3

F of X

Recognizing Functions and Function Families

MATERIALS

Graphs from *A Sort of Sorts*
Graphing technology
Glue sticks

Lesson Overview

The definition of *relation, function, function notation, domain,* and *range* are introduced in this lesson. For the remainder of the lesson, students use graphing technology to connect equations written in function form to their graph and then identify the function family to which they belong. The terms *Vertical Line Test, continuous graph,* and *discrete graph* are defined, and students sort the graphs from the previous lesson into functions and non-functions. Then, the terms *Vertical Line Test, increasing function, decreasing function*, *constant function, discrete function* and *continuous function* are defined, and students sort the graphs from the previous lesson into these groups and a group labeled for functions that include a combination of increasing and decreasing intervals. The terms *function family*, *linear function*, and *exponential function* are then defined, and students sort the increasing, constant, and decreasing functions into one of these families. Next, the terms *absolute minimum* and *absolute maximum* are defined, as well as the terms *quadratic function* and *linear absolute value function*. Students sort the functions with an absolute minimum or absolute maximum into one of these families. Finally, students recall the definition of *x*-intercept and *y*-intercept. Students then complete a graphic organizer for each function family that describes the graphical behavior and displays graphical examples. In the final activity, students use their knowledge of the function families to demonstrate how the families differ with respect to their *x*- and *y*-intercepts. Graphing technology is necessary to help students connect some equations and their graphs.

Algebra 1
Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:

(A) determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities.

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:

(C) graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems.

Quadratic Functions and Equations

(6) The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. The student is expected to:

(A) determine the domain and range of quadratic functions and represent the domain and range inequalities.

(7) The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to:

(A) graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including x-intercept, y-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry.

Exponential Functions and Equations

(9) The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:

(A) determine the domain and range of exponential functions of the form $f(x) = ab^x$ and represent the domain and range using inequalities.

(D) graph exponential functions that model growth and decay and identify key features, including y-intercept and asymptote, in mathematical and real-world problems.

Number and Algebraic Methods

(12) The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:

(A) decide whether relations represented verbally, tabularly, graphically, and symbolically define a function.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G

Essential Ideas

- A function is a relation that assigns to each element of the domain exactly one element of the range.
- The family of linear functions includes functions of the form $f(x) = ax + b$, where a and b are real numbers.
- The family of exponential functions includes functions of the form $f(x) = a \cdot b^x + c$, where a, b, and c are real numbers, and b is greater than 0 but is not equal to 1.
- The family of quadratic functions includes functions of the form $f(x) = ax^2 - bx + c$, where a, b, and c are real numbers, and a is not equal to 0.
- The family of linear absolute value functions includes functions of the form $f(x) = a|x + b| + c$, where a, b, and c are real numbers, and a is not equal to 0.

Lesson Structure and Pacing: 3 Days

Day 1

Engage

Getting Started: Odd One Out

Students are presented with four numberless graphs of relations and decide which of the graphs does not belong with the others. This activity solicits students' prior knowledge related to the characteristics of graphs, which they explored in the previous lesson. This activity has no correct answer. There are a variety of reasons related to graphical behavior to explain why each of the four graphs does not belong with the others.

Develop

Activity 3.1: Functions and Non-Functions

Students are provided the definitions of the terms *relation*, *domain*, *range, function,* and *function notation*. Multiple representations of relations are shown and then analyzed to determine which relations are functions. Students are given an equation written in function notation that models a scenario. They identify which expression in the function equation represents the domain and range and then determine the possible domain and range of the function in the context of the scenario. The *Vertical Line Test* is reviewed as a visual method used to determine whether a relation represented as a graph is a function, and the terms *discrete graph* and *continuous graph* are defined. Additional relations are analyzed to determine which are functions. Students then sort the graphs from the previous lesson into function and non-function groups.

Day 2

Activity 3.2: Domain and Range of a Function

Students revisit the graphs from the previous lesson and use different notations, including words and inequalities, to describe the domain and range of each graph.

Activity 3.3: Linear, Constant, and Exponential Functions

Students use a sorting activity and the graphs from the previous lesson to distinguish among *increasing, decreasing*, and *constant functions*, and functions that show a combination of increasing and decreasing behaviors. Next, focusing only on the graphs of increasing, decreasing, and constant functions, they match each graph with the appropriate equation written in function notation. They sort these graphs again into two groups based on the equation of each function. The terms *function family, linear functions,* and *exponential functions* are described, and students identify which group is best represented using these terms.

Day 3

Activity 3.4: Quadratic and Absolute Value Functions

Students sort the graphs that are both increasing and decreasing into three groups: having the characteristics of *absolute minimum,* having the characteristics of *absolute maximum*, and having no absolute minimum or absolute maximum. Focusing only on the graphs containing absolute minimums or absolute maximums, they match each graph with the appropriate equation written in function notation, and then sort these graphs again into two groups based on the feature of an absolute minimum or an absolute maximum. The terms *quadratic functions* and *linear absolute value functions* are defined, and students identify which graphs are best represented using these terms.

Activity 3.5: Function Families

If they have not done so already, students paste their equations and linear, exponential, quadratic, and linear absolute value into appropriate graphic organizers. Students then describe the graphical behavior of each function.

Demonstrate

Talk the Talk: Interception!

Students recall the definitions of *x-intercept* and *y-intercept*. They then use their knowledge about functions and function families to draw functions on numberless graphs, given only the *x*- and *y*-intercepts of the functions. This activity is designed to solicit students' reasoning about the possibilities for the graphs of functions.

Facilitation Notes

In this activity, students are presented with four numberless graphs of relations and decide which of the graphs does not belong with the others. This activity solicits students' prior knowledge related to the characteristics of graphs, which they explored in the previous lesson. This activity has no correct answer. There are a variety of reasons related to graphical behavior to explain why each of the four graphs does not belong with the others.

Have students complete this activity independently or with a partner. Share responses as a class.

As students work, look for

- Identification of different graphs as not belonging with the others.
- Use of math terminology related to the characteristics of the graphs.

Questions to ask

- The graph of the relation passes through how many different quadrants?
- How would you describe the *x*-values in this relation? The *y*-values?
- Does each individual point on the graph of the relation have a different *x*-value or do two or more points share the same *x*-value?
- Does each individual point on the graph of the relation have a different *y*-value or do two or more points share the same *y*-value?
- For each *x*-value, is there only one *y*-value or more than one *y*-value?
- For each *y*-value, is there only one *x*-value or more than one *x*-value?
- What characteristic did you use that applies to three of the graphs but not the fourth?

Summary

Graphical behaviors can help distinguish one type of relation from another.

DEVELOP

Activity 3.1
Functions and Non-Functions

Facilitation Notes

In this activity, the terms *relation*, *domain*, *range*, function and function notation are defined. Multiple representations of relations are shown and then analyzed to determine which relations are functions. Students are then given an equation written in function notation that models a scenario. They identify which expression in the function equation represents the domain and range and then determine the possible domain and range in the context of the scenario. The *Vertical Line Test* is reviewed as a visual method used to determine whether a relation represented as a graph is a function, and the terms *discrete graph* and *continuous graph* are defined. Additional relations are analyzed to determine which are functions. Students then sort the graphs from the previous lesson into function and non-function groups.

Have a student read the definition of *relation* aloud. As a class, discuss the six different representations of a relation provided.

Questions to ask
- What are the input values for this relation?
- What are the output values for this relation?
- Which relations have a limited number of values? An unlimited number of values?

Ask a student to read the definition of *function*.

Questions to ask
- Explain what a function is in your own words.
- What is the difference between a relation and a function?

Have students work independently or with a partner to complete Questions 1 through 3. Share responses as a class.

Questions to ask
- What are the elements of the domain in this representation?
- What are the elements of the range in this representation?
- Does each element in the domain correspond to exactly one element in the range?

Differentiation strategy

To extend the activity, have students create tables, mappings, and verbal representations that are functions. Then, ask students to modify each function to create a non-function.

Misconception

Students may connect the math term *relation* to its common usage as *relationship*; however, the math term *function* is more restrictive than its common usage. As an alternative method of understanding, address the fact that calculators can only perform operations involving functions; each input can only have one output.

Have a student read the information and definition following Question 3 aloud. Complete Questions 4 and 5 as a class.

Questions to ask

- Is the $f(x)$ the same as x or y in an equation?
- What is the advantage of using function notation?
- How can you tell the independent and dependent variable in the notation $C(s)$?

Differentiation strategy

To extend the activity, provide an equation, such as $y = 3x + 2$. Ask students to rewrite the statement using function notation.

- What is the value of the function when $x = 5$?
- What is the value of x when the function equals 10?

Have a student read the definitions of *Vertical Line Test, discrete graph,* and *continuous graph* aloud. Discuss as a class.

Questions to ask

- Why does the Vertical Line Test work?
- Why do you use a vertical line rather than a horizontal line?

Differentiation strategy

To assist all students, place a non-function on a coordinate plane. Have students identify coordinate pairs that demonstrate that it does not fit the definition of a function.

Have students work with a partner or in a group to complete Questions 6 through 10. Share responses as a class.

Questions to ask

- Do you have to graph the ordered pairs to apply the Vertical Line Test?
- Do you have to graph the equation to apply the Vertical Line Test?
- How could you rewrite Question 8, part (b) so that it is not a function?
- For each x-value, is there a unique y-value?
- How can you tell the relation represented as a table is not a function?
- How can you tell the relation represented as a mapping is not a function?
- Do Judy's graphs pass or fail the Vertical Line Test?
- For each x-value on one of Judy's graphs, how many y-values are there?
- After applying the Vertical Line Test, how many graphs did you sort into the function group? Non-function group?

- Did all of the graphs fit into one of the two groups or is there another category?
- How would you describe the graph of a non-function? A function?
- Are all curved graphs always non-functions?
- Sketch the curved graph that is a function and a curved graph that is not a function.
- Are all linear graphs considered functions?
- Sketch the graph of a line that is not a function.

Summary

A function is a relation that assigns to each element of the domain exactly one element of the range.

Activity 3.2

Domain and Range of a Function

Facilitation Notes

In this activity, students revisit their graphs from the previous lesson and use different notations, including words and inequalities, to describe the domain and range of each graph.

Have students work independently to complete Question 1. Share responses as a class. Discuss the worked example as a class.

Questions to ask

- How do the words relate to the graph?
- How do the inequalities relate to the graph?
- How is the inequality written differently when the graph has an arrow?

Differentiation strategy

To scaffold support, connect the symbols in the graphs and inequalities to those used with number lines and inequalities.

Misconception

Students may think that the range for $h(x)$ is $-2 \leq y \leq 8$. Have them extend the arrow in the graph to see that the y-values extend beyond -2.

Have students work with a partner or in a group to complete Question 2. Share responses as a class.

As students work, look for

Students using the correct inequality symbols when writing the domain and range in interval notation.

Questions to ask

- How did you decide which inequality symbols to use when writing the domain and range?
- What inequality symbols should you use if the domain or range includes negative or positive infinity?

Differentiation strategy

To extend the activity, introduce students to interval notation. An interval is defined as the set of real numbers between two given numbers. To describe an interval, use this notation:

- A closed interval $[a, b]$ describes the set of all numbers between a and b, including the endpoints a and b. Graphically, a closed interval is indicated by closed-circle endpoints.
- An open interval (a, b) describes the set of all numbers between a and b, but not including the endpoints a and b. Graphically, an open interval is indicated by open-circle endpoints.
- A half-closed or half-open interval $(a, b]$ describes the set of all numbers between a and b, including b but not including a. Or, $[a, b)$ describes the set of all numbers between a and b, including a but not including b.

Summary

The domain and range of a function can be represented in words or using inequalities.

Activity 3.3

Linear, Constant, and Exponential Functions

Facilitation Notes

In this activity, students use a sorting activity and the graphs from the previous lesson to distinguish among increasing and decreasing behaviors and functions that show a combination of increasing and decreasing behaviors. Next, focusing only on the graphs of increasing, decreasing, and constant functions, they match each graph with the appropriate equation written in function notation. They sort these graphs again into two groups based on the equation of each function. The terms *function family, linear functions,* and *exponential functions* are described, and students identify which group is best represented using these terms.

Ask a student to read the introduction and definitions aloud. Discuss the behaviors of increasing functions, decreasing functions, and constant functions as a class.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Questions to ask

- How would you describe the graphic behaviors of an increasing function?
- Are some graphs increasing at a faster rate than others? What does that look like?
- How would you describe the graphic behaviors of a decreasing function?
- Are some graphs decreasing at a faster rate than others? What does that look like?
- How would you describe the graphic behavior of a constant function?
- How many of your graphs are considered graphs of increasing functions? Decreasing functions? Constant functions?
- When entering the functions into your graphing technology, which functions require the use of parentheses? Explain why.
- What criteria did you use to sort the graphs into two groups?

Ask a student to read the definitions following Question 3 aloud. Discuss the definitions as a class.

Have students work with a partner or in a group to complete Questions 4 through 5. Share responses as a class.

Questions to ask

- What do a linear function and a constant function have in common? What are the differences?
- What do a linear function and an exponential function have in common? What are the differences?
- Is $f(x) = ax + b$ a constant function when $a = 0$? $a = 1$?
- Is $f(x) = ax + b$ a constant function when $b = 0$? $b = 1$?
- What kind of function is $f(x) = a \cdot b^x + c$ when $b = 1$?

Summary

The family of linear functions includes functions of the form $f(x) = ax + b$, where a and b are real numbers. The family of exponential functions includes functions of the form $f(x) = a \cdot b^x + c$, where a, b, and c are real numbers, and b is greater than 0 but is not equal to 1.

Activity 3.4
Quadratic and Absolute Value Functions

Facilitation Notes

In this activity, students sort the graphs that are both increasing and decreasing into two groups based on the characteristics of absolute minimum and absolute maximum. They match each graph with the appropriate equation written in function notation, and then sort these graphs again into two groups based on the equation of each function. The terms *quadratic functions* and *linear absolute value functions* are defined, and students identify which graphs are best represented using these terms.

Ask a student to read the definitions aloud. Discuss as a class.

Differentiation strategies

- To support students who struggle, have them connect the mathematical meaning of the terms minimum and maximum to the common usage of the terms, such as in mini dress and maxi dress.
- To extend the activity, discuss why *absolute* is used as an adjective for minimum and maximum. While there is not a current need to understand *relative* minimum or maximum, it may help students make better sense of why *absolute* is used. For an example that includes both a relative and absolute maximum, have students graph $f(x) = -0.5(x - 3)(x + 1)(x - 1)(x + 2)$.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Questions to ask

- What impact does the negative sign in front of the leading term have on the graph of the function?
- When entering the functions into your graphing technology, which functions require the use of parentheses? Explain why.
- What criteria did you use to sort the graphs into two groups?

Ask a student to read the definitions following Question 3 aloud, then complete Question 4 as a class.

Questions to ask

- How would you describe the graph of a quadratic function?
- How would you describe the graph of an absolute value function?
- What kind of function is $f(x) = ax^2 + bx + c$ when $a = 0$?
- What kind of function is $f(x) = a|x + b| + c$ when $a = 0$?

Summary

The family of quadratic functions includes functions of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers, and a is not equal to 0. The family of linear absolute value functions includes functions of the form $f(x) = a|x + b| + c$ where a, b, and c are real numbers, and a is not equal to 0.

Activity 3.5

Function Families

Facilitation Notes

In this activity, students paste their equations and linear, exponential, quadratic, and linear absolute value graphs into graphic organizers. They then describe the graphical behavior of each function.

Have students work with a partner or in a group to complete this activity.

Questions to ask

- Which families of functions contain straight lines?
- Does a linear function contain an absolute minimum or absolute maximum?
- Does an exponential function contain an absolute minimum or absolute maximum?

Summary

Functions can be classified as linear, exponential, quadratic, and linear absolute value.

Talk the Talk: Interception!

DEMONSTRATE

Facilitation Notes

In this activity, students review the definitions of x-*intercept* and y-*intercept*. They then use their knowledge about function families to draw functions on numberless graphs given only the x- and y-intercepts of the functions.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Questions to ask

- How does your function compare to the functions of others?
- What is another way to draw the function?
- When is a relation not a function?
- Can a function have more than one x-intercept? y-intercept? Explain.
- Do all functions have to have an x-intercept? y-intercept? Explain.

Summary

Different function families can have different numbers of x-intercepts and at most one y-intercept.

NOTES

3

F of X

Recognizing Functions and Function Families

Warm Up

Identify the domain and range of the relation described by each set of ordered pairs. Write an equation using the variables x and y that could map the domain to the range.

1. $\{(-3, -6), (-2, -4), (-1, -2), (0, 0), (1, 2)\}$

2. $\{(-3, 9), (-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$

3. $\{(-3, 3), (-2, 2), (-1, 1), (0, 0), (1, -1), (2, -2)\}$

Learning Goals

- Define a function as a relation that assigns each element of the domain to exactly one element of the range.
- Write equations using function notation.
- Recognize multiple representations of functions.
- Determine and recognize characteristics of functions.
- Determine and recognize characteristics of function families.

Key Terms

- relation
- domain
- range
- function
- function notation
- Vertical Line Test
- discrete graph
- continuous graph
- increasing function
- decreasing function
- constant function
- function family
- linear functions
- exponential functions
- absolute minimum
- absolute maximum
- quadratic functions
- linear absolute value functions
- x-intercept
- y-intercept

You have sorted graphs by their graphical behaviors. How can you describe the common characteristics of the graphs of the functions?

Warm Up Answers

1. Domain: $-3, -2, -1, 0, 1$
   Range: $-6, -4, -2, 0, 2$
   $y = -2x$
2. Domain: $-3, -2, -1, 0, 1, 2$
   Range: $9, 4, 1, 0$
   $y = x^2$
3. Domain: $-3, -2, -1, 0, 1, 2$
   Range: $3, 2, 1, 0, -1, -2$
   $y = -x$

Answers

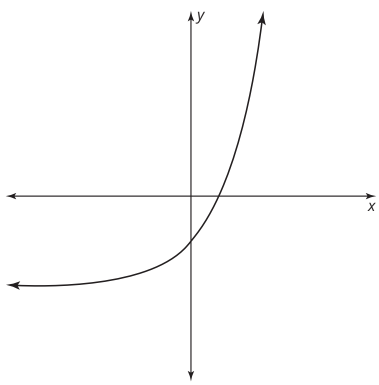
1. Answers will vary.

GETTING STARTED

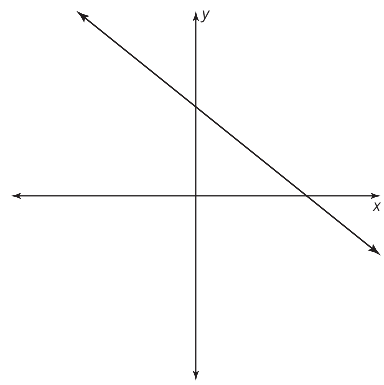
Odd One Out

1. Which of the graphs shown does not belong with the others? Explain your reasoning.

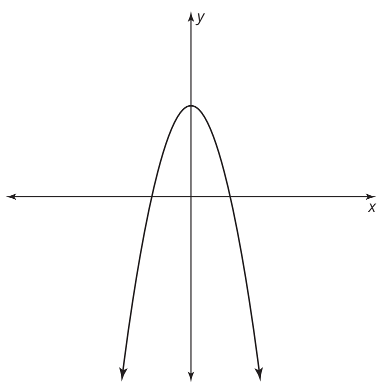
Graph A



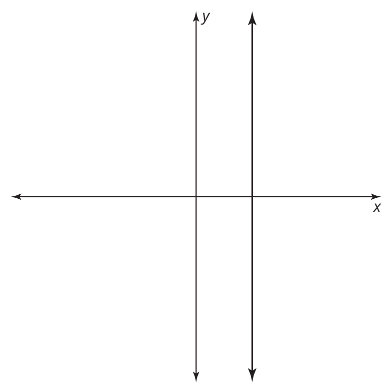
Graph B



Graph C



Graph D



ACTIVITY **3.1**

Functions and Non-Functions

A relation can be represented in the following ways.

> A **relation** is the mapping between a set of input values called the **domain** and a set of output values called the **range**.

Ordered Pairs

{(−2, 2), (0, 2), (3, −4), (3, 5)}

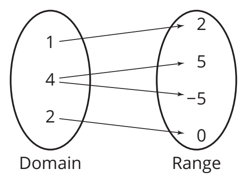
Equation

$y = \frac{2}{3}x - 1$

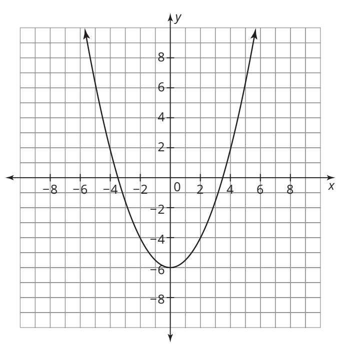
Verbal

The relation between students in your school and each student's birthday.

Mapping



Graph



Table

Domain	Range
−1	1
2	0
5	−5
6	−5
7	−8

ELL Tip

Ask students to identify what the prefix *non-* means in *non-function*. Follow up with additional examples of words with the prefix *non-* including nonsmoking, nonstop, and nonfat. Define these words and highlight the connection between the prefix *non-* and the words *not* and *no*. Encourage students to remember this connection to assist them in comprehension when they come across a word with this prefix.

Answers

1. Yes. Each element in the domain has exactly one element in the range.
2. No. An element in the domain maps to more than one element in the range.
3. Yes. Each student has one and only one birthday.

Think about:

So all functions are relations, but only some relations are functions.

A **function** is a relation that assigns to each element of the domain exactly one element of the range. Functions can be represented in a number of ways.

1. Analyze the relation represented as a table. Is the relation a function? Explain your reasoning.

2. Analyze the relation represented as a mapping. Is the relation a function? Explain your reasoning.

3. Analyze the relation represented verbally. Is the relation a function? Explain your reasoning.

You can write an equation representing a function using *function notation*. Let's look at the relationship between an equation and function notation.

Consider this scenario. U.S. Shirts charges \$8 per shirt plus a one-time charge of \$15 to set up a T-shirt design.

The equation $y = 8x + 15$ can be written to model this situation. The independent variable x represents the number of shirts ordered, and the dependent variable y represents the total cost of the order, in dollars.

This is a function because for each number of shirts ordered (independent value) there is exactly one total cost (dependent value) associated with it. Because this relationship is a function, you can write $y = 8x + 15$ in function notation.

$$f(x) = 8x + 15$$

name of function

independent variable

The cost, defined by f, is a function of x, defined as the number of shirts ordered.

You can write a function in a number of different ways. You could write the T-shirt cost function as $C(s) = 8s + 15$, where the cost, defined as C, is a function of s, the number of shirts ordered.

4. Consider the U.S. Shirts function, $C(s) = 8s + 15$. What expression in the function equation represents:

a. the domain of the function?

b. the range of the function?

5. Describe the possible domain and range for this situation.

Function notation is a way of representing functions algebraically. The function notation $f(x)$ is read as "f of x" and indicates that x is the independent variable.

If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x.

Answers

4a. The expression s represents the domain of the function.

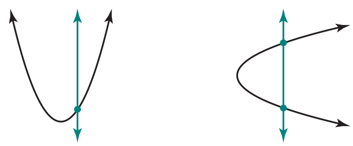
4b. The expressions $8s + 15$ and $C(s)$ each represent the range of the function.

5. The domain is the set of whole numbers. The range is the corresponding set of whole numbers that result from substituting values into the expression $8s + 15$. The range can be interpreted to either include the value at $C = 15$ or not depending on whether the design was set but no T-shirts were ordered yet.

Answers

6. A set of ordered pairs represents a function when each x-value in the set maps to exactly one y-value.
7. An equation represents a function when you can solve for y and get only one y-value for any given x-value.

The **Vertical Line Test** is a visual method used to determine whether a relation represented as a graph is a function. To apply the Vertical Line Test, consider all of the vertical lines that could be drawn on the graph of a relation. If any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function.



The Vertical Line Test applies for both *discrete* and *continuous graphs*.

A **discrete graph** is a graph of isolated points. A **continuous graph** is a graph of points that are connected by a line or smooth curve on the graph. Continuous graphs have no breaks.

6. How can you determine if a relation represented as a set of ordered pairs is a function? Explain your reasoning.

7. How can you determine if a relation represented as an equation is a function? Explain your reasoning.

ELL Tip

Review with students the difference between *continuous* and *discrete* functions. Create an anchor chart, and as a class, brainstorm different real-world functions that can be modeled either by a *continuous function* or by a *discrete function*.

8. Determine which relations represent functions. If the relation is not a function, state why not.

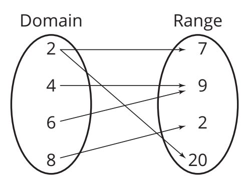
a. $y = 3^x$

b. For every house, there is one and only one street address.

c.

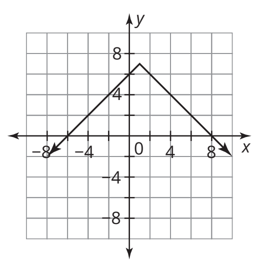
Domain	Range
–1	4
0	0
3	–2
0	4

d.



e. $\{(-7, 5), (-5, 5), (2, -2), (3, 5)\}$

f.



Answers

8a. The relation is a function.

8b. The relation is a function.

8c. The relation is not a function; the domain value of 0 has more than one range value.

8d. The relation is not a function; the domain value of 2 has more than one range value.

8e. The relation is a function.

8f. The relation is a function.

Answers

9. No. The graphs do not pass the Vertical Line Test.
10. Functions: A, B, C, E, F, G, H, I, J, L, M, N, P, Q, R, S

    Non-Functions: D, K, O,

NOTES

9. **Analyze the three graphs Judy grouped together in the previous lesson, graphs D, K, and O. Are the graphs she grouped functions? Explain your conclusion.**

10. **Use the Vertical Line Test to sort the graphs in the previous lesson into two groups: functions and non-functions. Record your results by writing the letter of each graph in the appropriate column in the table shown.**

Functions	Non-Functions

ACTIVITY
3.2

Domain and Range of a Function

You have identified the domain and range of a function given its equation.

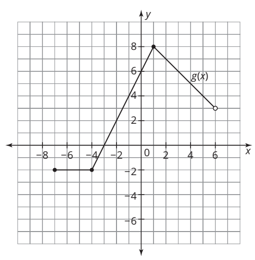
1. Explain how you can identify the domain and range of a function given:

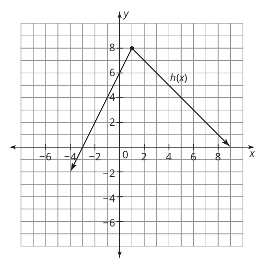
a. a verbal statement.

b. a graph.

Worked Example

There are different ways to write the domain and range of a function given its graph.





	Domain		Range	
	$g(x)$	$h(x)$	$g(x)$	$h(x)$
In Words	The domain is all real numbers greater than or equal to −7 and less than 6.	The domain is the set of all real numbers.	The range is all real numbers greater than or equal to −2 and less than or equal to 8.	The range is all real numbers less than or equal to 8.
Using Notation	$-7 \le x < 6$	$-\infty < x < \infty$	$-2 \le y \le 8$	$y \le 8$

2. Consider the Graph Cards from the previous activity that include continuous functions. Label each of these cards with the appropriate domain and range.

Answers

1a. Domain: the possible values that make sense as the independent quantities.

Range: the possible values that make sense as the dependent quantities.

1b. Domain: the set of *x*-values represented by the graph

Range: the set of *y*-values represented by the graph

2. Graph B

Domain: all real numbers

Range: $y \le 2$

Graph C

Domain: all real numbers

Range: $y \ge 0$

Graph F

Domain: all real numbers

Range: all real numbers

Graph G

Domain: all real numbers

Range: $y > -5$

Graph H

Domain: all real numbers

Range: $y \le 0$

Graph J

Domain: all real numbers

Range: $y \ge 0$

Graph M

Domain: all real numbers

Range: $y > 0$

Graph N

Domain: all real numbers

Range: $y \le 4$

Graph P

Domain: all real numbers

Range: $y \ge -2$

Graph Q

Domain: all real numbers

Range: $y \ge -4$

Answers

1. See table below.

ACTIVITY 3.3

Linear, Constant, and Exponential Functions

Gather all of the graphs that you identified as functions.

When determining whether a graph is increasing or decreasing, read the graph from left to right.

A function is described as increasing when the value of the dependent variable increases as the value of the independent variable increases. If a function increases across the entire domain, then the function is called an **increasing function**.

A function is described as decreasing when the value of the dependent variable decreases as the value of the independent variable increases. If a function decreases across the entire domain, then the function is called a **decreasing function**.

If the value of the dependent variable of a function remains constant over the entire domain, then the function is called a **constant function**.

1. Analyze each graph from left to right. Sort all the graphs into one of the four groups listed.

- **increasing function**
- **decreasing function**
- **constant function**
- **a combination of increasing and decreasing**

Record the function letter in the appropriate column of the table shown.

Increasing Function	Decreasing Function	Constant Function	Combination of Increasing and Decreasing

1.

Increasing Function	Decreasing Function	Constant Function	Combination of Increasing and Decreasing
F, I	G, L, M	A	B, C, E, H, J, N, P, Q

2. Each function shown represents one of the graphs in the increasing function, decreasing function, or constant function categories. Use graphing technology to determine the shape of its graph. Then match the function to its corresponding graph by writing the function directly on the graph that it represents.

- $f(x) = x$
- $f(x) = \left(\frac{1}{2}\right)^x$
- $f(x) = \left(\frac{1}{2}\right)^x - 5$
- $f(x) = 2$, where x is an integer
- $f(x) = 2^x$, where x is an integer
- $f(x) = -x + 3$, where x is an integer

Think about:

Be sure to correctly interpret the domain of each function. Also, remember to use parentheses when entering fractions into your calculator.

3. Consider the six graphs and functions that are increasing functions, decreasing functions, or constant functions.

   a. Sort the graphs into two groups based on the equations representing the functions and record the function letter in the table.

Group 1	Group 2

   b. What is the same about all the functions in each group?

Answers

2. $f(x) = x$: Graph F
   $f(x) = \left(\frac{1}{2}\right)^x - 5$: Graph G
   $f(x) = 2^x$, where x is an integer: Graph I
   $f(x) = -x + 3$, where x is an integer: Graph L
   $f(x) = \left(\frac{1}{2}\right)^x$: Graph M
   $f(x) = 2$ where x is an integer: Graph A

3a.

Group 1	Group 2
F, L, A	G, I, M

3b. Sample answer.
Group 1 represents lines. Group 2 represents smooth curves that are either increasing or decreasing.

Answers

4.

Group 1	Group 2
F, L, A Linear/ Constant	G, I, M Exponential

5. If $a = 0$ and b is any real number, then the result will be a constant function.

Ask yourself:

What other variables have you used to represent a linear function?

You have just sorted the graphs into their own *function families*. A **function family** is a group of functions that share certain characteristics.

The family of **linear functions** includes functions of the form $f(x) = ax + b$, where a and b are real numbers.

The family of **exponential functions** includes functions of the form $f(x) = a \cdot b^x + c$, where a, b, and c are real numbers, and b is greater than 0 but not equal to 1.

4. Go back to your table in Question 3 and identify which group represents linear and constant functions and which group represents exponential functions.

5. If $f(x) = ax + b$ represents a linear function, describe the a and b values that produce a constant function.

Place these two groups of graphs off to the side. You will need them again.

ELL Tip

Ensure that students understand the term *function family*. Because much of the focus of this lesson is on increasing, decreasing, and constant functions, students may think that all functions that are decreasing are part of the same function family. Display a graph of a decreasing linear function and a graph of a decreasing exponential function. Ask whether the functions belong to the same function family because they are both decreasing, and have students explain their answers.

ACTIVITY **3.4**

Quadratic and Absolute Value Functions

A function has an **absolute minimum** if there is a point on the graph of the function that has a *y*-coordinate that is less than the *y*-coordinate of every other point on the graph. A function has an **absolute maximum** if there is a point on the graph of the function that has a *y*-coordinate that is greater than the *y*-coordinate of every other point on the graph.

1. Sort the graphs from the combination of increasing and decreasing category in the previous activity into one of the two groups listed.

- **those that have an absolute minimum value**
- **those that have an absolute maximum value**

Then record the function letter in the appropriate column of the table shown.

Absolute Minimum	Absolute Maximum

Answers

1. See table below.

1.

Absolute Minimum	Absolute Maximum
C, J, P, Q	B, E, H, N

Answers

2. $f(x) = x^2 + 8x + 12$: Graph Q

   $f(x) = |x - 3| - 2$: Graph P

   $f(x) = x^2$: Graph J

   $f(x) = |x|$: Graph C

   $f(x) = -|x|$: Graph H

   $f(x) = -3x^2 + 4$, where x is integer: Graph E

   $f(x) = -\frac{1}{2}x^2 + 2x$: Graph B

   $f(x) = -2|x + 2| + 4$: Graph N

3a.

Group 1	Group 2
B, E, J, Q	C, H, N, P

3b. Sample answer.

Group 1 represents curves or points that make a U-shape. Group 2 represents graphs composed of 2 lines that make a V-shape.

2. Each function shown represents one of the graphs with an absolute maximum or an absolute minimum value. Use graphing technology to determine the shape of its graph. Then match the function to its corresponding graph by writing the function directly on the graph that it represents.

- $f(x) = x^2 + 8x + 12$
- $f(x) = -|x|$
- $f(x) = |x - 3| - 2$
- $f(x) = -3x^2 + 4$, where x is integer
- $f(x) = x^2$
- $f(x) = -\frac{1}{2}x^2 + 2x$
- $f(x) = |x|$
- $f(x) = -2|x + 2| + 4$

3. Consider the graphs of functions that have an absolute minimum or an absolute maximum.

a. Sort the graphs into two groups based on the equations representing the functions and record the function letter in the table.

Group 1	Group 2

b. What is the same about all the functions in each group?

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You have just sorted functions into two more function families.

The family of **quadratic functions** includes functions of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers, and a is not equal to 0.

The family of **linear absolute value functions** includes functions of the form $f(x) = a|x + b| + c$, where a, b, and c are real numbers, and a is not equal to 0.

4. Go back to your table in Question 3 and identify which group represents quadratic functions and which group represents linear absolute value functions.

Answers

4.

Group 1	Group 2
B, E, J, Q Quadratic	C, H, N, S Linear Absolute Value

ELL Tip

The visual nature of this activity provides a good opportunity to reinforce the vocabulary needed to describe functions. Write the words *increasing, decreasing, maximum, minimum, curve,* and *line* on the board. Use the words as you describe the graphs, and have students point to that feature of the graph to show they understand what you are saying.

Answers

1. Check students' graphic organizers.

ACTIVITY 3.5

Function Families

You have now sorted each of the graphs and equations representing functions into one of four function families: linear, exponential, quadratic, and linear absolute value.

1. Glue your sorted graphs and functions to the appropriate function family graphic organizer located at the end of the lesson. Write a description of the graphical behavior for each function family.

Hang on to your graphic organizers. They will be a great resource moving forward!

You've done a lot of work up to this point! You've been introduced to linear, exponential, quadratic, and linear absolute value functions. Don't worry—you don't need to know everything there is to know about these function families right now. As you progress through this course, you will learn more about each function family.

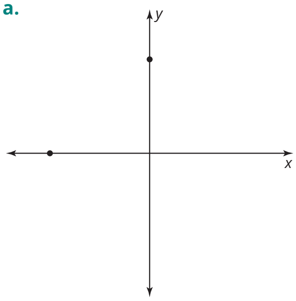
TALK the TALK

Interception!

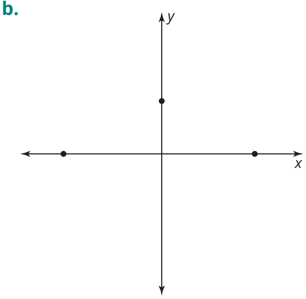
Recall that the ***x*-intercept** is the point where a graph crosses the *x*-axis. The ***y*-intercept** is the point where a graph crosses the *y*-axis.

1. The graphs shown represent relations with just the *x*- and *y*-intercepts plotted. If possible, draw a function that has the given intercepts. If it is not possible, explain why not.

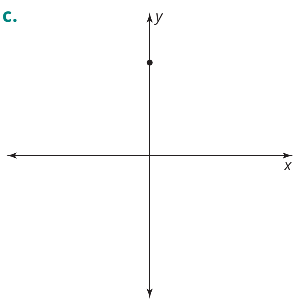
a.



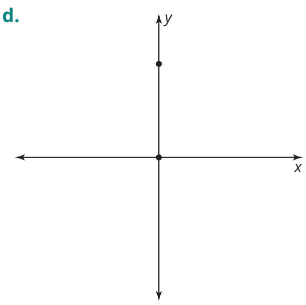
b.



c.



d.



NOTES

Answers

1a. Answers will vary.

1b. Answers will vary.

1c. Answers will vary.

1d. It is not possible. A function can have only one *y*-value for an *x*-value, but according to the graphed points, when $x = 0$ there are two different *y*-values.

The family of **linear functions** includes functions of the form $f(x) = ax + b$, where a and b are real numbers.

Graphs

Linear Functions

Increasing/Decreasing/Constant:

Domain and Range:

Maximum/Minimum:

Curve/Line:

Graphical Behaviors

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The family of **exponential functions** includes functions of the form $f(x) = a \cdot b^x + c$, where a, b, and c are real numbers, and b is greater than 0 but not equal to 1.

Graphs

Exponential Functions

Increasing/Decreasing/Constant:

Domain and Range:

Maximum/Minimum:

Curve/Line:

Graphical Behaviors

The family of **quadratic functions** includes functions of the form $f(x) = ax^2 + bx + c$ where a, b, and c are real numbers, and a is not equal to 0.

Graphs

Quadratic Functions

Increasing/Decreasing/Constant:

Domain and Range:

Maximum/Minimum:

Curve/Line:

Graphical Behaviors

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The family of **linear absolute value functions** includes functions of the form $f(x) = a|x + b| + c$, where a, b, and c are real numbers, and a is not equal to 0.

Graphs

Linear Absolute Value Functions

Increasing/Decreasing/Constant:

Domain and Range:

Maximum/Minimum:

Curve/Line:

Graphical Behaviors