## Lesson 2

# The Password Is... Operations!:

### Arithmetic and Geometric Sequences

### Lesson Overview

Given 16 numeric sequences, students generate several additional terms for each sequence and describe the rule they used for each sequence. They sort the sequences into groups based upon common characteristics of their choosing and explain their rationale. The terms *arithmetic sequence*, *common difference*, *geometric sequence*, and *common ratio* are then defined, examples are provided, and students respond to clarifying questions. They then categorize the sequences from the beginning of the lesson as arithmetic, geometric, or neither and identify the common difference or common ratio where appropriate. Students begin to identify four different representations for each arithmetic and geometric sequence. They identify each arithmetic and geometric sequence in the first activity, and in the second activity, the corresponding graph is added. The remaining representations are completed in the following lessons. This lesson concludes with students writing sequences given a first term and a common difference or common ratio and identifying whether the sequences are arithmetic or geometric.

**Lesson Video(s):** The aligned lesson overview video(s) provide additional instruction for students on the key concepts in this lesson and can be found alongside the digital interactive student lesson.

**TEKS:** A.12A, A.12D

## Lesson Structure and Pacing: 2 Days

### Day 1

**Engage**

Getting Started: What comes Next, and How Do You Know?

**Develop**

Activity 2.1: Defining Arithmetic and Geometric Sequences

### Day 2

Activity 2.2: Matching Graphs and Sequences

**Demonstrate**

Talk the Talk: Name That Sequence!

# Getting Started: What Comes Next, and How Do You Know?

## Asynchronous Facilitation Notes

In this activity, students generate additional terms for 16 different numeric sequences, and then they describe the rule they used for each sequence by filling in blanks in a provided table. Students sort the sequences based upon common characteristics and identify the mathematical operations they used to determine the next terms in each sequence in free response questions.

## Synchronous Facilitation Notes

In this activity, students generate additional terms for 16 different numeric sequences, and then describe the rule they used for each sequence. A sort activity is used to categorize the sequences based upon common characteristics.

Have students work with a partner or in a group to complete Questions 1 through 3. Make sure that students understand that they are just describing a pattern; they do not have to write a rule. Share responses as a class.

**Differentiation strategy**

To scaffold instruction, reduce the number of sequences while maintaining variety.

**As students work, look for**

Strategies and phrases they use to determine the next terms of the sequences.

**Questions to ask**

- How did you determine the next term in the sequence?
- Is there another rule that can be used to determine that same sequence?
- Is the sequence increasing or decreasing? How do you know?
- How many sequences involve addition or subtraction?
- Which sequences involve addition by the same number each time?
- Which sequences involve addition by numbers in a pattern each time?
- How many sequences involve multiplication or division?
- What other operations are used to generate the sequences?

## Summary

Different operations can be used to generate sequences.

# Activity 2.1:
## Defining Arithmetic and Geometric Sequences

## Asynchronous Facilitation Notes

In this activity, students are provided the definitions of *arithmetic sequence*, *common difference*, *geometric sequence*, and *common ratio*. Examples are provided, and students respond to clarifying free response questions. They also categorize the sequences from the

beginning of the lesson as arithmetic, geometric, or neither in multiple choice questions and identify the common difference or common ratio where appropriate in fill in the blank tables.

## Synchronous Facilitation Notes

In this activity, students are provided the definitions of *arithmetic sequence*, *common difference*, *geometric sequence*, and *common ratio*. Examples are provided, and students respond to clarifying questions. They then categorize the sequences from the beginning of the lesson as arithmetic, geometric, or neither and identify the common difference or common ratio where appropriate.

Ask a student to read the introduction and definitions aloud. Review the Worked Example as a class. Have students work individually or with a partner to complete Question 1 and discuss as a class. Then have students work with a partner or in a group to complete Question 2. Share responses as a class.

**Misconception**

Students may confuse the term *arithmetic* (noun) with the term *arithmetic* (adjective). Emphasize how to pronounce *arithmetic* when it is an adjective rather than a noun.

**Questions to ask for Question 1**

- Think about a sequence such as 1, 2, 3, 4 . . . where $x$ is any real number. Is there a difference between adding a negative $x$ to each term of the sequence and subtracting a positive $x$ from each term of the sequence?
- Is there a difference between adding 2 to each term of the sequence and subtracting 2 from each term in the sequence?
- If the common difference of the sequence is 4, how would you describe the rule used to generate the next terms using addition?
- If the common difference of the sequence is 4, how would you describe the rule used to generate the next terms using subtraction?

**Questions to ask for Question 2**

- How many of the sixteen sequences used a rule that is described by the use of addition or subtraction?
- How is the common difference evident in the description of each pattern?

Ask a student to read the definitions following Question 2 aloud. Review the Worked Example as a class.

Have students work with a partner or in a group to complete Questions 3 through 5. Share responses as a class.

**Misconception**

Students already have an understanding of the terms *arithmetic* and *geometry*. Address how previous use of these terms is the same and different as how they are used with sequences.

**Questions to ask**

- Explain the difference between a common ratio and a common difference.
- If the common ratio is changed from 2 to 3, will the terms increase more rapidly or more slowly? Why? Is the new sequence increasing or decreasing?
- If the common ratio is changed from 2 to 3, what will be the first 5 terms? Is the new sequence increasing or decreasing?
- If the common ratio is changed from 2 to $\frac{1}{3}$, will the terms increase more rapidly or more slowly? Why? Is the new sequence increasing or decreasing?
- If the common ratio is changed from 2 to $-2$, will the terms increase more rapidly or more slowly? Why? Is the new sequence increasing or decreasing?
- Is the common ratio of a sequence the number which each term is divided by or multiplied by?

Have students work with a partner or in a group to complete Questions 6 through 12. Share responses as a class.

**Questions to ask**

- Is each term of this sequence multiplied by 3 or multiplied by $\frac{1}{3}$ ?
- How many of the sixteen sequences used a rule that is described by the use of multiplication?
- Is the common ratio stated in the description of each pattern? Where?
- Can you think of a sequence that is different from Dante's and Kira's?
- Describe a third sequence that would also begin with these first two terms. How would you describe the pattern? Does it have a common ratio or a common difference?
- Is there a different arithmetic sequence that satisfies these first two terms?
- Is there a different geometric sequence that satisfies these first two terms?
- Are all sequences considered either geometric or arithmetic sequences? Why or why not?
- If every term in a sequence is the same number, what is the common difference?

### Summary

An arithmetic sequence is a sequence of numbers in which a positive or negative constant, called the constant difference, is added to each term to produce the next term. A geometric sequence is a sequence of numbers in which you multiply each term by a constant, called the common ratio, to determine the next term.

## Activity 2.2:
### Matching Graphs and Sequences

### Asynchronous Facilitation Notes

In this activity, students match several graphs to the appropriate numeric sequence by filling in the corresponding graph number for each sequence in a table. Students also explain the

strategies they used to match each graph and describe how to verify that all sequences are functions in free response questions.

### Synchronous Facilitation Notes

In this activity, students match several graphs to the appropriate numeric sequence.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

**Questions to ask**

- Which graphs appear to be linear? What information does this give you about the sequence?
- Which graphs appear to be exponential? What information does this give you about the sequence?
- Which graphs appear to be increasing? What information does this give you about the sequence?
- Which graphs appear to be decreasing? What information does this give you about the sequence?
- How can determining the bounds of the *y*-axis be helpful in matching the graphs to the appropriate sequence?
- How can determining the *y*-intercept be helpful in matching the graphs to the appropriate sequence?
- How can the coordinates of the first term be helpful in matching the graphs to the appropriate sequence?

### Summary

All sequences are functions. The graph of a sequence is a set of discrete points. The points of an arithmetic sequence lie on a line. When the common difference is a positive, the graph is increasing, and when the common difference is a negative, the graph is decreasing. The points of a geometric sequence do not lie on a line. When the common ratio is greater than 1, the graph is increasing; when the common ratio is between 0 and 1, the graph is decreasing; and when the common ratio is less than 0, the graph alternates between increasing and decreasing between consecutive points.

## Talk the Talk: Name That Sequence!

### Asynchronous Facilitation Notes

In this activity, students are given a first term and a common difference or common ratio. Using those criteria, they write the first five terms of a unique sequence and state whether the sequence is arithmetic or geometric in free response questions.

## Synchronous Facilitation Notes

In this activity, students are given a first term and a common difference or common ratio. Using those criteria, they write the first five terms of a unique sequence and state whether the sequence is arithmetic or geometric.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

**Differentiation strategy**

To extend the activity, have students design their own problems.

- Ask students to write a first term and either common difference or common ratio. Give the information to their partner and ask them to generate the first few terms in the sequence.
- Ask students to create a sequence using their own rule, then ask their partner to identify the rule.

**Questions to ask**

- What two pieces of information are needed to generate a sequence?
- Explain why this information always provides a unique sequence.
- How can you determine whether a sequence is arithmetic or geometric from the sequence of numbers? From its graph?

## Summary

A unique sequence can be described by a first term and common difference or common ratio.