1

Is There a Pattern Here?

Recognizing Patterns and Sequences

Warm Up

Write the next three terms in each pattern and explain how you generated each term.

1. J, F, M, A, M, J, J, A, S, . . .

2. S, M, T, W, . . .

3. 5, 10, 15, 20, . . .

4. 100, 81, 64, 49, . . .

Learning Goals

- Recognize and describe patterns.
- Represent patterns as sequences.
- Predict the next term in a sequence.
- Represent a sequence as a table of values.

Key Terms

- sequence
- term of a sequence
- infinite sequence
- finite sequence

Since early elementary school, you have been recognizing and writing patterns involving shapes, colors, letters, and numbers. How are patterns related to sequences and how can sequences be represented using a table of values?

A Pyramid of Patterns

Pascal's Triangle is a famous pattern named after the French mathematician and philosopher Blaise Pascal. A portion of the pattern is shown.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

1. List at least 3 patterns that you notice.

2. Describe the pattern for the number of terms in each row.

3. Describe the pattern within each row.

4. Describe the pattern that results from determining the sum of each row.

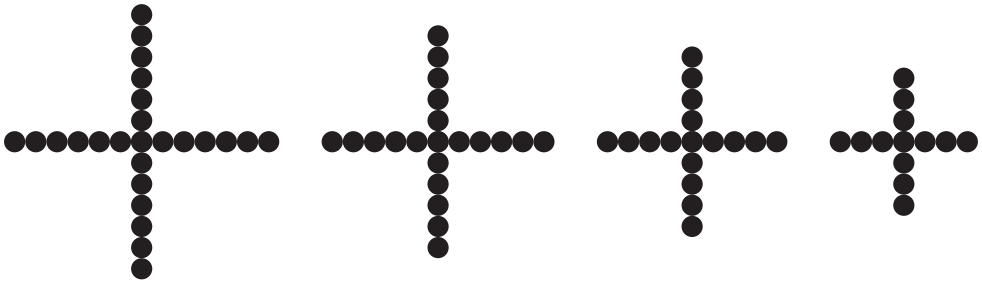
5. Determine the next two rows in Pascal's Triangle. Explain your reasoning.

ACTIVITY
1.1 Patterns to Sequences to Tables

A **sequence** is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. A **term of a sequence** is an individual number, figure, or letter in the sequence.

Ten examples of sequences are given in this activity. For each sequence, describe the pattern, draw or describe the next terms, and represent each sequence numerically.

1. Positive Thinking



a. Analyze the number of dots. Describe the pattern.

b. Draw the next three figures of the pattern.

c. Represent the number of dots in each of the seven figures as a numeric sequence.

d. Represent the number of dots in each of the first seven figures as a function using a table of values.

Term Number	1	2	3	4	5	6	7
Term Value							

All numeric sequences can be represented as functions. The independent variable is the term number beginning with 1, and the dependent variable is the term of the sequence.

2. Family Tree

Jessica is investigating her family tree by researching each generation, or set, of parents. She learns all she can about the first four generations, which include her two parents, her grandparents, her great-grandparents, and her great-great-grandparents.

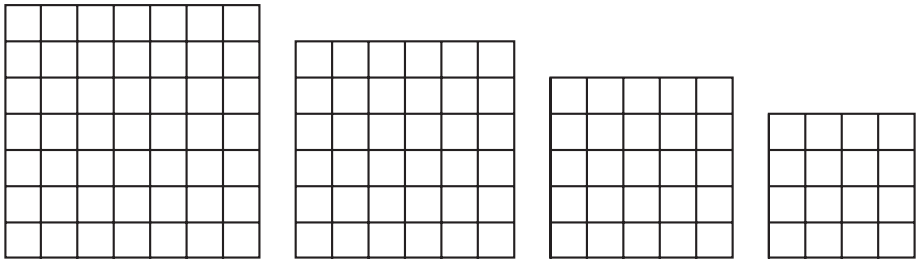
Term Number	Term Value

a. Think about the number of parents. Describe the pattern.

b. Determine the number of parents in the fifth and sixth generations.

c. Represent the number of parents in each of the 6 generations as a numeric sequence. Then represent the sequence using a table of values.

3. A Collection of Squares



Term Number	Term Value

a. Analyze the number of small squares in each figure. Describe the pattern.

b. Draw the next three figures of the pattern.

c. Represent the number of small squares in each of the first seven figures as a numeric sequence. Then represent the sequence using a table of values.

4. Al's Omelets

Al's House of Eggs N'at makes omelets. Al begins each day with 150 eggs to make his famous *Bestern Western Omelets*. After making 1 omelet, he has 144 eggs left. After making 2 omelets, he has 138 eggs left. After making 3 omelets, he has 132 eggs left.

a. Think about the number of eggs Al has left after making each omelet. Describe the pattern.

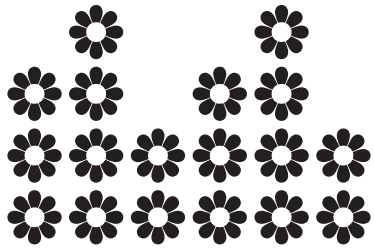
b. Determine the number of eggs left after Al makes the next two omelets.

c. Represent the number of eggs left after Al makes each of the first 5 omelets as a numeric sequence. Be sure to include the number of eggs he started with. Then represent the sequence using a table of values.

Term Number	Term Value

5. Donna's Daisies

Donna is decorating the top border of her bedroom walls with a daisy pattern. She is applying decals with each column having a specific number of daisies.



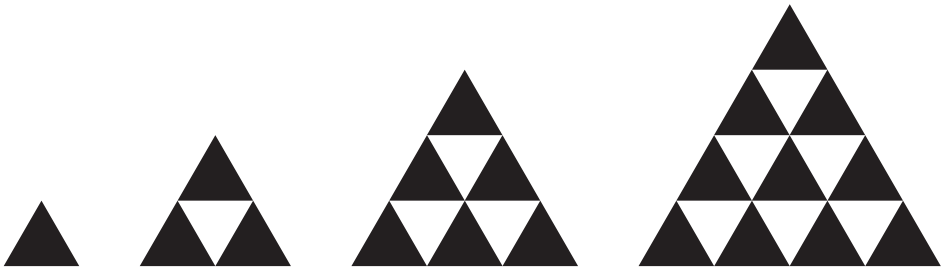
a. Think about the number of daisies in each column. Describe the pattern.

b. Determine the number of daisies in each of the next two columns.

c. Represent the number of daisies in each of the first 8 columns as a numeric sequence. Then represent the sequence using a table of values.

Term Number	Term Value

6. Troop of Triangles



a. Analyze the number of dark triangles. Describe the pattern.

Term Number	Term Value

b. Draw the next two figures of the pattern.

c. Represent the number of dark triangles in each of the first 6 figures as a numeric sequence. Then represent the sequence using a table of values.

7. Gamer Guru

Mica is trying to beat his high score on his favorite video game. He unlocks some special mini-games where he earns points for each one he completes. Before he begins playing the mini-games, Mica has 500 points. After completing 1 mini-game he has a total of 550 points, after completing 2 mini-games he has 600 points, and after completing 3 mini-games he has 650 points.

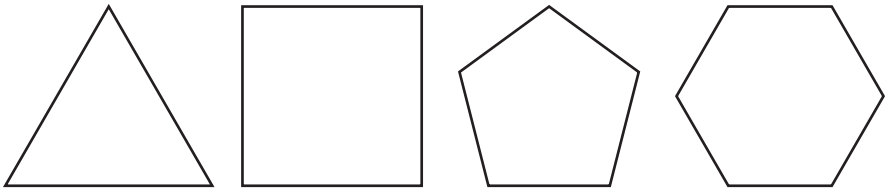
a. Think about the total number of points Mica gains from mini-games. Describe the pattern.

b. Determine Mica's total points after he plays the next two mini-games.

c. Represent Mica's total points after completing each of the first 5 mini-games as a numeric sequence. Be sure to include the number of points he started with. Then represent the sequence using a table of values.

Term Number	Term Value

8. Polygon Party



a. Analyze the number of sides in each polygon. Describe the pattern.

Term Number	Term Value

b. Draw the next two figures of the pattern.

c. Represent the number of sides of each of the first 6 polygons as a numeric sequence. Then represent the sequence using a table of values.

9. Pizza Contest

Jacob is participating in a pizza-making contest. Each contestant has to bake the largest and most delicious pizza they can. Jacob's pizza has a 6-foot diameter! After the contest, he plans to cut the pizza so that he can pass the slices out to share. He begins with 1 whole pizza. Then, he cuts it in half. After that, he cuts each of those slices in half. Then he cuts each of those slices in half, and so on.

a. Think about the size of each slice in relation to the whole pizza. Describe the pattern.

b. Determine the size of each slice compared to the whole pizza after the next two cuts.

c. Represent the size of each slice compared to the whole pizza after each of the first 5 cuts as a numeric sequence. Include the whole pizza before any cuts. Then represent the sequence using a table of values.

Term Number	Term Value

10. Coin Collecting

Miranda's uncle collects rare coins. He recently purchased a rare coin for $5. He claims that the value of the coin will triple each year. So even though the coin is currently worth $5, next year it will be worth $15. In 2 years it will be worth $45, and in 3 years it will be worth $135.

a. Think about how the value of the coin changes each year. Describe the pattern.

Term Number	Term Value

b. Determine the value of the coin after 4 years and after 5 years.

c. Represent the value of the coin after each of the first 5 years as a numeric sequence. Include the current value. Then represent the sequence using a table of values.

ACTIVITY
1.2 Looking at Sequences More Closely

There are many different patterns that can generate a sequence of numbers. For example, you may have noticed that some of the sequences in the previous activity were generated by performing the same operation using a constant number. In other sequences, you may have noticed a different pattern.

The next term in a sequence is calculated by determining the pattern of the sequence, and then using that pattern on the last known term of the sequence.

1. For each sequence in the previous activity, write the numeric sequence, record whether the sequence increases or decreases, and describe the sequence by stating the first term and the operation(s) used to create the sequence. The first one has been completed for you.

Problem Name	Numeric Sequence	Increases or Decreases	Sequence Description
Positive Thinking	25, 21, 17, 13, 9, 5, 1	Decreases	Begin at 25. Subtract 4 from each term.
Family Tree			
A Collection of Squares			
Al's Omelets			
Donna's Daisies			
Troop of Triangles			
Gamer Guru			
Polygon Party			
Pizza Contest			
Coin Collecting			

2. **Which sequences are similar? Explain your reasoning.**

3. **What do all sequences have in common?**

4. **Consider a sequence in which the first term is 64 and each term after that is calculated by dividing the previous term by 4. Margaret says that this sequence ends at 1 because there are no whole numbers that come after 1. Jasmine disagrees and says that the sequence continues beyond 1. Who is correct? If Margaret is correct, explain why. If Jasmine is correct, predict the next two terms of the sequence.**

5. **What is the domain of a sequence? What is the range?**

If a sequence continues on forever, it is called an **infinite sequence**. If a sequence terminates, it is called a **finite sequence**.

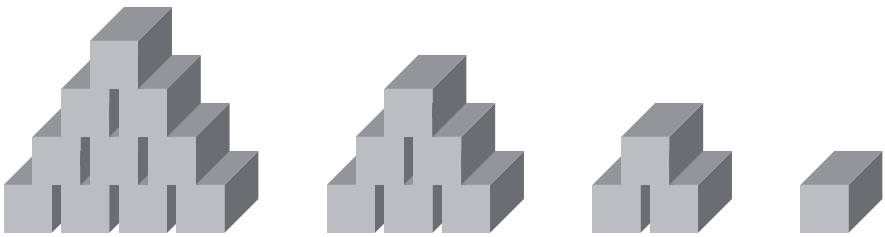
For example, consider an auditorium where the seats are arranged according to a specific pattern. There are 22 seats in the first row, 26 seats in the second row, 30 seats in the third row, and so on. Numerically, the sequence is 22, 26, 30, . . . , which continues infinitely. However, in the context of the problem, it does not make sense for the number of seats in each row to increase infinitely. Eventually, the auditorium would run out of space! Suppose that this auditorium can hold a total of 10 rows of seats. The correct sequence for this problem situation is:

22, 26, 30, 34, 38, 42, 46, 50, 54, 58.

Therefore, because of the problem situation, the sequence is a finite sequence.

An ellipsis is three periods, which means "and so on." An infinite sequence can be represented using an ellipsis.

6. Does the pattern shown represent an infinite or finite sequence? Explain your reasoning.



NOTES

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Searching for a Sequence

In this lesson you have seen that many different patterns can generate a sequence of numbers.

1. **Explain why the definition of a function applies to all sequences.**

2. **Create a sequence to fit the given criteria. Describe your sequence using figures, words, or numbers. Provide the first four terms of the sequence. Explain how you know that it is a sequence.**

   a. **Create a sequence that begins with a positive integer, is decreasing by multiplication, and is finite.**

   b. **Create a sequence that begins with a negative rational number, is increasing by addition, and is infinite.**