 15$. What expression in the function equation represents:

a. the domain of the function?

b. the range of the function?

5. Describe the possible domain and range for this situation.

The **Vertical Line Test** is a visual method used to determine whether a relation represented as a graph is a function. To apply the Vertical Line Test, consider all of the vertical lines that could be drawn on the graph of a relation. If any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function.



The Vertical Line Test applies for both *discrete* and *continuous graphs*.

A **discrete graph** is a graph of isolated points. A **continuous graph** is a graph of points that are connected by a line or smooth curve on the graph. Continuous graphs have no breaks.

6. How can you determine if a relation represented as a set of ordered pairs is a function? Explain your reasoning.

7. How can you determine if a relation represented as an equation is a function? Explain your reasoning.

8. **Determine which relations represent functions. If the relation is not a function, state why not.**

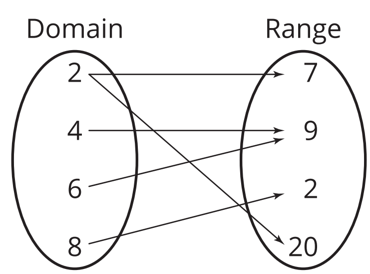
a. $y = 3^x$

b. **For every house, there is one and only one street address.**

c.

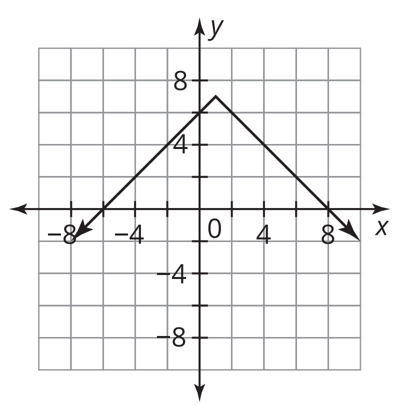
Domain	Range
−1	4
0	0
3	−2
0	4

d.



e. **{(−7, 5), (−5, 5), (2, −2), (3, 5)}**

f.



NOTES

9. **Analyze the three graphs Judy grouped together in the previous lesson, graphs D, K, and O. Are the graphs she grouped functions? Explain your conclusion.**

10. **Use the Vertical Line Test to sort the graphs in the previous lesson into two groups: functions and non-functions. Record your results by writing the letter of each graph in the appropriate column in the table shown.**

Functions	Non-Functions

ACTIVITY 3.2 Domain and Range of a Function

You have identified the domain and range of a function given its equation.

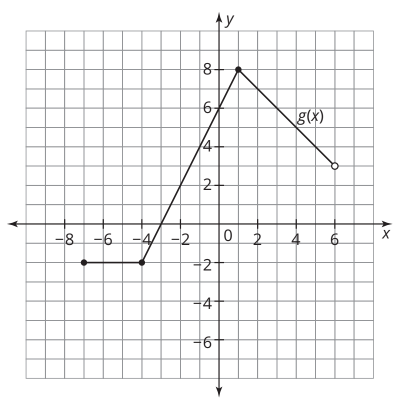
1. Explain how you can identify the domain and range of a function given:

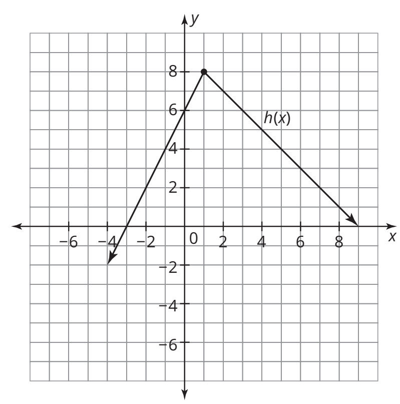
a. a verbal statement.

b. a graph.

Worked Example

There are different ways to write the domain and range of a function given its graph.





	Domain		Range	
	$g(x)$	$h(x)$	$g(x)$	$h(x)$
In Words	The domain is all real numbers greater than or equal to −7 and less than 6.	The domain is the set of all real numbers.	The range is all real numbers greater than or equal to −2 and less than or equal to 8.	The range is all real numbers less than or equal to 8.
Using Notation	$-7 \le x < 6$	$-\infty < x < \infty$	$-2 \le y \le 8$	$y \le 8$

2. Consider the Graph Cards from the previous activity that include continuous functions. Label each of these cards with the appropriate domain and range.

ACTIVITY
3.3

Linear, Constant, and Exponential Functions

Gather all of the graphs that you identified as functions.

When determining whether a graph is increasing or decreasing, read the graph from left to right.

A function is described as increasing when the value of the dependent variable increases as the value of the independent variable increases. If a function increases across the entire domain, then the function is called an **increasing function**.

A function is described as decreasing when the value of the dependent variable decreases as the value of the independent variable increases. If a function decreases across the entire domain, then the function is called a **decreasing function**.

If the value of the dependent variable of a function remains constant over the entire domain, then the function is called a **constant function**.

1. **Analyze each graph from left to right. Sort all the graphs into one of the four groups listed.**
    - **increasing function**
    - **decreasing function**
    - **constant function**
    - **a combination of increasing and decreasing**

    Record the function letter in the appropriate column of the table shown.

Increasing Function	Decreasing Function	Constant Function	Combination of Increasing and Decreasing

2. Each function shown represents one of the graphs in the increasing function, decreasing function, or constant function categories. Use graphing technology to determine the shape of its graph. Then match the function to its corresponding graph by writing the function directly on the graph that it represents.

- $f(x) = x$
- $f(x) = \left(\frac{1}{2}\right)^x$
- $f(x) = \left(\frac{1}{2}\right)^x - 5$
- $f(x) = 2$, where x is an integer
- $f(x) = 2^x$, where x is an integer
- $f(x) = -x + 3$, where x is an integer

Think about:

Be sure to correctly interpret the domain of each function. Also, remember to use parentheses when entering fractions into your calculator.

3. Consider the six graphs and functions that are increasing functions, decreasing functions, or constant functions.

a. Sort the graphs into two groups based on the equations representing the functions and record the function letter in the table.

Group 1	Group 2

b. What is the same about all the functions in each group?

Ask yourself:

What other variables have you used to represent a linear function?

You have just sorted the graphs into their own *function families*. A **function family** is a group of functions that share certain characteristics.

The family of **linear functions** includes functions of the form $f(x) = ax + b$, where a and b are real numbers.

The family of **exponential functions** includes functions of the form $f(x) = a \cdot b^x + c$, where a, b, and c are real numbers, and b is greater than 0 but not equal to 1.

4. Go back to your table in Question 3 and identify which group represents linear and constant functions and which group represents exponential functions.

5. If $f(x) = ax + b$ represents a linear function, describe the a and b values that produce a constant function.

Place these two groups of graphs off to the side. You will need them again.

ACTIVITY **3.4**

Quadratic and Absolute Value Functions

A function has an **absolute minimum** if there is a point on the graph of the function that has a y-coordinate that is less than the y-coordinate of every other point on the graph. A function has an **absolute maximum** if there is a point on the graph of the function that has a y-coordinate that is greater than the y-coordinate of every other point on the graph.

1. Sort the graphs from the combination of increasing and decreasing category in the previous activity into one of the two groups listed.

- **those that have an absolute minimum value**
- **those that have an absolute maximum value**

Then record the function letter in the appropriate column of the table shown.

Absolute Minimum	Absolute Maximum

2. **Each function shown represents one of the graphs with an absolute maximum or an absolute minimum value. Use graphing technology to determine the shape of its graph. Then match the function to its corresponding graph by writing the function directly on the graph that it represents.**

- $f(x) = x^2 + 8x + 12$
- $f(x) = -|x|$
- $f(x) = |x - 3| - 2$
- $f(x) = -3x^2 + 4$, where x is integer
- $f(x) = x^2$
- $f(x) = -\frac{1}{2}x^2 + 2x$
- $f(x) = |x|$
- $f(x) = -2|x + 2| + 4$

3. **Consider the graphs of functions that have an absolute minimum or an absolute maximum.**

   a. **Sort the graphs into two groups based on the equations representing the functions and record the function letter in the table.**

Group 1	Group 2

   b. **What is the same about all the functions in each group?**

You have just sorted functions into two more function families.

The family of **quadratic functions** includes functions of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers, and a is not equal to 0.

The family of **linear absolute value functions** includes functions of the form $f(x) = a|x + b| + c$, where a, b, and c are real numbers, and a is not equal to 0.

4. Go back to your table in Question 3 and identify which group represents quadratic functions and which group represents linear absolute value functions.

ACTIVITY

3.5

Function Families

You have now sorted each of the graphs and equations representing functions into one of four function families: linear, exponential, quadratic, and linear absolute value.

1. **Glue your sorted graphs and functions to the appropriate function family graphic organizer located at the end of the lesson. Write a description of the graphical behavior for each function family.**

Hang on to your graphic organizers. They will be a great resource moving forward!

You've done a lot of work up to this point! You've been introduced to linear, exponential, quadratic, and linear absolute value functions. Don't worry—you don't need to know everything there is to know about these function families right now. As you progress through this course, you will learn more about each function family.

NOTES

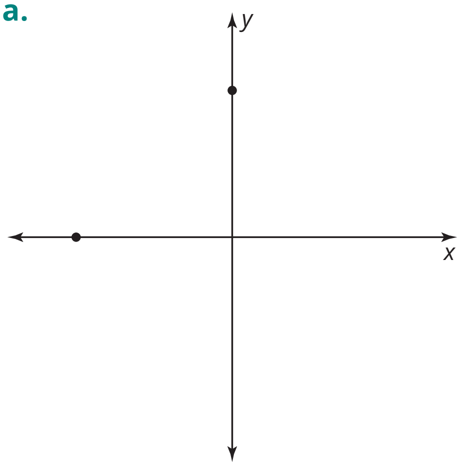
TALK the TALK

Interception!

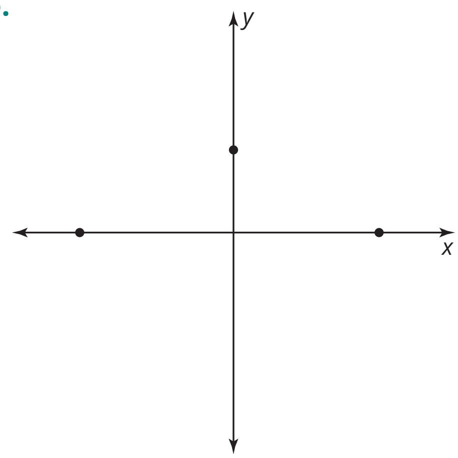
Recall that the ***x*-intercept** is the point where a graph crosses the *x*-axis. The ***y*-intercept** is the point where a graph crosses the *y*-axis.

1. The graphs shown represent relations with just the *x*- and *y*-intercepts plotted. If possible, draw a function that has the given intercepts. If it is not possible, explain why not.

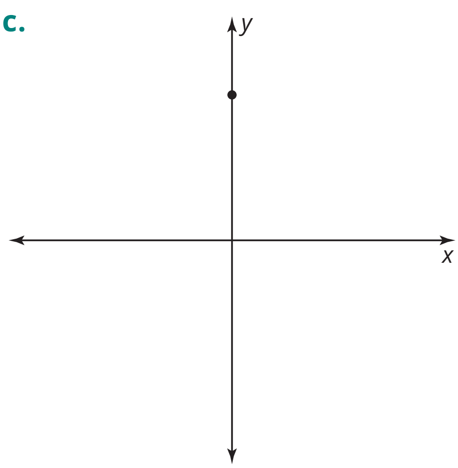
a.



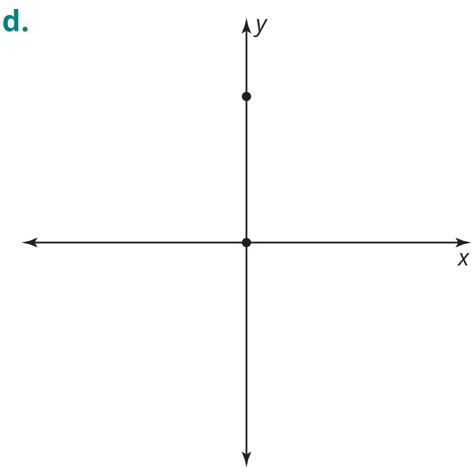
b.



c.



d.



The family of **linear functions** includes functions of the form $f(x) = ax + b$, where a and b are real numbers.

Graphs

Linear Functions

Increasing/Decreasing/Constant:

Domain and Range:

Maximum/Minimum:

Curve/Line:

Graphical Behaviors

The family of **exponential functions** includes functions of the form $f(x) = a \cdot b^x + c$, where a, b, and c are real numbers, and b is greater than 0 but not equal to 1.

Graphs

Exponential Functions

Increasing/Decreasing/Constant:

Domain and Range:

Maximum/Minimum:

Curve/Line:

Graphical Behaviors

The family of **quadratic functions** includes functions of the form $f(x) = ax^2 + bx + c$ where a, b, and c are real numbers, and a is not equal to 0.

Graphs

Quadratic Functions

Increasing/Decreasing/Constant:

Domain and Range:

Maximum/Minimum:

Curve/Line:

Graphical Behaviors

The family of **linear absolute value functions** includes functions of the form $f(x) = a|x + b| + c$, where a, b, and c are real numbers, and a is not equal to 0.

Graphs

Linear Absolute Value Functions

Increasing/Decreasing/Constant:

Domain and Range:

Maximum/Minimum:

Curve/Line:

Graphical Behaviors