2

The Password Is. . . Operations!

Arithmetic and Geometric Sequences

Warm Up

Write the next three terms in each sequence and explain how you generated each term.

1. $-2, 4, -8, 16, \ldots$

2. $60, 53, 46, 39, 32, \ldots$

3. $1, 5, 17, 53, 161, 485, \ldots$

4. $4, 10, 16, 22, \ldots$

Learning Goals

- Determine the next term in a sequence.
- Recognize arithmetic sequences and geometric sequences.
- Determine the common difference or common ratio for a sequence.
- Graph arithmetic and geometric sequences.
- Recognize graphical behavior of sequences.
- Sort sequences that are represented graphically.

Key Terms

- arithmetic sequence
- common difference
- geometric sequence
- common ratio

You have represented patterns as sequences of numbers—a relationship between term numbers and term values. What patterns appear when sequences are represented as graphs?

What Comes Next, and How Do You Know?

Cut out Sequences A through P located at the end of the lesson.

1. **Determine the unknown terms of each sequence. Then describe the pattern under each sequence.**

2. **Sort the sequences into groups based on common characteristics. In the space provided, record the following information for each of your groups.**

   - **List the letters of the sequences in each group.**

   - **Provide a rationale as to why you created each group.**

3. **What mathematical operation(s) did you perform in order to determine the next terms of each sequence?**

Defining Arithmetic and Geometric Sequences

For some sequences, you can describe the pattern as adding a constant to each term to determine the next term. For other sequences, you can describe the pattern as multiplying each term by a constant to determine the next term. Still other sequences cannot be described either way.

An **arithmetic sequence** is a sequence of numbers in which the difference between any two consecutive terms is a constant. In other words, it is a sequence of numbers in which a constant is added to each term to produce the next term. This constant is called the **common difference**. The common difference is typically represented by the variable d.

The common difference of a sequence is positive if the same positive number is added to each term to produce the next term. The common difference of a sequence is negative if the same negative number is added to each term to produce the next term.

Remember:

When you add a negative number, it is the same as subtracting a positive number.

Worked Example

Consider the sequence shown.

$$11, 9, 7, 5, \ldots$$

The pattern is to add the same negative number, -2, to each term to determine the next term.

Sequence: ___11___, ___9___, ___7___, ___5___, . . .

This sequence is arithmetic and the common difference d is -2.

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1. Suppose a sequence has the same starting number as the sequence in the worked example, but its common difference is 4.

   a. How would the pattern change?

   b. Is the sequence still arithmetic? Why or why not?

   c. If possible, write the first 5 terms of the new sequence.

2. Analyze the sequences you cut out in the Getting Started.

   a. List the sequences that are arithmetic.

   b. Write the common difference of each arithmetic sequence you identified.

A **geometric sequence** is a sequence of numbers in which the ratio between any two consecutive terms is a constant. In other words, it is a sequence of numbers in which you multiply each term by a constant to determine the next term. This integer or fraction constant is called the common ratio. The **common ratio** is represented by the variable r.

Worked Example

Consider the sequence shown.

$$1, 2, 4, 8, \ldots$$

The pattern is to multiply each term by the same number, 2, to determine the next term.

| | multiply by 2 | multiply by 2 | multiply by 2 |

Sequence: 1 ,   2 ,   4 ,   8 , . . .
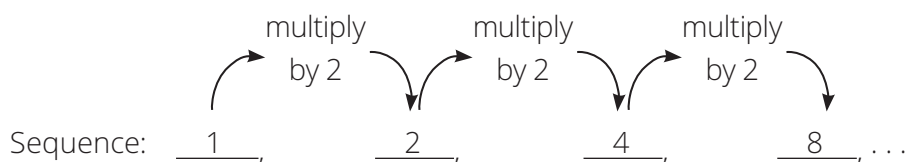
This sequence is geometric and the common ratio r is 2.

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3. **Suppose a sequence has the same starting number as the sequence in the worked example, but its common ratio is 3.**

   a. **How would the pattern change?**

   b. **Is the sequence still geometric? Explain your reasoning.**

   c. **Write the first 5 terms of the new sequence.**

4. Suppose a sequence has the same starting number as the sequence in the worked example, but its common ratio is $\frac{1}{3}$.

   a. How would the pattern change?

   b. Is the sequence still geometric? Why or why not?

   c. Write the first 6 terms of the new sequence.

5. Suppose a sequence has the same starting number as the sequence in the worked example, but its common ratio is −2.

   a. How would the pattern change?

   b. Is the sequence still geometric? Explain your reasoning.

   c. Write the first 6 terms of the new sequence.

6. Consider the sequence shown.

$$270, 90, 30, 10, \ldots$$

Devon says that he can determine each term of this sequence by multiplying each term by $\frac{1}{3}$, so the common ratio is $\frac{1}{3}$. Chase says that he can determine each term of this sequence by dividing each term by 3, so the common ratio is 3. Who is correct? Explain your reasoning.

7. Consider the sequences you cut out in the Getting Started. List the sequences that are geometric. Then write the common ratio on each Sequence Card.

8. Consider the sequences that are neither arithmetic nor geometric. List these sequences. Explain why these sequences are neither arithmetic nor geometric.

9. Consider the first two terms of the sequence 3, 6, . . .

Dante says, "This is how I wrote the sequence for the given terms."

$$3, 6, 9, 12, . . .$$

Kira says, "This is the sequence I wrote."

$$3, 6, 12, 24, . . .$$

Who is correct? Explain your reasoning.

10. Using the terms given in Question 9, write a sequence that is neither arithmetic nor geometric. Then, have your partner tell you what the pattern is in your sequence.

11. How many terms did your partner need before the pattern was recognized?

12. Consider the sequence 2, 2, 2, 2, 2. . . Identify the type of sequence it is and describe the pattern.

13. Begin to complete the graphic organizers located at the end of the lesson to identify arithmetic and geometric sequences. Glue each arithmetic sequence and each geometric sequence to a separate graphic organizer according to its type. Discard all other sequences.

Matching Graphs and Sequences

As you have already discovered when studying functions, graphs can help you see trends of a sequence—and at times can help you predict the next term in a sequence.

1. **The graphs representing the arithmetic and geometric sequences from the previous activity are located at the end of this lesson. Cut out these graphs. Match each graph to its appropriate sequence and glue it into the Graph section of its graphic organizer.**

2. **What strategies did you use to match the graphs to their corresponding sequences?**

3. **How can you use the graphs to verify that all sequences are functions?**

TALK the TALK

Name That Sequence!

Write the first five terms of each sequence described and identify the sequence as arithmetic or geometric.

1. **The first term of the sequence is 8 and the common difference is 12.**

2. **The first term of the sequence is −9 and the common ratio is −2.**

3. **The first term of the sequence is 0 and the common difference is −6.**

4. **The first term of the sequence is −3 and the common ratio is $-\frac{1}{4}$.**

Sequence Cards

A

45, 90, 180, 360, _____, _____,

_____, . . .

B

−4, −2, 0, 2, _____, _____,

_____, . . .

C

−2, −6, −18, −54, _____, _____,

_____, . . .

D

2, 5, 10, 17, _____, _____,

_____, . . .

E

$4, \frac{7}{4}, -\frac{1}{2}, -\frac{11}{4}$ _____, _____,

_____, . . .

F

1234, 123.4, 12.34, 1.234, _____,

_____, _____, . . .

G

1, −2, 3, −4, 5 _____, _____,

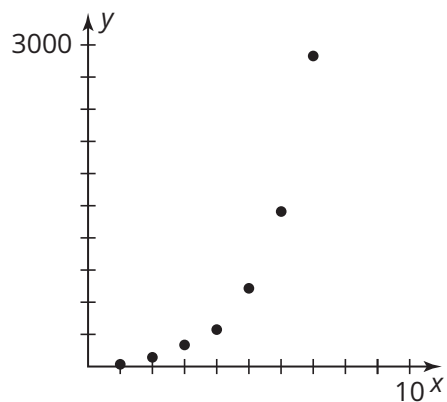_____, _____, . . .

H

−20, −16, −12, −8, −4, _____,

_____, _____, . . .

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So you can cut out the Sequence Cards on the other side.

I

−1, 2, 7, 14, _____, _____, . . .

J

−5, $-\frac{5}{2}$, $-\frac{5}{4}$, $-\frac{5}{8}$, _____, _____,

_____, . . .

K

6.5, 5, 3.5, 2, _____, _____,

_____, . . .

L

86, 85, 83, 80, 76, _____, _____, . . .

M

−16, 4, −1, $\frac{1}{4}$, _____, _____, . . .

N

1473.2, 1452.7, 1432.2, 1411.7,

_____, _____, _____, . . .

O

$\sqrt{5}$, 2, $\sqrt{3}$, $\sqrt{2}$, 1, 0, $\sqrt{-1}$, _____,

_____, . . .

P

−4, 12, −36, 108, _____, _____, . . .

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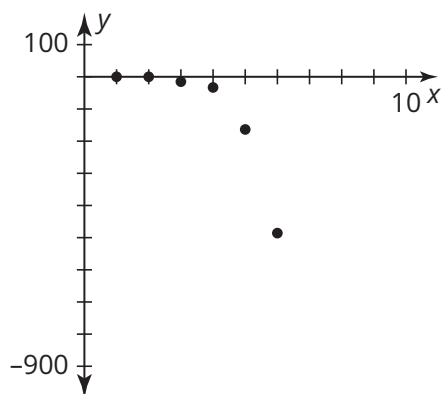
So you can cut out the Sequence Cards on the other side.
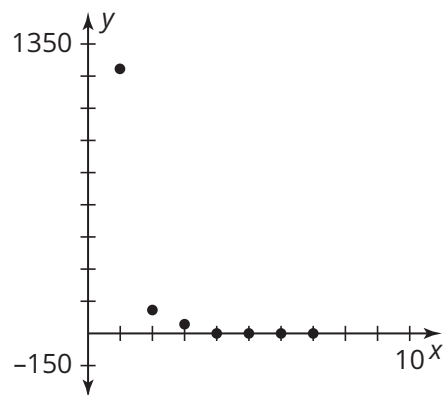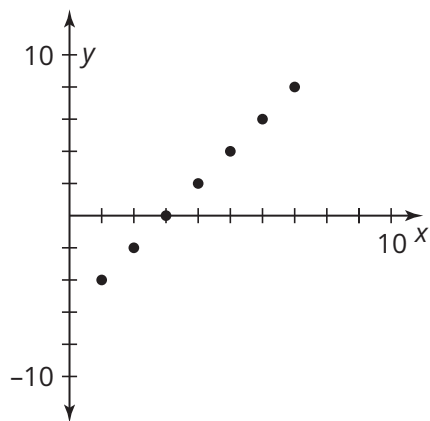
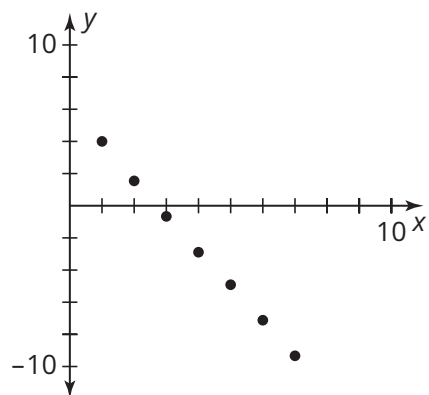# Graph Cards
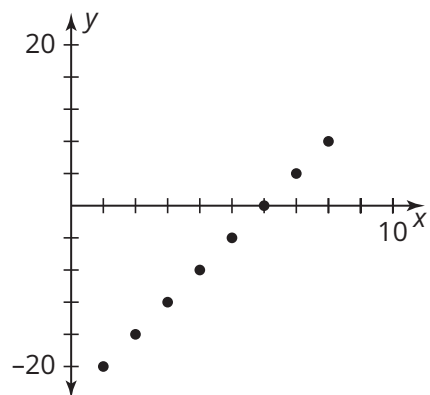
Graph 1

Graph 2

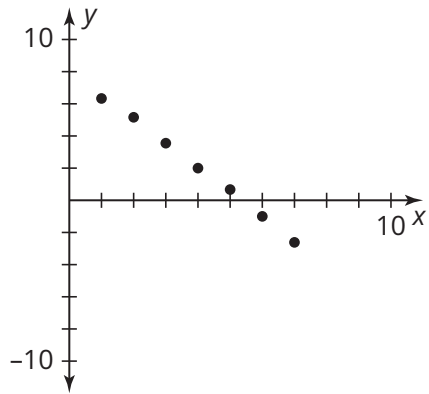**Graph 3**

Graph 4

Graph 5

Graph 6

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So you can cut out the Graph Cards on the other side.

Graph 7

Graph 8
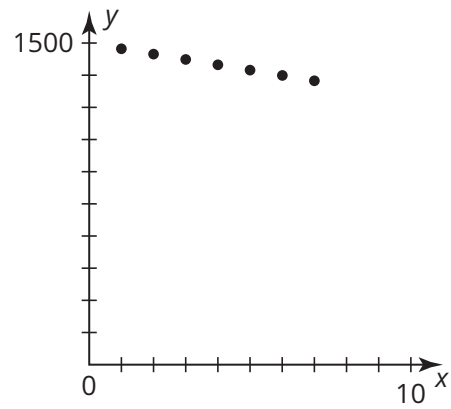
Graph 9
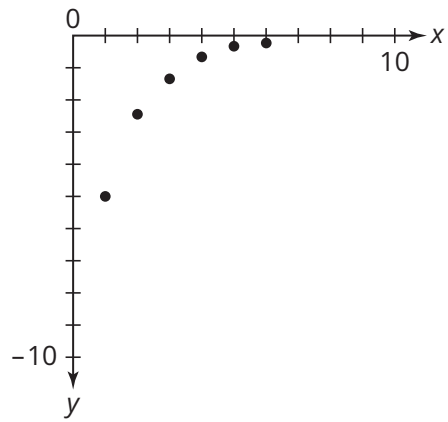
Graph 10
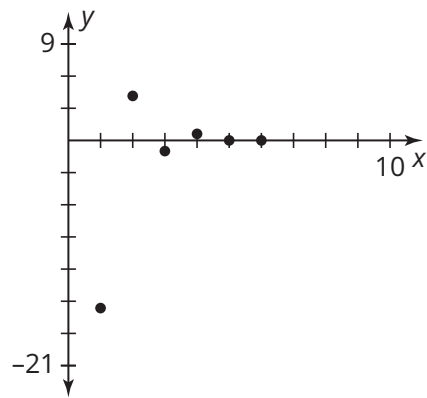
Graph 11

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So you can cut out the Graph Cards on the other side.

Sequence

Graph

Arithmetic Sequence

Recursive Formula

Explicit Formula

Sequence

Graph

Arithmetic Sequence

Recursive Formula

Explicit Formula

Sequence

Graph

Arithmetic Sequence

Recursive Formula

Explicit Formula

Sequence

Graph

Arithmetic Sequence

Recursive Formula

Explicit Formula

Sequence

Graph

Arithmetic Sequence

Recursive Formula

Explicit Formula

Sequence

Graph

Geometric Sequence

Recursive Formula

Explicit Formula

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Recursive Formula

Explicit Formula