4

3 Pegs, N Discs
Modeling Using Sequences

Warm Up

Write an explicit formula for each arithmetic sequence.

1. {9, 8, 7, 6, 5, . . .}

2. {20, 40, 60, 80 . . .}

3. {1, $\frac{5}{2}$, 4, 5.5, 7, $\frac{17}{2}$ . . .}

Learning Goals

- Model situations using recursive and explicit formulas.
- Translate between recursive and explicit expressions of a mathematical model.
- Explore the process of mathematical modeling.

Key Term

- mathematical modeling

You have written recursive and explicit formulas for arithmetic and geometric sequences. How can you model a real-world situation using both recursive and explicit formulas for sequences?

Notice and Wonder

Mathematical modeling is explaining patterns in the real world based on mathematical ideas.

In this lesson, you will explore the process of *mathematical modeling*. The first step in modeling a situation mathematically is to gather information, notice patterns, and formulate mathematical questions about what you notice.

Let's play a game.

The object of the game is to move an entire stack of discs or coins from the start circle to any of the other circles.

The rules of the game are simple:
- You can only move one disc at a time.
- You cannot put a larger disc on top of a smaller disc.

Let's first play with 3 discs. To begin, place a quarter, nickel, and dime on top of each other in that order in a stack in the Start circle. Or, use the cutout discs at the end of the lesson, stacked from largest to smallest inside the Start circle.

Start
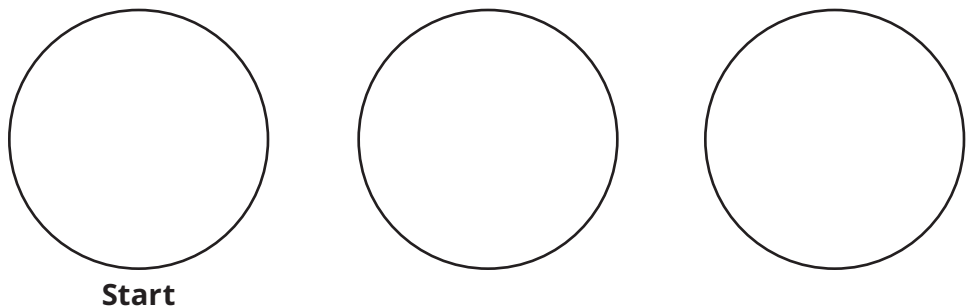
Think about:

Is there a relationship between the number of discs and the number of moves it takes to complete the game?

1. **Play this game several times with a partner. Record any patterns you notice.**

Organize and Mathematize

The second step in the modeling process is to organize your information and represent it using mathematical notation.

Consider the question from the previous activity. Is there a relationship between the number of discs and the minimum number of moves?

1. Play the game again and record your results in the table.

Number of Discs	Minimum Number of Moves
1	
2	
3	

Ask yourself:

How do you know you did it in the least number of moves?

2. What pattern do you notice in your results?

3. Use mathematical notation to represent the pattern you have identified in your results. Explain your reasoning.

Predict and Analyze

Step 3 of the modeling process is to extend the patterns you created, complete operations, make predictions, and analyze the mathematical results.

1. **Use your results to extend the pattern in the table in the previous activity.**

2. **Write a recursive formula to represent the pattern shown in your table. What predictions does this formula make for the minimum number of moves required for 4 and 5 discs?**

3. **Write an explicit formula to represent the pattern shown in your table. What predictions does this formula make for the minimum number of moves required for 4 and 5 discs?**

Test and Interpret

The final step in the modeling process is to interpret your results and test your mathematical predictions in the real world. If your predictions are incorrect, you can revisit your mathematical work and make adjustments—or start all over!

1. **Play the game again to demonstrate that your prediction for 4 discs and 5 discs is accurate. Record your observations.**

2. **Construct a graph to represent your explicit formula. Describe the characteristics of the graph in terms of the situation.**

Ask yourself:

What is the level of accuracy appropriate for this situation?
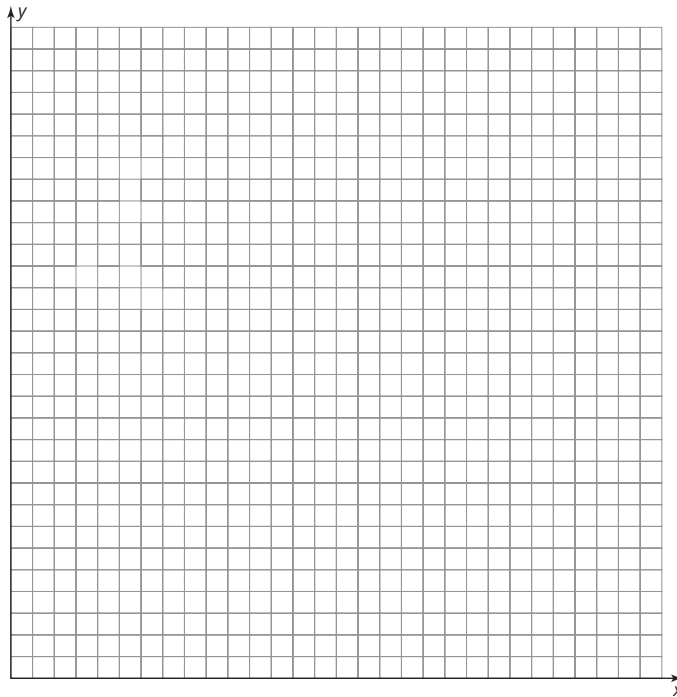
3. **Suppose you could make 1 move every second. How long would it take to complete a game with 25 discs? Show your work.**

TALK the TALK

A Modeling Process

In this lesson, you used a modeling process to figure out whether the number of moves in the disc game is related to the number of discs. The basic steps of the mathematical modeling process are summarized in the diagram.

Summarize what is involved in each phase of this modeling process.

Notice and Wonder

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Organize and Mathematize

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Predict and Analyze

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Test and Interpret

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The Modeling Process

+ NOTICE | WONDER

+ ORGANIZE | MATHEMATIZE

+ PREDICT | ANALYZE

+ TEST | INTERPRET

+ REPORT
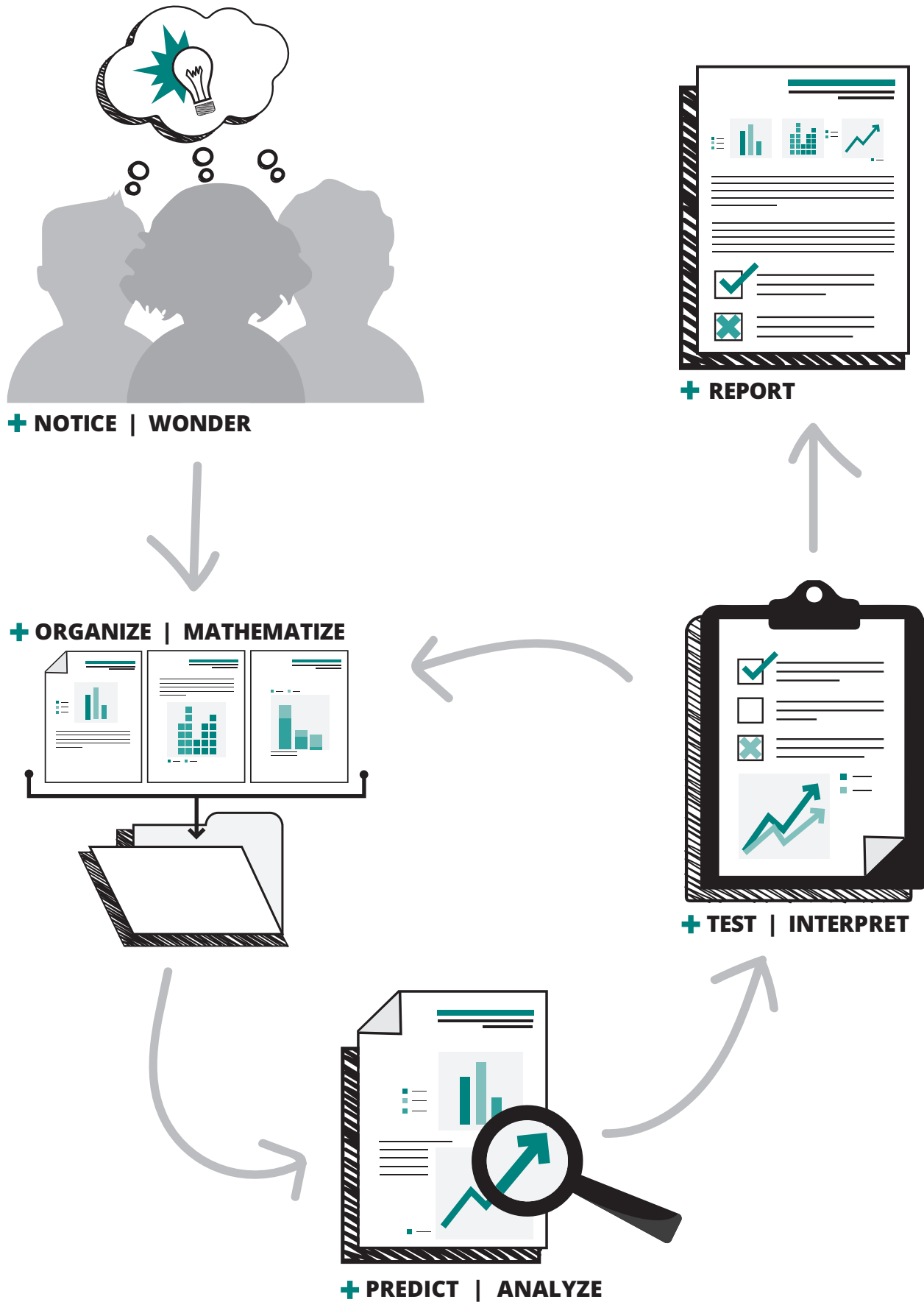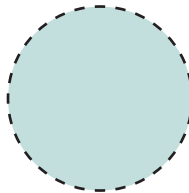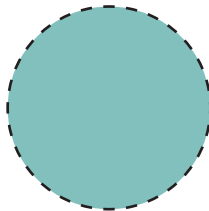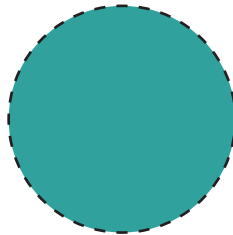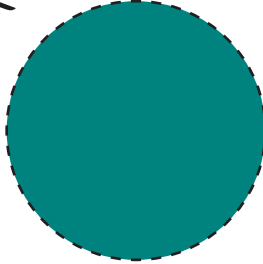
Discs

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So you can cut out the figures on the other side.