Sequences Summary

KEY TERMS

- sequence
- term of a sequence
- infinite sequence
- finite sequence
- arithmetic sequence
- common difference
- geometric difference
- common ratio
- recursive formula
- explicit formula
- mathematical modeling

LESSON 1 Is There a Pattern Here?

A **sequence** is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. A **term in a sequence** is an individual number, figure, or letter in the sequence. Many different patterns can generate a sequence of numbers.

A sequence that continues on forever is called an **infinite sequence**. A sequence that terminates is called a **finite sequence**.

For example, consider the situation in which an album that can hold 275 baseball cards is filled with 15 baseball cards at the end of each week. A sequence to represent how many baseball cards can fit into the album after 6 weeks is 275 cards, 260 cards, 245 cards, 230 cards, 215 cards, and 200 cards. This sequence begins at 275 and decreases by 15 with each term. The pattern cannot continue forever since you cannot have a negative number of cards, so this is a finite sequence.

LESSON 2

The Password Is...Operations!

An **arithmetic sequence** is a sequence of numbers in which the difference between any two consecutive terms is a constant. This constant is called the **common difference** and is typically represented by the variable *d*. The common difference of a sequence is positive if the same positive number is added to each term to produce the next term. The common difference of a sequence is negative if the same negative number is added to each term to produce the next term.

For example, consider the sequence 14, $16\frac{1}{2}$, 19, $21\frac{1}{2}$, ... The pattern of this sequence is to add $2\frac{1}{2}$ to each term to produce the next term. This is an arithmetic sequence, and the common difference d is $2\frac{1}{2}$.

A **geometric sequence** is a sequence of numbers in which the ratio between any two consecutive terms is a constant. The constant, which is either an integer or a fraction, is called the **common ratio** and is typically represented by the variable *r*.

For example, consider the sequence 27, 9, 3, 1, $\frac{1}{3}$, $\frac{1}{9}$. The pattern is to multiply each term by the same number, $\frac{1}{3}$, to determine the next term. Therefore, this sequence is geometric and the common ratio, r, is $\frac{1}{3}$.

LESSON 3

Did You Mean: *Recursion*?

A **recursive formula** expresses each new term of a sequence based on a preceding term of the sequence. The recursive formula to determine the *n*th term of an arithmetic sequence is $a_n = a_{n-1} + d$. The recursive formula to determine the *n*th term of a geometric sequence is $g_n = g_{n-1} \cdot r$. When using the recursive formula, it is not necessary to know the first term of the sequence.

For example, consider the geometric sequence 32, 8, 2, $\frac{1}{2}$, . . . with a common ratio of $\frac{1}{4}$. The 5th term of the sequence can be determined using the recursive formula.

$$g_n = g_{n-1} \cdot r$$
$$g_5 = g_4 \cdot r$$
$$g_5 = \frac{1}{2} \cdot \frac{1}{4}$$
$$g_5 = \frac{1}{8}$$

The 5th term of the sequence is $\frac{1}{8}$.

An **explicit formula** for a sequence is a formula for calculating each term of the sequence using the index, which is a term's position in the sequence. The explicit formula to determine the *n*th term of an arithmetic sequence is $a_n = a_1 + d(n - 1)$. The explicit formula to determine the *n*th term of a geometric sequence is $g_n = g_1 \cdot r^{n-1}$.

For example, consider the situation of a cactus that is currently 3 inches tall and will grow $\frac{1}{4}$ inch every month. The explicit formula for arithmetic sequences can be used to determine how tall the cactus will be in 12 months.

$$a_n = a_1 + d(n - 1)$$
$$a_{12} = 3 + \frac{1}{4}(12 - 1)$$
$$a_{12} = 3 + \frac{1}{4}(11)$$
$$a_{12} = 5\frac{3}{4}$$

In 12 months, the cactus will be $5\frac{3}{4}$ inches tall.

LESSON
4

3 Pegs, N Discs

A process called **mathematical modeling** involves explaining patterns in the real world based on mathematical ideas. The four basic steps of the mathematical modeling process are Notice and Wonder, Organize and Mathematize, Predict and Analyze, and Test and Interpret.

For example, consider a theater that has 25 rows of seats. The first three rows have 16, 18, and 20 seats, respectively. The ushers working at this theater need to know how many seats their sections have when they are directing people.

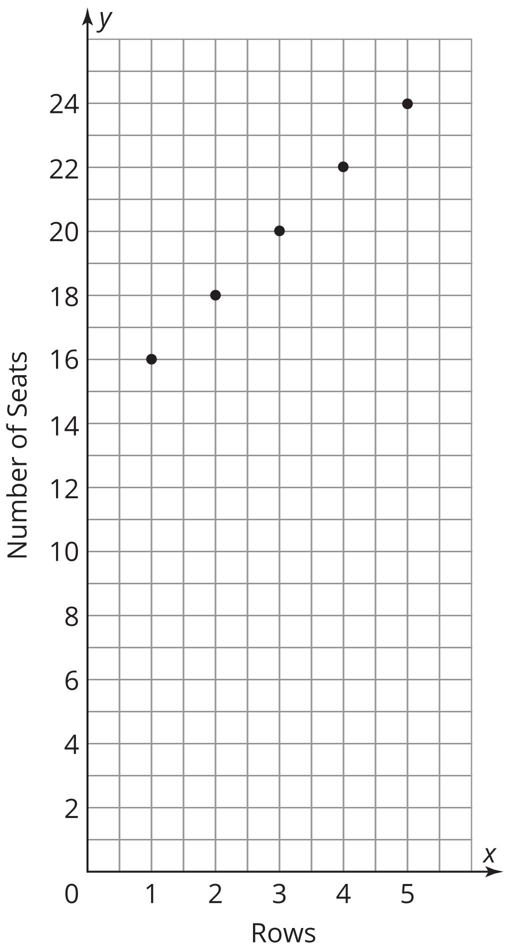
The first step of the modeling process, Notice and Wonder, is to gather information, look for patterns, and formulate mathematical questions about what you notice. In the example, each row seems to have 2 more seats than the previous row.

The second step of the modeling process, Organize and Mathematize, is to organize the information and express any patterns you notice using mathematical notation. A table can be used to represent the given information about the first three rows in the theater. The recursive pattern shown in the table can be expressed as $S_n = S_{n-1} + 2$.

Row	Number of Seats
1	16
2	18
3	20

The third step of the modeling process, Predict and Analyze, is to analyze the mathematical notation and make predictions. The fourth row will have 22 seats and the fifth row will have 24 seats. The pattern can be expressed using the explicit formula $S_n = 16 + 2(n - 1)$.

The fourth and final step of the modeling process, Test and Interpret, is to test and interpret the information. A graph can be constructed for the explicit formula. The graph is discrete because rows and seats are integer values.



This information can be used to determine that an usher working in rows 15 and 16 will have 44 and 46 seats, respectively.