Quantities and Relationships
Topic 1 Overview

How is *Quantities and Relationships* organized?

In *Quantities and Relationships*, students encounter different scenarios representing the functions they will study throughout the course. The intent is merely to introduce these new functions, providing an overview but not a deep understanding at this point. The topic is designed to help students recognize that different function families have different key characteristics. In later study—both in this course and in future courses—they will formalize their understanding of the defining characteristics of each type of function.

Students begin by analyzing real-world scenarios. These scenarios move beyond the linear relationships familiar from middle school to include various nonlinear functions. Students connect the scenarios to corresponding graphs. They examine the graphical behavior of different function types by exploring a wide variety of graphs. Students search for patterns in the graphs' shape and structure, and then sort them according to defined characteristics.

Students review the definition of *function, domain*, and *range*. Building on their knowledge from middle school, they formalize their representations of functions by writing equations in function notation. They use graphical behavior and the structure of the corresponding equations to classify each function according to its function family. Finally, with a more thorough understanding of the key characteristics of graphs of functions, students return to the scenarios from the first lesson and define each in terms of function family and graphical behavior.

At the end of *Quantities and Relationships*, students create a graphic organizer with the equation representation and graphical behavior of each function family studied in this course: linear, exponential, quadratic, and linear absolute value.

What is the entry point for students?

Throughout middle school, students explored relationships between quantities. In grade 6, students defined independent and dependent variables and used them to write equations and create tables and graphs for various relationships. In grade 8, students defined a function and used linear functions to model the relationship between two quantities. Students have written linear functions in slope-intercept form and should be able to identify the slope and *y*-intercept in the equation. *Quantities and Relationships* introduces formal function notation as a natural extension of the informal expression evaluation mastered in grades 6 through 8. In grade 8, students learned that a graph of a function is the set of ordered pairs consisting of an input and the corresponding output. They characterized graphs as functions using the terms *increasing, decreasing, constant, discrete, continuous, linear*, and *nonlinear*. In *Quantities and Relationships*, students build on these characteristics to define new function families.

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in *Quantities and Relationships* if they can:

- Choose appropriate scale and origin for graphs.
- Identify the appropriate unit of measure for each variable or quantity.
- Analyze a graph and state the key characteristics of the graph.
- Use a problem situation to explain what the key features of a graph mean in real-world context.
- Decide whether relations represented verbally, tabularly, graphically, and symbolically define a function.
- Recognize a linear, exponential, quadratic, or linear absolute value function by its equation or graph.
- Evaluate functions, expressed in function notation, given one or more elements in their domain.
- Determine the domain and range and the independent and dependent quantities in a relationship.

Why is *Quantities and Relationships* important?

The study of functions is a main focus of high school mathematics. This topic builds the foundation for future, more in-depth study by familiarizing students with the concept of a function. With a function approach, students see functions as objects that can be represented by scenarios, equations, tables, and graphs. Recognizing patterns and structure in these representations will allow them to connect different representations and to generalize patterns across function families. As students explore linear, piecewise, absolute value, exponential, and quadratic functions in future topics, the background acquired in *Quantities and Relationships* will help them to recognize and differentiate among mathematical relationships, and to compare and contrast the key characteristics of graphs and equations. Students will continue to use formal function notation throughout this course and in higher-level math courses. Facility with this notation helps students to see relationships between a function's input, x, and its corresponding output, $f(x)$.

Ultimately, searching for patterns in graphs is critical as students learn to recognize, generalize, and use patterns that exist in numbers, in shapes, and in the world around them. This work helps students to become better problem solvers and make sense of the mathematics.

How do the activities in *Quantities and Relationships* promote student expertise in the mathematical process standards?

All Carnegie Learning topics are written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work towards solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

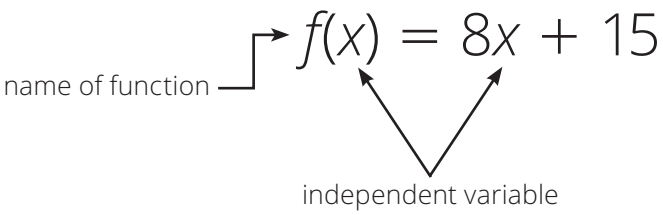
Throughout *Quantities and Relationships*, students search for patterns in tables, equations, and scenarios. They examine the structure of these function representations to identify common characteristics of function types. They should notice that the equations of graphs in the same family all take the same general form.

Materials Needed

Glue
Graphing technology
Scissors

New Notation

An equation representing a function can be written using function notation. The function notation $f(x)$ is read as "f of x" and indicates that x is the independent variable.



In the function shown, the cost, defined by f, is a function of x, the number of shirts ordered. The name of the function is f. It is not a variable.

Learning Together

ELPS: 1.A, 1.C,1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G

Lesson	Lesson Name	TEKS	Days	Highlights
1	A Picture Is Worth a Thousand Words Understanding Quantities and Their Relationships	A.3C A.7A A.9D	2	Students are presented with various scenarios and identify the independent and dependent quantities for each. They then match a graph to the appropriate scenario, label the axes using the independent and dependent quantities, and create the scale for the axes. Students make basic observations about the similarities and differences in the graphs. They then look more deeply at pairs of scenarios along with their graphs to focus on characteristics of the graphs, such as intercepts, increasing, and decreasing intervals, and maximum and minimum points. The lesson concludes with students creating their own scenario and a sketch of a graph to model the scenario.
2	A Sort of Sorts Analyzing and Sorting Graphs	A.3C A.7A A.9D	1	Students begin this lesson by cutting out 17 different graphs. They sort the graphs into different groups based on their own rationale, compare their groupings with their classmates' and discuss the reasoning behind their choices. Next, four different groups of graphs are given, and students analyze the groupings and explain possible rationales behind the choices made. Students explore different representations of relations. Students need to keep their graphs as they will be used in lessons that follow.

Lesson	Lesson Name	TEKS	Days	Highlights
3	F of X Recognizing Functions and Function Families	A.2A A.3C A.6A A.7A A.9A A.9D A.12A	3	The definitions *function* and *function notation* are introduced in this lesson. For the remainder of the lesson, students use graphing technology to connect equations written in function forms to its graph and then identify the function family to which they belong. The terms *increasing function, decreasing function,* and *constant function* are defined, and students sort the graphs from the previous lesson into these groups and a group labeled for functions that include a combination of increasing, decreasing, and constant intervals. The terms *function family, linear function,* and *exponential function* are then defined, and students sort the increasing constant and decreasing functions into one of these families. Next, the terms *absolute minimum* and *absolute maximum* are defined, as well as the terms *quadratic function* and *linear absolute value function.* Students sort the functions with an absolute minimum or absolute maximum into one of these families. Students then complete a graphic organizer for each function family that describes the graphical behavior and displays graphical examples. In the final activity, students use their knowledge of the function families to demonstrate how the families differ with respect to their *x*- and *y*-intercepts. Graphing technology is necessary to help students connect some equations and their graphs.
4	Function Families for 2000, Alex Recognizing Functions by Characteristics	A.2A A.3C A.6A A.7A A.9A A.9D A.12A	3	Given characteristics describing the graphical behavior of specific functions, students name the possible function family/families that fit each description. Students revisit the scenarios and graphs from the first lesson, name the function family associated with each scenario, identify the domain, and describe the graph. Students then write equations and sketch graphs to satisfy a list of characteristics. They conclude by determining that a function or equation, not just a list of characteristics, is required to generate a unique graph.

Suggested Topic Plan

*1 Day Pacing = 45 min. Session

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: A.3C, A.7A, A.9D **LESSON 1** **A Picture is Worth a Thousand Words** GETTING STARTED ACTIVITY 1	**LESSON 1** continued ACTIVITY 2 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress	TEKS: A.3C, A.7A, A.9D **LESSON 2** **A Sort of Sorts** GETTING STARTED ACTIVITY1 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
TEKS: A.2A, A.3C, A.6A, A.7A, A.9A, A.9D, A.12A **LESSON 3** **F of X** GETTING STARTED ACTIVITY 1	**LESSON 3** continued ACTIVITY 2 ACTIVITY 3	**LESSON 3** continued ACTIVITY 4 ACTIVITY 5 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress	TEKS: A.2A, A.3C, A.6A, A.7A, A.9A, A.9D, A.12A **LESSON 4** **Function Families for 2000, Alex** GETTING STARTED ACTIVITY 1
Day 11	**Day 12**	**Day 13**		
LESSON 4 continued ACTIVITY 2 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress	**END OF TOPIC ASSESSMENT**		

Assessments

There is one assessment aligned to this topic: End of Topic Assessment.