

The Password Is... Operations!

Arithmetic and Geometric Sequences

MATERIALS

Scissors
Glue

Lesson Overview

Given 16 numeric sequences, students generate several additional terms for each sequence and describe the rule they used for each sequence. They sort the sequences into groups based upon common characteristics of their choosing and explain their rationale. The terms *arithmetic sequence*, *common difference*, *geometric sequence*, and *common ratio* are then defined, examples are provided, and students respond to clarifying questions. They then categorize the sequences from the beginning of the lesson as arithmetic, geometric, or neither and identify the common difference or common ratio where appropriate. Students begin to create graphic organizers, identifying four different representations for each arithmetic and geometric sequence. In the first activity, they glue each arithmetic and geometric sequence to a separate graphic organizer and label them, and in the second activity, the corresponding graph is added. The remaining representations are completed in the following lessons. This lesson concludes with students writing sequences given a first term and a common difference or common ratio and identifying whether the sequences are arithmetic or geometric.

Algebra 1

Number and Algebraic Methods

(12) The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:

- (A) decide whether relations represented verbally, tabularly, graphically, and symbolically define a function.
- (D) write a formula for the n^{th} term of arithmetic and geometric sequences, given the value of several of their terms.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G

Essential Ideas

- An arithmetic sequence is a sequence of numbers in which the difference between any two consecutive terms is a positive or negative constant. This constant is called the common difference and is represented by the variable d .
- A geometric sequence is a sequence of numbers in which the ratio between any two consecutive terms is a constant. This constant is called the common ratio and is represented by the variable r .
- The graph of a sequence is a set of discrete points.
- The points of an arithmetic sequence lie on a line. When the common difference is a positive, the graph is increasing, and when the common difference is a negative, the graph is decreasing.
- The points of a geometric sequence do not lie on a line. When the common ratio is greater than 1, the graph is increasing; when the common ratio is between 0 and 1, the graph is decreasing; and when the common ratio is less than 0, the graph alternates between increasing and decreasing between consecutive points.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: What Comes Next, and How Do You Know?

Students generate several additional terms for 16 different numeric sequences and describe the rule they used for each sequence. They sort the sequences into groups based upon common characteristics of their choosing and explain their rationale.

Develop

Activity 2.1: Defining Arithmetic and Geometric Sequences

Students are provided the definitions of *arithmetic sequence*, *common difference*, *geometric sequence*, and *common ratio*. Examples are provided, and students respond to clarifying questions. They then categorize the sequences from the beginning of the lesson as arithmetic, geometric, or neither and identify the common difference or common ratio where appropriate. Students begin to create graphic organizers, identifying four different representations for each sequence. In this activity, students glue each arithmetic and geometric sequence to a separate graphic organizer.

Day 2

Activity 2.2: Matching Graphs and Sequences

Students match graphs to their corresponding numeric sequence and then add the graphs to each graphic organizer.

Demonstrate

Talk the Talk: Name That Sequence!

Students are given a first term and a common difference or common ratio, and they must identify the unique sequence it describes and state whether the sequence is arithmetic or geometric.

Getting Started: What Comes Next, and How Do You Know?

ENGAGE

Facilitation Notes

In this activity, students cut out sequence cards, generate additional terms for 16 different numeric sequences, and then describe the rule they used for each sequence. A sort activity is used to categorize the sequences based upon common characteristics.

Have students work with a partner or in a group to complete Questions 1 through 3. Make sure that students understand that they are just describing a pattern; they do not have to write a rule. Share responses as a class.

Differentiation strategy

To scaffold support, reduce the number of sequences while maintaining variety.

As students work, look for

Strategies and phrases they use to determine the next terms of the sequences.

Questions to ask

- How did you determine the next term in the sequence?
- Is there another rule that can be used to determine that same sequence?
- Is the sequence increasing or decreasing? How do you know?
- How many sequences involve addition or subtraction?
- Which sequences involve addition by the same number each time?
- Which sequences involve addition by numbers in a pattern each time?
- How many sequences involve multiplication or division?
- What other operations are used to generate the sequences?

Summary

Different operations can be used to generate sequences.

Activity 2.1 Defining Arithmetic and Geometric Sequences



DEVELOP

Facilitation Notes

In this activity, students are provided the definitions of *arithmetic sequence*, *common difference*, *geometric sequence*, and *common ratio*. Examples are provided, and students respond to clarifying questions. They then

categorize the sequences from the beginning of the lesson as arithmetic, geometric, or neither and identify the common difference or common ratio where appropriate. Students begin to create graphic organizers, identifying four different representations for each sequence. In this activity, students glue each arithmetic and geometric sequence to a separate graphic organizer.

Ask a student to read the introduction and definitions aloud. Review the Worked Example as a class. Have students work individually or with a partner to complete Question 1 and discuss as a class. Then have students work with a partner or in a group to complete Question 2. Share responses as a class

Misconception

Students may confuse the term *arithmetic* (noun) with the term *arithmetic* (adjective). Emphasize how to pronounce *arithmetic* when it is an adjective rather than a noun.

Questions to ask for Question 1

- Think about a sequence such as $1, 2, 3, 4 \dots$ where x is any real number. Is there a difference between adding a negative x to each term of the sequence and subtracting a positive x from each term of the sequence?
- Is there a difference between adding 2 to each term of the sequence and subtracting 2 from each term in the sequence?
- If the common difference of the sequence is 4, how would you describe the rule used to generate the next terms using addition?
- If the common difference of the sequence is 4, how would you describe the rule used to generate the next terms using subtraction?

Questions to ask for Question 2

- How many of the sixteen sequences used a rule that is described by the use of addition or subtraction?
- How is the common difference evident in the description of each pattern?

Ask a student read the definitions following Question 2 aloud. Review the worked example as a class.

Have students work with a partner or in a group to complete Questions 3 through 5. Share responses as a class.

Misconception

Students already have an understanding of the terms *arithmetic* and *geometry*. Address how previous use of these terms is the same and different as how they are used with sequences.

Questions to ask

- Explain the difference between a common ratio and a common difference.
- If the common ratio is changed from 2 to 3, will the terms increase more rapidly or more slowly? Why? Is the new sequence increasing or decreasing?
- If the common ratio is changed from 2 to 3, what will be the first 5 terms? Is the new sequence increasing or decreasing?
- If the common ratio is changed from 2 to $\frac{1}{3}$, will the terms increase more rapidly or more slowly? Why? Is the new sequence increasing or decreasing?
- If the common ratio is changed from 2 to -2 , will the terms increase more rapidly or more slowly? Why? Is the new sequence increasing or decreasing?
- Is the common ratio of a sequence the number which each term is divided by or multiplied by?

Have students work with a partner or in a group to complete Questions 6 through 13. Share responses as a class.

Questions to ask

- Is each term of this sequence multiplied by 3 or multiplied by $\frac{1}{3}$?
- How many of the sixteen sequences used a rule that is described by the use of multiplication?
- Is the common ratio stated in the description of each pattern? Where?
- Can you think of a sequence that is different than Dante's and Kira's?
- Describe a third sequence that would also begin with these first two terms. How would you describe the pattern? Does it have a common ratio or a common difference?
- Is there a different arithmetic sequence that satisfies these first two terms?

- Is there a different geometric sequence that satisfies these first two terms?
- Are all sequences considered either geometric or arithmetic sequences? Why or why not?
- If every term in a sequence is the same number, what is the common difference?

Summary

An arithmetic sequence is a sequence of numbers in which a positive or negative constant, called the constant difference, is added to each term to produce the next term. A geometric sequence is a sequence of numbers in which you multiply each term by a constant, called the common ratio, to determine the next term.

Activity 2.2

Matching Graphs and Sequences



Facilitation Notes

In this activity, students cut out and match several graphs to the appropriate numeric sequence and then attach the graphs to each graphic organizer.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Questions to ask

- Which graphs appear to be linear? What information does this give you about the sequence?
- Which graphs appear to be exponential? What information does this give you about the sequence?
- Which graphs appear to be increasing? What information does this give you about the sequence?
- Which graphs appear to be decreasing? What information does this give you about the sequence?
- How can determining the bounds of the y -axis be helpful in matching the graphs to the appropriate sequence?
- How can determining the y -intercept be helpful in matching the graphs to the appropriate sequence?
- How can the coordinates of the first term be helpful in matching the graphs to the appropriate sequence?

Summary

All sequences are functions. The graph of a sequence is a set of discrete points. The points of an arithmetic sequence lie on a line. When the common difference is a positive, the graph is increasing, and when the common difference is a negative, the graph is decreasing. The points of a geometric sequence do not lie on a line. When the common ratio is greater than 1, the graph is increasing; when the common ratio is between 0 and 1, the graph is decreasing; and when the common ratio is less than 0, the graph alternates between increasing and decreasing between consecutive points.

Talk the Talk: Name That Sequence!

DEMONSTRATE

Facilitation Notes

In this activity, students are given a first term and a common difference or common ratio. Using those criteria, they write the first five terms of a unique sequence and state whether the sequence is arithmetic or geometric.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

Differentiation strategy

To extend the activity, have students design their own problems.

- Ask students to write a first term and either common difference or common ratio. Give the information to their partner and ask them to generate the first few terms in the sequence.
- Ask students to create a sequence using their own rule, then ask their partner to identify the rule.

Questions to ask

- What two pieces of information are needed to generate a sequence?
- Explain why this information always provides a unique sequence.
- How can you determine whether a sequence is arithmetic or geometric from the sequence of numbers? From its graph?

Summary

A unique sequence can be described by a first term and common difference or common ratio.

NOTES

The Password Is... Operations!

Arithmetic and Geometric Sequences

2

Warm Up

Write the next three terms in each sequence and explain how you generated each term.

1. $-2, 4, -8, 16, \dots$
2. $60, 53, 46, 39, 32, \dots$
3. $1, 5, 17, 53, 161, 485, \dots$
4. $4, 10, 16, 22, \dots$

Learning Goals

- Determine the next term in a sequence.
- Recognize arithmetic sequences and geometric sequences.
- Determine the common difference or common ratio for a sequence.
- Graph arithmetic and geometric sequences.
- Recognize graphical behavior of sequences.
- Sort sequences that are represented graphically.

Key Terms

- arithmetic sequence
- common difference
- geometric sequence
- common ratio

You have represented patterns as sequences of numbers—a relationship between term numbers and term values. What patterns appear when sequences are represented as graphs?

Warm Up Answers

1. $-32, 64, -128$;
multiply the previous term by -2
2. $25, 18, 11$;
subtract 7 from the previous term
3. $1457, 4373, 13,121$;
multiply the previous term by 3, then add 2
4. $28, 34, 40$;
add 6 to the previous term

1. A: 720, 1440, 2880;
multiply by 2
B: 4, 6, 8; add 2
C: $-162, -486, -1458$;
multiply by 3
D: 26, 37, 50; square the
term and then add 1
E: $-5, -\frac{29}{4}, -\frac{19}{2}$;
subtract $\frac{9}{4}$
F: 0.1234, 0.01234,
0.001234; multiply by 0.1
G: $-6, 7, -8, 9$;
consecutive numbers,
every other number
negative
H: 0, 4, 8; add 4
I: 23, 34; add consecutive
odd numbers
J: $-\frac{5}{16}, -\frac{5}{32}$; multiply by $\frac{1}{2}$
K: 0.5, $-1, -2.5$; subtract
1.5
L: 71, 65; subtract 1,
then 2, then 3,...
M: $-\frac{1}{16}, \frac{1}{64}$; divide by -4
N: 1391.2, 1370.7,
1350.2; subtract 20.5
O: $\sqrt{2}, \sqrt{3}$; square
roots of decreasing
consecutive integers
P: $-324, 972$; multiply
by -3
2. Sample answer.
A, C, F, J, M, and P;
sequences that change
by multiplying or
dividing by the same
number each time
B, E, H, K, and N;
sequences that change
by adding or subtracting
by the same number
each time
D, G, I, L, and O:
sequences that change
in some other way
3. Some sequences
required addition,

What Comes Next, and How Do You Know?

Cut out Sequences A through P located at the end of the lesson.

1. Determine the unknown terms of each sequence. Then describe the pattern under each sequence.
2. Sort the sequences into groups based on common characteristics. In the space provided, record the following information for each of your groups.
 - List the letters of the sequences in each group.
 - Provide a rationale as to why you created each group.
3. What mathematical operation(s) did you perform in order to determine the next terms of each sequence?

2 • TOPIC 2: Sequences

subtraction, multiplication or division by the same number each time. Some sequences involved operations such as squaring or taking the square root, operations with consecutive numbers, or switching signs each time.

ELL Tip

Review the term *rationale* and create a list of synonyms for the term. Ask students for examples of when *rationale* is used in different contexts.

ACTIVITY
2.1

Defining Arithmetic and Geometric Sequences



For some sequences, you can describe the pattern as adding a constant to each term to determine the next term. For other sequences, you can describe the pattern as multiplying each term by a constant to determine the next term. Still other sequences cannot be described either way.

An **arithmetic sequence** is a sequence of numbers in which the difference between any two consecutive terms is a constant. In other words, it is a sequence of numbers in which a constant is added to each term to produce the next term. This constant is called the **common difference**. The common difference is typically represented by the variable d .

The common difference of a sequence is positive if the same positive number is added to each term to produce the next term. The common difference of a sequence is negative if the same negative number is added to each term to produce the next term.

Remember:

When you add a negative number, it is the same as subtracting a positive number.

Worked Example

Consider the sequence shown.

11, 9, 7, 5, ...

The pattern is to add the same negative number, -2 , to each term to determine the next term.

Sequence: $\underline{11}, \quad \underline{9}, \quad \underline{7}, \quad \underline{5}, \dots$

Diagram showing the pattern: $11 \xrightarrow{\text{add } -2} 9 \xrightarrow{\text{add } -2} 7 \xrightarrow{\text{add } -2} 5, \dots$

This sequence is arithmetic and the common difference d is -2 .

LESSON 2: The Password Is... Operations! • 3

ELL Tip

Ask students what is meant by *consecutive terms*. Discuss how the word *consecutive* is used in mathematical and non-mathematical situations. Have students provide mathematical examples of *consecutive* and *non-consecutive* terms relating to sequences.

Answers

- 1a. The sequence would increase by 4 instead of decreasing by 2.
- 1b. Yes. The sequence is still arithmetic because the difference between each consecutive term is constant.
- 1c. 11, 15, 19, 23, 27
- 2a. Sequences B, E, H, K, and N
- 2b. Sequence B: $d = 2$
Sequence E: $d = -\frac{9}{4}$
Sequence H: $d = 4$
Sequence K: $d = -1.5$
Sequence N: $d = -20.5$

NOTES

1. Suppose a sequence has the same starting number as the sequence in the worked example, but its common difference is 4.

a. How would the pattern change?

b. Is the sequence still arithmetic? Why or why not?

c. If possible, write the first 5 terms of the new sequence.

2. Analyze the sequences you cut out in the Getting Started.

a. List the sequences that are arithmetic.

b. Write the common difference of each arithmetic sequence you identified.

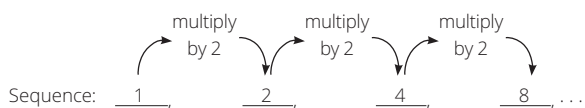
A **geometric sequence** is a sequence of numbers in which the ratio between any two consecutive terms is a constant. In other words, it is a sequence of numbers in which you multiply each term by a constant to determine the next term. This integer or fraction constant is called the common ratio. The **common ratio** is represented by the variable r .

Worked Example

Consider the sequence shown.

1, 2, 4, 8, ...

The pattern is to multiply each term by the same number, 2, to determine the next term.



This sequence is geometric and the common ratio r is 2.

3. Suppose a sequence has the same starting number as the sequence in the worked example, but its common ratio is 3.

a. How would the pattern change?

b. Is the sequence still geometric? Explain your reasoning.

c. Write the first 5 terms of the new sequence.

Answers

3a. The sequence would still increase, but the terms would be different. The sequence would increase more rapidly.

3b. Yes. The sequence is still geometric because the ratio between any two consecutive terms is constant.

3c. 1, 3, 9, 27, 81

Answers

4a. The sequence would decrease.

4b. Yes. The sequence is still geometric because the ratio between any two consecutive terms is constant.

4c. $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \frac{1}{243}$

5a. The sequence would decrease and increase and contain alternating positive and negative integers.

5b. Yes. The sequence is still geometric because the ratio between any two consecutive terms is constant.

5c. $1, -2, 4, -8, 16, -32$

NOTES

4. Suppose a sequence has the same starting number as the sequence in the worked example, but its common ratio is $\frac{1}{3}$.

a. How would the pattern change?

b. Is the sequence still geometric? Why or why not?

c. Write the first 6 terms of the new sequence.

5. Suppose a sequence has the same starting number as the sequence in the worked example, but its common ratio is -2 .

a. How would the pattern change?

b. Is the sequence still geometric? Explain your reasoning.

c. Write the first 6 terms of the new sequence.

6. Consider the sequence shown.

270, 90, 30, 10, . . .

Devon says that he can determine each term of this sequence by multiplying each term by $\frac{1}{3}$, so the common ratio is $\frac{1}{3}$. Chase says that he can determine each term of this sequence by dividing each term by 3, so the common ratio is 3. Who is correct? Explain your reasoning.



7. Consider the sequences you cut out in the Getting Started. List the sequences that are geometric. Then write the common ratio on each Sequence Card.
8. Consider the sequences that are neither arithmetic nor geometric. List these sequences. Explain why these sequences are neither arithmetic nor geometric.

Answers

6. Devon is correct. The next term in the sequence can be determined by multiplying the previous term by $\frac{1}{3}$. Chase is correct in that he can determine the sequence by dividing each term by 3, but the common ratio represents the number by which each term is multiplied. Each term in this sequence is not multiplied by 3, it is multiplied by $\frac{1}{3}$.
7. Sequence A: $r = 2$
Sequence C: $r = 3$
Sequence F: $r = 0.1$
Sequence J: $r = \frac{1}{2}$
Sequence M: $r = -\frac{1}{4}$
Sequence P: $r = -3$
8. Sequences D, G, I, L, and O are neither arithmetic nor geometric because there is no common difference or common ratio for any of these sequences.

Answers

9. Both are correct.
From the first two terms, Dante or Kira did not know whether the sequence was arithmetic or geometric. Dante assumed it was arithmetic with a common difference of 3. Kira assumed it was geometric with a common ratio of 2.
10. Sample answer.
Sequence 3, 6, 9, 15, 24, . . . ; each term is the sum of the two previous terms.
11. Answers will vary.
12. Sample answers.
This sequence could be arithmetic in that you could add 0 to each term.
This sequence could be geometric in that you could multiply each term by 1.
This sequence could be neither arithmetic nor geometric in that the term 2 could just be repeating.
13. Sequence A: geometric
Sequence B: arithmetic
Sequence C: geometric
Sequence E: arithmetic
Sequence F: geometric
Sequence H: arithmetic
Sequence J: geometric
Sequence K: arithmetic
Sequence M: geometric
Sequence N: arithmetic
Sequence P: geometric



9. Consider the first two terms of the sequence 3, 6, . . .

Dante says, "This is how I wrote the sequence for the given terms."

3, 6, 9, 12, . . .

Kira says, "This is the sequence I wrote."

3, 6, 12, 24, . . .

Who is correct? Explain your reasoning.

10. Using the terms given in Question 9, write a sequence that is neither arithmetic nor geometric. Then, have your partner tell you what the pattern is in your sequence.
11. How many terms did your partner need before the pattern was recognized?
12. Consider the sequence 2, 2, 2, 2, 2. . . Identify the type of sequence it is and describe the pattern.
13. Begin to complete the graphic organizers located at the end of the lesson to identify arithmetic and geometric sequences. Glue each arithmetic sequence and each geometric sequence to a separate graphic organizer according to its type. Discard all other sequences.

ACTIVITY
2.2

Matching Graphs and Sequences



As you have already discovered when studying functions, graphs can help you see trends of a sequence—and at times can help you predict the next term in a sequence.

1. The graphs representing the arithmetic and geometric sequences from the previous activity are located at the end of this lesson. Cut out these graphs. Match each graph to its appropriate sequence and glue it into the Graph section of its graphic organizer.

2. What strategies did you use to match the graphs to their corresponding sequences?

3. How can you use the graphs to verify that all sequences are functions?



LESSON 2: The Password Is... Operations! • 9

Answers

1. Sequence A, Graph 1
Sequence B, Graph 4
Sequence C, Graph 2
Sequence E, Graph 5
Sequence F, Graph 3
Sequence H, Graph 6
Sequence J, Graph 9
Sequence K, Graph 7
Sequence M, Graph 10
Sequence N, Graph 8
Sequence P, Graph 11
2. Answers may vary.
3. Sample answer.
The graphs all pass the vertical line test.

Answers

1. 8, 20, 32, 44, 56;
arithmetic
2. -9, 18, -36, 72, -144;
geometric
3. 0, -6, -12, -18, -24;
arithmetic
4. $-3, \frac{3}{4}, -\frac{3}{16}, \frac{3}{64}, -\frac{3}{256}$;
geometric

NOTES

TALK the TALK

Name That Sequence!

Write the first five terms of each sequence described and identify the sequence as arithmetic or geometric.

1. The first term of the sequence is 8 and the common difference is 12.

2. The first term of the sequence is -9 and the common ratio is -2.

3. The first term of the sequence is 0 and the common difference is -6.

4. The first term of the sequence is -3 and the common ratio is $-\frac{1}{4}$.

Sequence Cards



<p>A</p> <p>45, 90, 180, 360, _____, _____,</p> <p>_____, ...</p>	<p>B</p> <p>-4, -2, 0, 2, _____, _____,</p> <p>_____, ...</p>
<p>C</p> <p>-2, -6, -18, -54, _____, _____,</p> <p>_____, ...</p>	<p>D</p> <p>2, 5, 10, 17, _____, _____,</p> <p>_____, ...</p>
<p>E</p> <p>$4, \frac{7}{4}, -\frac{1}{2}, -\frac{11}{4}$ _____, _____,</p> <p>_____, ...</p>	<p>F</p> <p>1234, 123.4, 12.34, 1.234, _____,</p> <p>_____, _____, ...</p>
<p>G</p> <p>1, -2, 3, -4, 5 _____, _____,</p> <p>_____, _____, ...</p>	<p>H</p> <p>-20, -16, -12, -8, -4, _____,</p> <p>_____, _____, ...</p>

Why is this page blank?

So you can cut out the Sequence Cards on the other side.

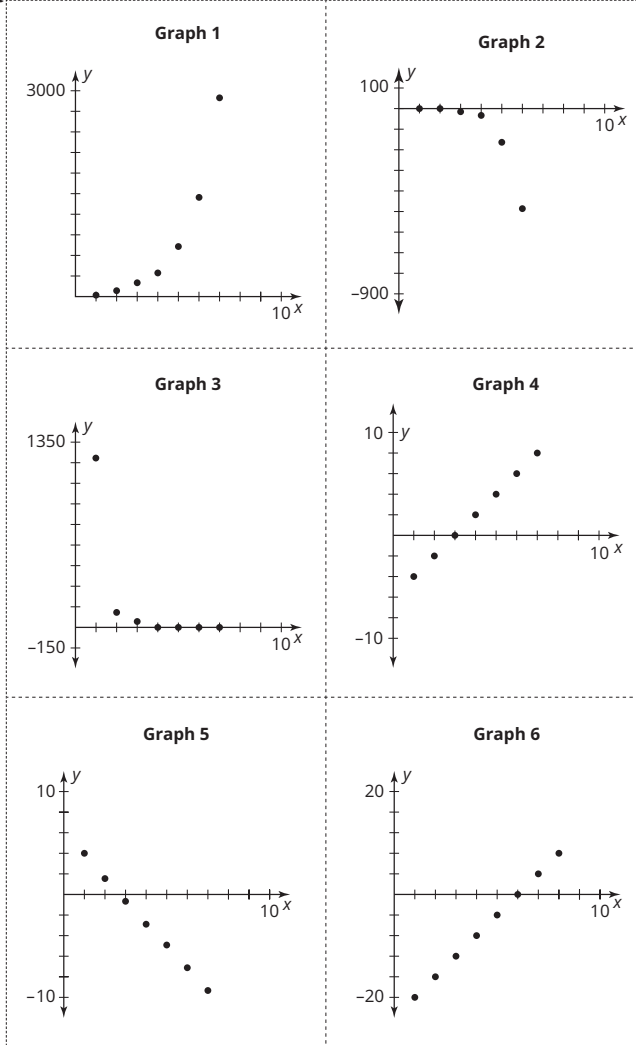


I -1, 2, 7, 14, _____, _____, ...	J $-5, -\frac{5}{2}, -\frac{5}{4}, -\frac{5}{8},$ _____, _____, _____, ...
K 6.5, 5, 3.5, 2, _____, _____, _____, ...	L 86, 85, 83, 80, 76, _____, _____, ...
M $-16, 4, -1, \frac{1}{4},$ _____, _____, ...	N 1473.2, 1452.7, 1432.2, 1411.7, _____, _____, _____, ...
O $\sqrt{5}, 2, \sqrt{3}, \sqrt{2}, 1, 0, \sqrt{-1},$ _____, _____, ...	P -4, 12, -36, 108, _____, _____, ...

Why is this page blank?

So you can cut out the Sequence Cards on the other side.

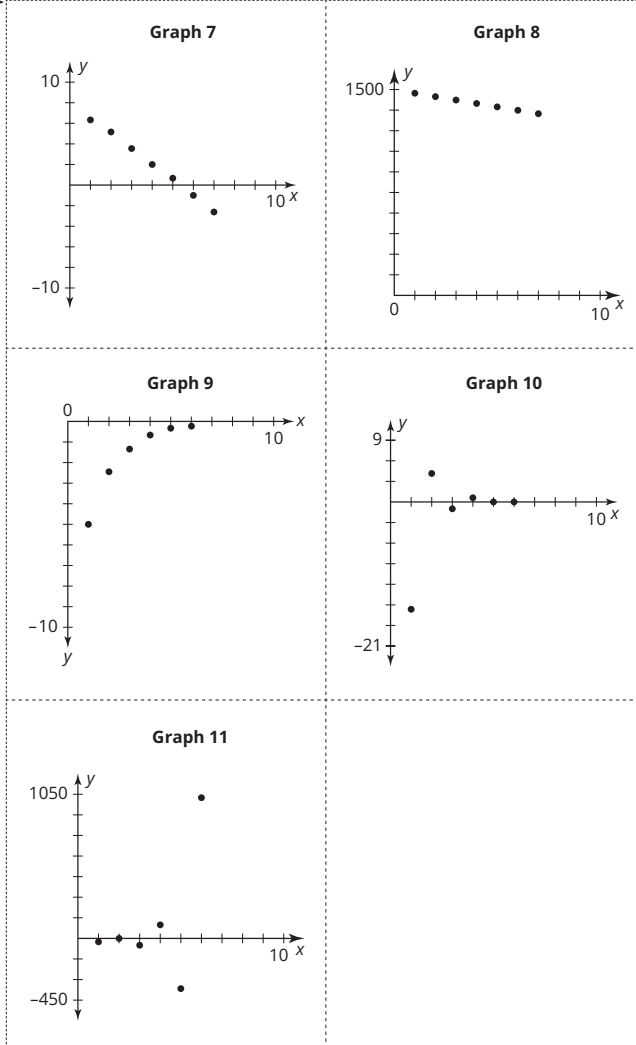
Graph Cards



LESSON 2: The Password Is... Operations! • 15

Why is this page blank?

So you can cut out the Graph Cards on the other side.



Why is this page blank?

So you can cut out the Graph Cards on the other side.

