4

Function Families for 2000, Alex

Recognizing Functions by Characteristics

MATERIALS

None

Lesson Overview

Given characteristics describing graphical behavior, students name the possible function family or families that fit each description. Using the scenarios and their graphs from the first lesson of the topic, they complete a table by naming the function family associated with each scenario, identifying the domain, and describing the graphical behavior as increasing, decreasing, constant, or both increasing and decreasing. Students then work with a partner and write equations and sketch graphs to satisfy different lists of characteristics. They conclude the lesson by creating their own list of characteristics, providing two graphs that include those characteristics, and determining that an equation, not just a list of characteristics, is required to generate a unique graph.

Algebra 1

Linear functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:

(A) determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities.

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:

(C) graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems.

Quadratic Functions and Equations

(6) The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. The student is expected to:

(A) determine the domain and range of quadratic functions and represent the domain and range using inequalities.

(7) The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to:

(A) graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including *x*-intercept, *y*-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry.

Exponential Functions and Equations

(9) The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:

(A) determine the domain and range of exponential functions of the form $f(x) = ab^x$ and represent the domain and range using inequalities.

(D) graph exponential functions that model growth and decay and identify key features, including *y*-intercept and asymptote, in mathematical and real-world problems.

Number and Algebraic Methods

(12) The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:

(A) decide whether relations represented verbally, tabularly, graphically, and symbolically define a function.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G

Essential Ideas

- The graph of an exponential or quadratic function is a curve.
- The graph of a linear or linear absolute value function is a line or pair of lines, respectively.
- The graph of a linear or exponential function is either increasing or decreasing.
- The graph of a quadratic function or a linear absolute value function has intervals where it is increasing and intervals where it is decreasing. Each function also has an absolute maximum or absolute minimum.
- Key characteristics of graphs help to determine the function family to which it belongs.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Name that Function!

Students are given one or two characteristics of a graph and determine whether the function could be a member of the linear, exponential, quadratic, and/or linear absolute value function family.

Develop

Activity 4.1: Categorizing Scenarios Into Their Function Families

Students revisit the scenarios and their graphs from the first lesson of the topic to complete a table naming the function family associated with each scenario, identifying the domain, and describing the graphical behavior as increasing, decreasing, constant, or both increasing and decreasing.

Day 2

Activity 4.2: Building Graphs From Characteristics

Students write equations and sketch graphs to satisfy different lists of characteristics. They then create their own function and describe characteristics of the function so that another person can sketch the graph. They exchange descriptions with a partner and sketch their partner's function.

Demonstrate

Talk the Talk: Trying to Be Unique

Students create their own list of characteristics, provide two graphs that include those characteristics, and determine that an equation, not just a list of characteristics, is required to generate a unique graph.

ENGAGE

Getting Started: Name That Function!

Facilitation Notes

In this activity, students are given one or two characteristics of a graph and determine whether the function could be a member of the linear, exponential, quadratic, and/or linear absolute value function family.

Differentiation strategy

To scaffold support, prior to beginning the lesson, review the four function families as a class: linear, exponential, quadratic, and linear absolute.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- What does the graph of a smooth curve look like? Not a smooth curve?
- What does the graph of a function that increases over the entire domain look like?
- What does the graph of a function that decreases over the entire domain look like?
- What does the graph of a function that has an absolute maximum look like?
- Can the graph of a function have more than one maximum? Absolute maximum?
- What does a graph that has symmetry look like?

Summary

Graphs described as straight lines may be associated with a linear function or linear absolute value function, while those described as smooth curves may be associated with an exponential or quadratic function. The graph of a linear or exponential function is either increasing or decreasing, while the graph of a quadratic function or a linear absolute value function has an interval where it is increasing and an interval where it is decreasing.

Activity 4.1
Categorizing Scenarios into Their Function Families

DEVELOP

Facilitation Notes

In this activity, students revisit the scenarios and their graphs from the first lesson of the topic to complete a table naming the function family associated with each scenario, identifying the domain, and describing the graphical behavior as increasing, decreasing, constant or both increasing and decreasing.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- What is a function?
- How can you tell from a scenario whether or not it represents a function?
- How can you tell from the scenario that Graph B represents an exponential rather than half of the graph of a quadratic function?
- How many of the scenarios are associated with an exponential function? How did you tell them apart?
- How can you tell from the scenario that it is associated with an absolute value function?
- How can you tell from the scenario whether the domain should be continuous or discrete?
- How many of the scenarios contain an absolute maximum? Absolute minimum?
- How many of the scenarios can be described as increasing? As decreasing?

Misconception

Students may think the graphs are incorrect because they all are continuous, while some of the scenarios have domains that are discrete. Discuss the fact that the graphs relate to functions that are mathematical models of the scenarios; the scenarios require an interpretation of the necessary components of the mathematical model.

Summary

A scenario and its graph provide the necessary characteristics to determine the function family to which it belongs.

Activity 4.2
Building Graphs from Characteristics

Facilitation Notes

In this activity, students write equations and sketch graphs to satisfy different lists of characteristics. They then create their own function and describe characteristics of the function so that another person can sketch the graph. They exchange descriptions with a partner and sketch their partner's function.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Differentiation strategy

To scaffold support, have students complete Question 1 with a partner. Decide who begins with part (a) and who begins with part (b), then create an equation based on the given criteria. Have students swap equations with their partner, check each other's equations, and together correct any errors. Then, have students graph the equation written by their partner, then swap graphs and check for correctness again. Repeat this process with parts (c) and (d). Both partners should write an equation for part (e), and graph their partner's equation.

As student work, look for

- Students who efficiently create a correct equation and graph on their first attempt.
- Students who must self-correct as they attempt to create an equation and graph.
- Students who reorder the characteristics prior to creating a graph.
- Students who attempt to graph first and then write the equation from the graph.

Questions to ask

- If the equation is described as a function, what does that imply?
- If the equation is described an exponential function, what does that imply? As a continuous function? As a decreasing function?
- Is there more than one correct equation and graph that fits this list of criteria?
- If the equation described contains a minimum, what does that imply?
- If the equation described is discrete, what does that imply?
- If the equation is described as a linear absolute value function, what does that imply?
- If the equation is described as linear, what does that imply? Is it a function?
- If the equation is described as increasing, what does that imply?

- If the equation is described as continuous, what does that imply? Is it a function?
- If the equation is described as quadratic, what does that imply?
- How many characteristics did you list for your function?
- Is it possible to compose a list of characteristics that do not describe a function?
- Do any of the characteristics on your list contradict each other?
- What is an example of two characteristics that contradict each other?
- Is there more than one correct sketch that matches all of the characteristics on your list?
- How is your list of characteristics different than your partner's list of characteristics?

Summary

A list of characteristics can be used to write an equation and the equation can then be used to generate a graph.

Talk the Talk: Trying to Be Unique

DEMONSTRATE

Facilitation Notes

In this activity, students create their own list of characteristics, provide two graphs that include those characteristics, and determine that an equation, not just a list of characteristics, is required to generate a unique graph.

Have students work with a partner or in a group to complete this activity. Share responses as a class.

Questions to ask

- Did you include the characteristic function or non-function?
- Did you include the characteristic continuous or discrete?
- Did you include the characteristic smooth curve or straight line?
- Did you include the characteristic absolute minimum or absolute maximum?

Summary

An equation, not just a list of characteristics, is required to generate a unique graph.

NOTES

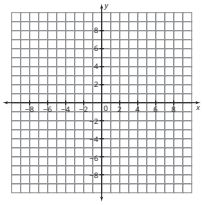
4

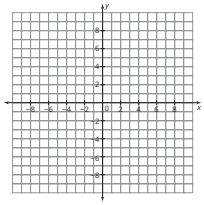
Function Families for 2000, Alex

Recognizing Functions by Characteristics

Warm Up

1. Sketch a graph and write an equation for each function.
   a. decreasing linear function
   b. increasing exponential function





Learning Goals

- Recognize similar characteristics among function families.
- Recognize different characteristics among function families.
- Determine function types given certain characteristics.

You have identified key characteristics of graphs. How can the key characteristics help you sketch the graph of a function?

Warm Up Answers

1a. Sample answer.
$f(x) = -2x + 1$

1b. Sample answer.
$f(x) = 3^x$

ELL Tip

Create an anchor chart to identify the various characteristics of functions. Ask students what they think of when they hear the word *characteristics*. Compare and contrast *characteristics* of a person to *characteristics* of a function. Ask students to sketch examples of graphs that represent different types of functions and label the graphs with the *characteristics* of each function.

Answers

1a. exponential function or quadratic function

1b. linear function or linear absolute value function

1c. linear function or exponential function

1d. quadratic function or linear absolute value function

2a. quadratic function

2b. linear function

2c. exponential function

2d. linear absolute value function

GETTING STARTED

Name That Function!

You have sorted graphs according to their function family. Now, consider which function families have the given characteristics.

Function Families
linear
exponential
quadratic
linear absolute value

1. Which function families can be described by the characteristic provided? Choose from the given list.

a. The graph is a smooth curve.

b. The graph is made up of one or more straight lines.

c. The graph increases or decreases over the entire domain.

d. The graph has an absolute maximum or minimum.

2. One or more characteristics have been added to the graphical description of each function. Name the possible function families.

a. The graph has an absolute minimum or absolute maximum and is a smooth curve.

b. The graph either increases or decreases over the entire domain and is a straight line.

c. The graph is a smooth curve, and either increases or decreases over the entire domain.

d. The graph has either an absolute minimum or an absolute maximum, has symmetry, and is made up of 2 straight lines.

Each function family has certain graphical behaviors, with some behaviors common among different function families. Notice, the more specific characteristics that are given, the more specifically you can name that function!

ELL Tip

Review the difference between behaviors of functions and characteristics of functions. Discuss as a class the characteristics of a person compared to the behaviors of a person. Help students make the connection that characteristics are usually nouns and behaviors are usually verbs. Create a list of words that are characteristics of a function and a list of words that can be described as behaviors of a function. Have students add to the list as they come across the different functions.

ACTIVITY
4.1

Categorizing Scenarios into Their Function Families

You have been introduced to several function families: linear, exponential, quadratic, and linear absolute value. Let's revisit the first lesson: *A Picture Is Worth a Thousand Words*. Each of the scenarios in that lesson represents one of these function families.

1. Describe how each scenario represents a function.

Remember:

Each of the graphs representing the scenarios was drawn with either a continuous line or a continuous smooth curve to model the problem situation.

2. Complete the table on the following pages to describe each scenario.

a. Identify the appropriate function family under the scenario name.

b. Based on the context, identify the domain.

c. Describe the graphical behavior as increasing, decreasing, constant, or a combination.

Answers

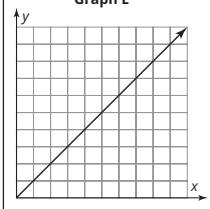
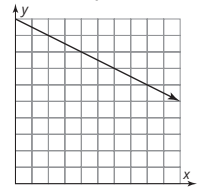
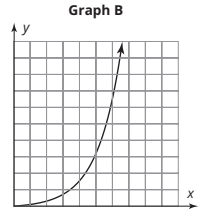
1. Each scenario describes a function because there is one unique output value for each input value.
2. See answers beside the table.

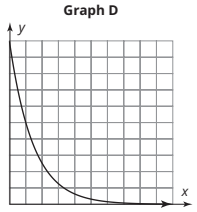
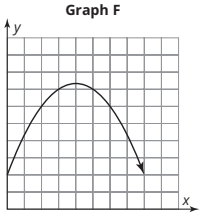
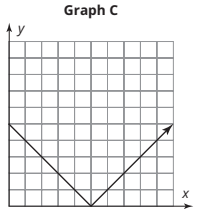
Answers

Music Club: linear, discrete, increasing

Something's Fishy: linear, continuous, decreasing

Smart Phone, But Is It a Smart Deal?: exponential, discrete, increasing

Scenario	Domain of the Real-World Situation	Graph of the Mathematical Model	Graphical Behavior
Music Club		Graph E 	
Something's Fishy		Graph A 	
Smart Phone, but Is It a Smart Deal?		Graph B 	

Scenario	Domain of the Real-World Situation	Graph of the Mathematical Model	Graphical Behavior
It's Magic		Graph D 	
Baton Twirling		Graph F 	
Jelly Bean Challenge		Graph C 	

Answers

It's Magic: exponential, discrete, decreasing

Baton Twirling: quadratic, continuous, increasing and decreasing

Jelly Bean Challenge: linear absolute value function, discrete, decreasing and increasing

Answers

1a. Sample answer.
$f(x) = 3\left(\frac{1}{2}\right)^x$

ACTIVITY
4.2

Building Graphs from Characteristics

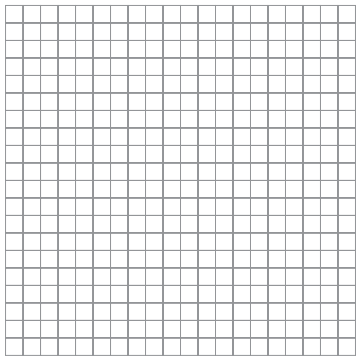
In this activity, you will write equations and sketch a graph based on given characteristics.

1. Use the given characteristics to create an equation and sketch a graph. Use the equations given in the box as a guide. When creating your equation, use *a*, *b*, and *c* values that are any real numbers between −3 and 3. Do not use any functions that were used previously in this topic.

Linear Function
$f(x) = ax + b$
Exponential Function
$f(x) = a \cdot b^x + c$
Quadratic Function
$f(x) = ax^2 + bx + c$
Linear Absolute Value Function
$f(x) = a|x + b| + c$

a. Create an equation and sketch a graph that is:
- **a function,**
- **exponential,**
- **continuous, and**
- **decreasing.**

Equation: _____



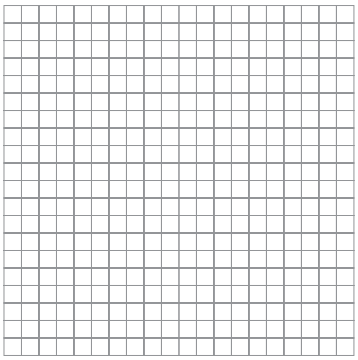
Think about:

Don't forget about the function family graphic organizers you created if you need some help.

b. Create an equation and sketch a graph that:

- **has a minimum,**
- **is discrete, and**
- **is a linear absolute value function.**

Equation: _____



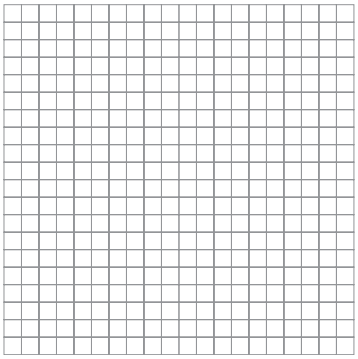
Ask yourself:

Is the domain the same or different for each function?

c. Create an equation and sketch a graph that is:

- **linear,**
- **discrete,**
- **increasing, and**
- **a function.**

Equation: _____



Answers

1b. Sample answer.

$f(x) = 2|x|$

Domain is set of integers.

Graph should show discrete values.

1c. Sample answer.

$f(x) = 2x - 1$

Domain is set of integers.

Graph should show discrete values.

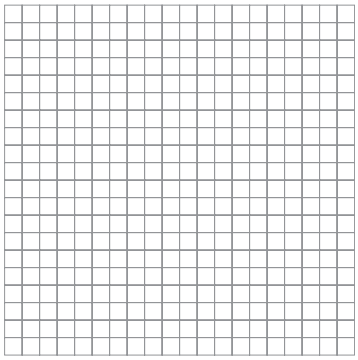
Answers

1d. Sample answer.
$f(x) = -x^2$

1e. Sample answer.
$x = 3$

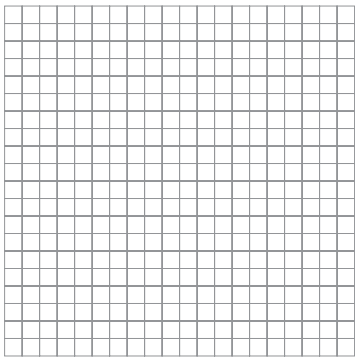
d. Create an equation and sketch a graph that:
- **is continuous,**
- **has a maximum,**
- **is a function, and**
- **is quadratic.**

Equation: _____

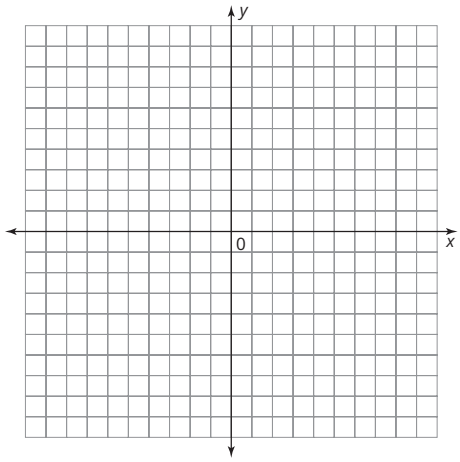


e. Create an equation and sketch a graph that is:
- **not a function,**
- **continuous, and**
- **a straight line**

Equation: _____



2. **Create your own function. Describe certain characteristics of the function and see if your partner can sketch it. Then sketch your partner's function based on characteristics provided.**



Answers

2. Answers will vary.

Answers

1. Answers will vary.
2. Graphs should match the response to Question 1.
3. Change the directions to write an equation, or provide a specific number of points that lie on the graph.

NOTES

TALK the TALK

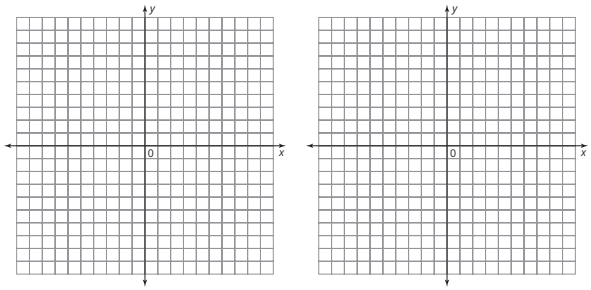
Trying to Be Unique

Throughout this lesson, you used characteristics to describe graphs.

1. Write a list of four characteristics to describe a graph.

- _____
- _____
- _____
- _____

2. Sketch two possible graphs based on your characteristics.



3. How could you modify your list of characteristics to describe a unique graph?

ELL Tip

Ask students what they think of when they hear the word *unique*. Have them define the term with examples of how it would be used in their culture. Discuss examples of ways to make objects *unique*, and then connect the concept of *uniqueness* to graphs. Ask for volunteers to discuss how to make a particular type of graph *unique*. For example, show a graph of two linear functions, one with a negative slope and one with a positive slope. Ask students, "Although both of these graphs represent linear functions, how is each one *unique*?"