Module 1: Searching for Patterns

TOPIC 2: SEQUENCES

In this topic, students explore sequences represented as lists of numbers, in tables of values, by equations, and as graphs on the coordinate plane. Students move from an intuitive understanding of patterns to a more formal approach of representing sequences as functions. In the final lesson of the topic, students are introduced to the modeling process. Defined in four steps—Notice and Wonder, Organize and Mathematize, Predict and Analyze, and Test and Interpret—the modeling process gives students a structure for approaching real-world mathematical problems.

Where have we been?

Students have been analyzing and extending numeric patterns since elementary school. They have discovered and explained features of patterns. They have formed ordered pairs with terms of two sequences and compared the terms. In middle school, students have connected term numbers and term values as the inputs and outputs of a function.

Where are we going?

As students deepen their understanding of functions throughout this course and beyond, recognizing that all sequences are functions is an important building block. A rich understanding of arithmetic sequences is the foundation for linear functions. As students gain experience with more complex functions, the modeling process will help them approach and solve problems they encounter in the real world.

Formulas for Sequences

A formula can be written to determine any number in an arithmetic or geometric sequence. You just need to know the first number in the sequence, the position of the number in the sequence, and the common difference or common ratio.

Arithmetic Sequence

*n*th term

common difference

$$a_n = a_1 + d\underbrace{(n - 1)}_{\text{previous term number}}$$

1st term

Geometric Sequence

*n*th term

$$g_n = g_1 \cdot r^{\overbrace{n-1}^{\text{previous term number}}}$$

1st term

common ratio

Given the arithmetic sequence 2, 5, 8, 11, 14 . . . , the 10th number is $2 + 3(10 - 1)$, or 29.

Given the geometric sequence 2, 6, 18, 54, 162 . . . , the 10th number is $2 \cdot 3^{10-1}$, or 39,366.

The Dress, with Sequence

Not too long ago, a picture of a dress caused a lot of controversy on the internet. Was it white and gold or blue and black? (Spoiler: It was actually blue and black.) As a result of all the attention, the company that made the dress saw sales go up nearly 350%.

Public demand for products usually changes much more slowly than this. But still, companies have to be ready to deliver. So production managers use sequences to determine how much of their product to have available to meet the demand.

These sequences never match the perfect mathematical sequences you see in math class, but companies analyze the mathematical sequences to model the actual demand for products and make predictions and estimates for their inventory.

Talking Points

Sequences are an important topic to know about for college admissions tests.

Here is a sample question:

What is the second term in this geometric sequence?

$\frac{1}{3}$, ______, $\frac{1}{48}$, $\frac{1}{192}$, ...

To solve this, students need to know that each term in a geometric sequence is calculated by using the same multiplier, or constant ratio. The multiplier can be determined by dividing a term by the term before it.

In this case, $192 \div 48 = 4$. Therefore, $\frac{1}{192} \div \frac{1}{48} = \frac{1}{4}$. This means the multiplier is $\frac{1}{4}$. The second term can be calculated by multiplying the first term by $\frac{1}{4}$. Because $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$, the second term is $\frac{1}{12}$.

Key Terms

sequence
A sequence is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects.

arithmetic sequence
An arithmetic sequence is a sequence of numbers in which the difference between any two consecutive terms is a constant.

geometric sequence
A geometric sequence is a sequence of numbers in which you multiply each term by a constant to determine the next term.

mathematical modeling
Mathematical modeling is explaining patterns in the real world based on mathematical ideas.