3

Did You Mean: *Recursion*?

Determining Recursive and Explicit Expressions from Contexts

Warm Up

The local bank has agreed to donate $250 to the annual turkey fund to help feed families in need. In addition, for every bank customer that donates $50, the bank will donate $25.

1. A sequence describes the relationship between the number of $50 donations and the amount of the bank's donation. Is the sequence arithmetic or geometric?
2. How can you calculate the 10th term based on the 9th term?
3. What is the 20th term?

Learning Goals

- Write recursive formulas for arithmetic and geometric sequences from contexts.
- Write explicit expressions for arithmetic and geometric sequences from contexts.
- Use formulas to determine unknown terms of a sequence.

Key Terms

- recursive formula
- explicit formula

You have learned that arithmetic and geometric sequences always describe functions. How can you write equations to represent these functions?

Can I Get a Formula?

While a common ratio or a common difference can help you determine the next term in a sequence, how can they help you determine the thousandth term of a sequence? The ten-thousandth term of a sequence?

Consider the sequence represented in this situation.

Rico owns a sporting goods store. He has agreed to donate $125 to the Centipede Valley High School baseball team for their equipment fund. In addition, he will donate $18 for every home run the Centipedes hit during the season. The sequence shown represents the possible dollar amounts that Rico could donate for the season.

125, 143, 161, 179, . . .

Think about:

Notice that the 1st term in this sequence is the amount Rico donates if the team hits 0 home runs.

Number of Home Runs	Term Number (n)	Donation Amount (dollars)
0	1	
1		
2		
3		
4		
5		
6		
7		
8		
9		

1. **Identify the sequence type. Describe how you know.**

2. **Determine the common difference or common ratio for the sequence.**

3. **Complete the table.**

4. **Explain how you can calculate the tenth term based on the ninth term.**

ACTIVITY
3.1 Writing Formulas for Arithmetic Sequences

A **recursive formula** expresses each new term of a sequence based on the preceding term in the sequence. The recursive formula to determine the *n*th term of an arithmetic sequence is:

$$a_n = \underbrace{a_{n-1}}_{\text{previous term}} + d$$

*n*th term ⟶ a_n ⟵ common difference

You only need to know the previous term and the common difference to use the recursive formula.

Worked Example

Consider the sequence −2, −9, −16, −23, . . .
You can use the recursive formula to determine the 5th term.

$$a_n = a_{n-1} + d$$
$$a_5 = a_{5-1} + (-7)$$

The expression a_5 represents the 5th term. The previous term is −23, and the common difference is −7.

$$a_5 = a_4 + (-7)$$
$$a_5 = -23 + (-7)$$
$$a_5 = -30$$

The 5th term of the sequence is −30.

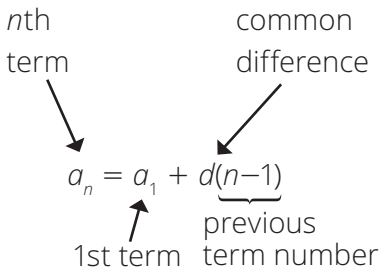
Consider the sequence showing Rico's contribution to the Centipedes baseball team in terms of the number of home runs hit.

1. **Use a recursive formula to determine the 11th term in the sequence. Explain what this value means in terms of this problem situation.**

2. **Is there a way to calculate the 20th term without first calculating the 19th term? If so, describe the strategy.**

You can determine the 93rd term of the sequence by calculating each term before it, and then adding 18 to the 92nd term, but this will probably take a while! A more efficient way to calculate any term of a sequence is to use an *explicit formula*.

An **explicit formula** of a sequence is a formula to calculate the *n*th term of a sequence using the term's position in the sequence. The explicit formula for determining the *n*th term of an arithmetic sequence is:



Remember:

The 1st term in this sequence is the amount Rico donates if the team hits 0 home runs. So, the 93rd term represents the amount Rico donates if the team hits 92 home runs.

Worked Example

You can use the explicit formula to determine the 93rd term in this problem situation.

$$a_n = a_1 + d(n - 1)$$

$$a_{93} = 125 + 18(93 - 1)$$

The expression a_{93} represents the 93rd term. The first term is 125, and the common difference is 18.

$$a_{93} = 125 + 18(92)$$

$$a_{93} = 125 + 1656$$

$$a_{93} = 1781$$

The 93rd term of the sequence is 1781.

This means Rico will contribute a total of $1781 if the Centipedes hit 92 home runs.

NOTES

3. **Use the explicit formula to determine the amount of money Rico will contribute for each number of home runs hit.**

   a. **35 home runs**

   b. **48 home runs**

   c. **86 home runs**

   d. **214 home runs**

Rico decides to increase his initial contribution and amount donated per home run hit. He decides to contribute $500 and will donate $75 for every home run the Centipedes hit.

4. **Write the first 5 terms of the sequence representing the new contribution Rico will donate to the Centipedes.**

5. **Determine Rico's contribution for each number of home runs hit.**

   a. **39 home runs**

   b. **50 home runs**

ACTIVITY
3.2
Writing Formulas for Geometric Sequences

When it comes to bugs, bats, spiders, and—ugh, any other creepy crawlers—finding one in your house is finding one too many! Then again, when it comes to cells, the more the better. Animals, plants, fungi, slime, molds, and other living creatures are composed of eukaryotic cells. During growth, generally there is a cell called a "mother cell" that divides itself into two "daughter cells." Each of those daughter cells then divides into two more daughter cells, and so on.

Notice that the 1st term in this sequence is the total number of cells after 0 divisions (that is, the mother cell).

1. The sequence shown represents the growth of eukaryotic cells.

1, 2, 4, 8, 16, . . .

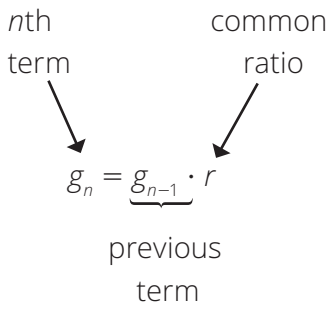
a. Describe why this sequence is geometric and identify the common ratio.

Number of Cell Divisions	Term Number (n)	Total Number of Cells
0	1	
1		
2		
3		
4		
5		
6		
7		
8		
9		

b. Complete the table of values. Use the number of cell divisions to identify the term number and the total number of cells after each division.

c. Explain how you can calculate the tenth term based on the ninth term.

The recursive formula to determine the *n*th term of a geometric sequence is:



Worked Example

Consider the sequence shown.

$$4, 12, 36, 108, \ldots$$

You can use the recursive formula to determine the 5th term.

$$g_n = g_{n-1} \cdot r$$

$$g_5 = g_{5-1} \cdot (3)$$

The expression g_5 represents the 5th term. The previous term is 108, and the common ratio is 3.

$$g_5 = g_4 \cdot (3)$$

$$g_5 = 108 \cdot (3)$$

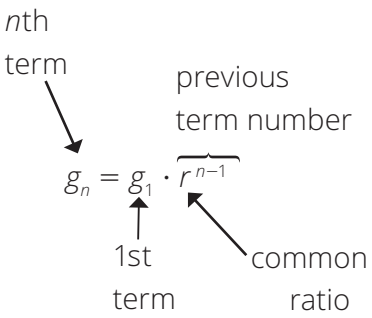
$$g_5 = 324$$

The 5th term of the sequence is 324.

Consider the sequence of cell divisions and the total number of resulting cells.

2. Write a recursive formula for the sequence and use the formula to determine the 12th term in the sequence. Explain what your result means in terms of this problem situation.

The explicit formula to determine the *n*th term of a geometric sequence is:



Remember:

The 1st term in this sequence is the total number of cells after 0 divisions. So, the 20th term represents the total number of cells after 19 divisions.

Worked Example

You can use the explicit formula to determine the 20th term in this problem situation.

$$g_n = g_1 \cdot r^{n-1}$$
$$g_{20} = 1 \cdot 2^{20-1}$$

The expression g_{20} represents the 20th term. The first term is 1, and the common ratio is 2.

$$g_{20} = 1 \cdot 2^{19}$$
$$g_{20} = 1 \cdot 524{,}288$$
$$g_{20} = 524{,}288$$

The 20th term of the sequence is 524,288.

This means that after 19 cell divisions, there are a total of 524,288 cells.

3. **Use the explicit formula to determine the total number of cells for each number of divisions.**

   a. **11 divisions**

   b. **14 divisions**

   c. **18 divisions**

   d. **22 divisions**

Suppose that a scientist has 5 eukaryotic cells in a petri dish. She wonders how the growth pattern would change if each mother cell divided into 3 daughter cells.

4. **Write the first 5 terms of the sequence for the scientist's hypothesis.**

5. **Determine the total number of cells in the petri dish for each number of divisions.**

   a. **13 divisions**

   b. **16 divisions**

ACTIVITY
3.3

Writing Recursive and Explicit Formulas

In the previous lesson you identified sequences as either arithmetic or geometric and then matched a corresponding graph.

1. **Go back to the graphic organizers from the previous lesson. Write the recursive and explicit formulas for each sequence.**

NOTES

TALK the TALK

Pros and Cons

1. **Explain the advantages and disadvantages of using a recursive formula.**

2. **Explain the advantages and disadvantages of using an explicit formula.**