TEXAS MATH SOLUTION

How to support your student as they learn about

Searching for Patterns

Mathematics is a connected set of ideas, and your student knows a lot. Encourage them to use the mathematics they already know when seeing new concepts in this module.

Module Introduction

In this module, your student will deepen their understanding of functions to explore function families, including linear, exponential, quadratic, and absolute value. There are 3 topics in this module: *Quantities and Relationships, Sequences,* and *Linear Regressions.* Your student will use what they already know about patterns in this module.

Academic Glossary

Each module will highlight an important term. Knowing and using these terms will help your student think, reason, and communicate their math ideas.

Term	Analyze
Definition	• To study or look closely for patterns • To break a concept down into smaller parts to gain a better understanding of it
Questions to Ask Your Student	• Do you see any patterns? • Have you seen something like this before? • What happens if the shape, model, or numbers change?
Related Phrases	• Examine • Evaluate • Determine • Observe • Consider • Investigate • What do you notice?

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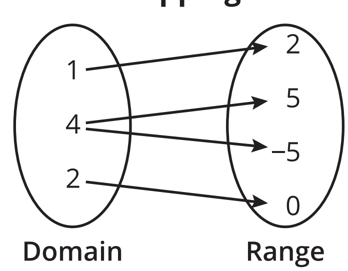
Example: Topic 1 Lesson 3

Analyze the relation represented as a mapping. Is the relation a function? Explain your reasoning.

Mapping



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Math Process Standards

Each module will focus on a process (or a pair of processes) that will help your student become a mathematical thinker. The "I can" statements listed below help your student to develop their mathematical learning and understanding.

Analyze mathematical relationships to connect and communicate mathematical ideas.

I can:

- identify important relationships in a problem situation.
- use what I know to solve new problems.
- analyze and organize information.
- look closely to identify patterns or structure.
- look for general methods and more efficient ways to solve problems.

Look for examples of these processes in the Topic Summaries.

The Carnegie Learning Way

Our Instructional Approach

Carnegie Learning's instructional approach is based on how people learn and real-world understandings. It is based on three key components:

ENGAGE	DEVELOP	DEMONSTRATE
Purpose: Provide an introduction that creates curiosity and uses what students already know and have experienced.	**Purpose:** Build a deep understanding of mathematics through different activities.	**Purpose:** Reflect on and evaluate what was learned.
Questions to Ask: How does this problem look like something you did in class?	**Questions to Ask:** Do you know another way to solve this problem? Does your answer make sense?	**Questions to Ask:** Is there anything you do not understand?

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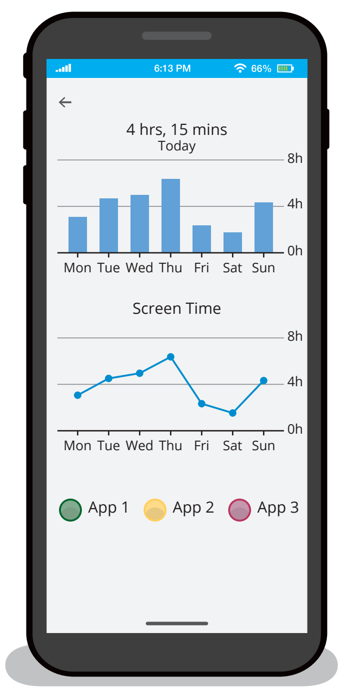
Module Overview

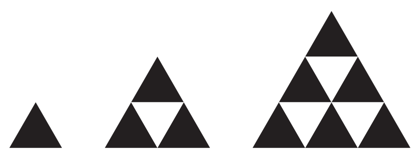
TOPIC 1	TOPIC 2	TOPIC 3
Quantities and Relationships	**Sequences**	**Linear Regressions**
13 Days	14 Days	7 Days
Your student will analyze scenarios and graphs representing the functions they will study in the course.	**Your student will explore sequences represented as lists of numbers, tables of values, equations, and graphs.**	**Your student will learn how to use lines of best fit to model data.**

What in the world?

Graphs allow us to see data in new ways so that we can find patterns and make predictions about the things we do not know. They can even be used to track daily habits and learn more about ourselves.



Did you know that?

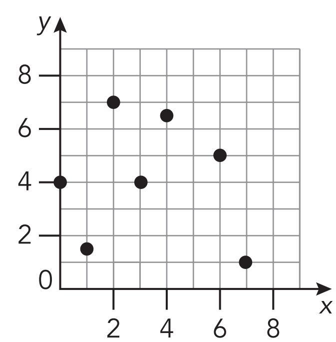


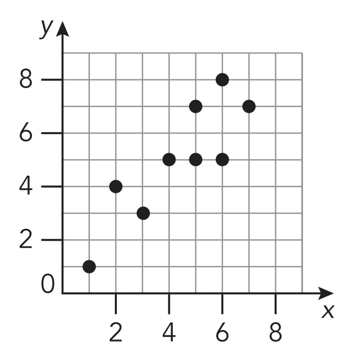
A sequence is a pattern of numbers, geometric figures, letters, or other objects that are placed in an exact order

What would the next figure look like in the sequence?

Did you know that?

The closer the *r*-value gets to 0, the data appears more random and less like a straight line.





Can you tell which set of data has an *r*-value closer to 0?

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TEXAS MATH SOLUTION

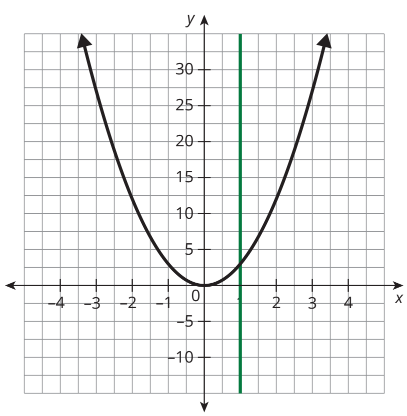
Topic 1: Quantities and Relationships

Key Terms

- dependent quantity
- independent quantity
- relation
- domain
- range
- function
- function notation
- Vertical Line Test
- discrete graph
- continuous graph
- increasing function
- decreasing function
- constant function
- function family
- linear functions
- exponential functions
- absolute maximum
- absolute minimum
- quadratic functions
- linear absolute value functions
- *x*-intercept
- *y*-intercept

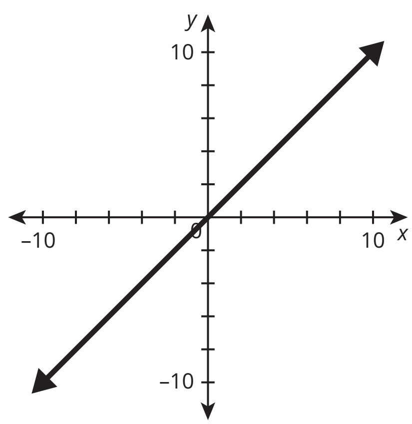
The **Vertical Line Test** is a way to determine if a relation on a graph is a function.

The equation $y = 3x^2$ is a function. The graph passes the vertical line test because there are no vertical lines you can draw that would cross the graph at more than one point.



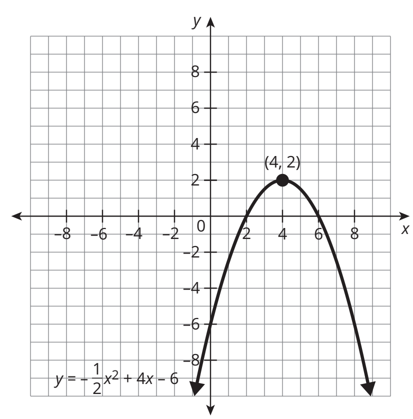
A **continuous graph** is a graph of points connected by a line or smooth curve. Continuous graphs have no breaks.

The graph shown is a continuous graph.



A function has an **absolute maximum** when there is a point that has a *y*-coordinate that is greater than the *y*-coordinates of every other point on the graph. It is the highest point that the curve reaches on the graph.

The absolute maximum of the graph of the function $f(x) = -\frac{1}{2}x^2 + 4x - 6$ is $y = 2$.



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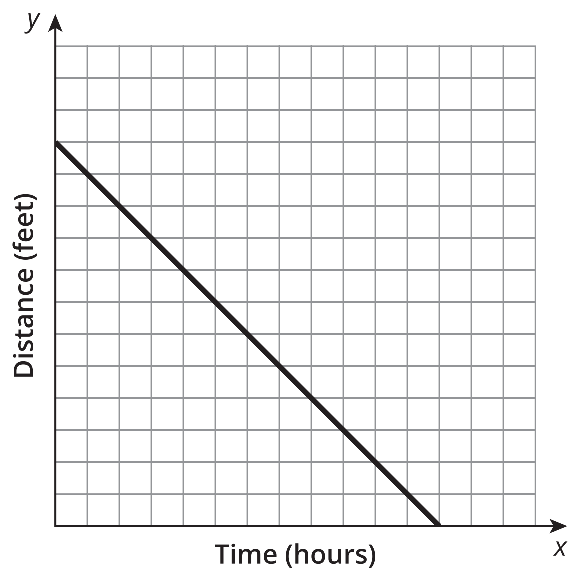
Dependent and Indepentent Quantities

Many problem situations include two quantities that change. When one quantity depends on another, it is said to be the **dependent quantity**. The quantity that changes the other quantity is called the **independent quantity**.

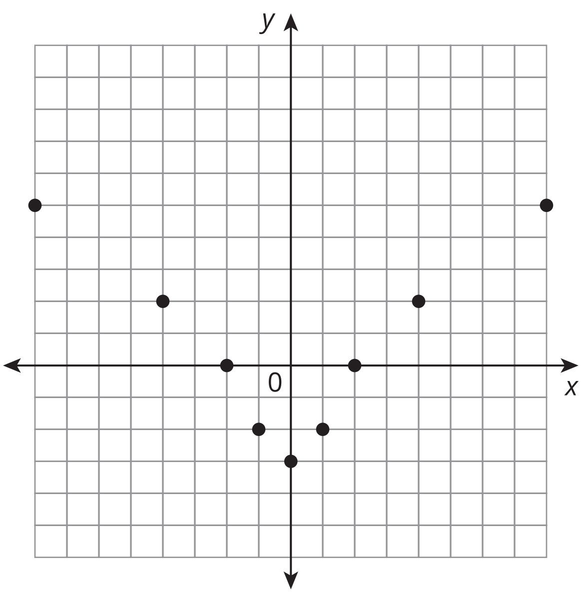


For example, consider the graph which models the situation where Pedro is hiking in a canyon. At the start of his hike, he was at 3500 feet. During the first 20 minutes of the hike, he walked 500 feet down at a constant rate. Then he rested for half an hour before continuing the hike at the same rate. Time is the independent quantity and distance is the dependent quantity.

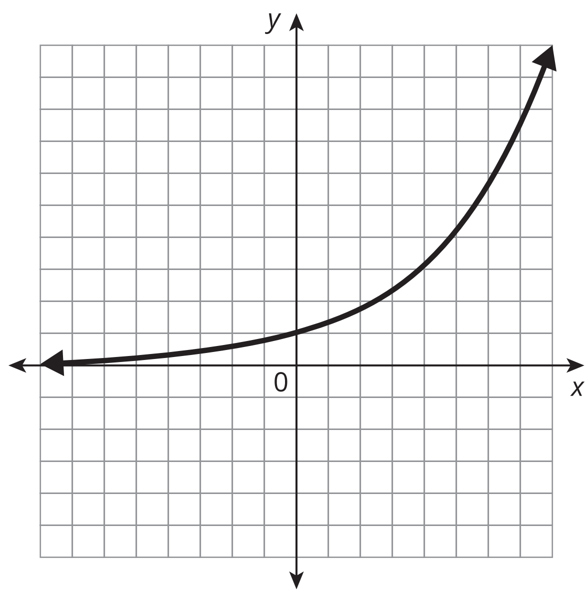
Patterns and Graphs

Looking for patterns can help when sorting and comparing graphs. Some graphs, such as Graph A, show vertical symmetry (if a vertical line were drawn through the middle of the graph, the image is the same on both sides). Graphs that show vertical symmetry can have a U-shape or a straight line. Other patterns to look for include: always increasing from left to right, such as Graph B below, always decreasing from left to right and smooth curves.

Graph A



Graph B



For example, Graph *A* has vertical symmetry. Graph *B* is a smooth curve that increases from left to right.

MATH PROCESS STANDARDS

How do the activities in *Quantities and Relationships* promote student expertise in the math process standards?

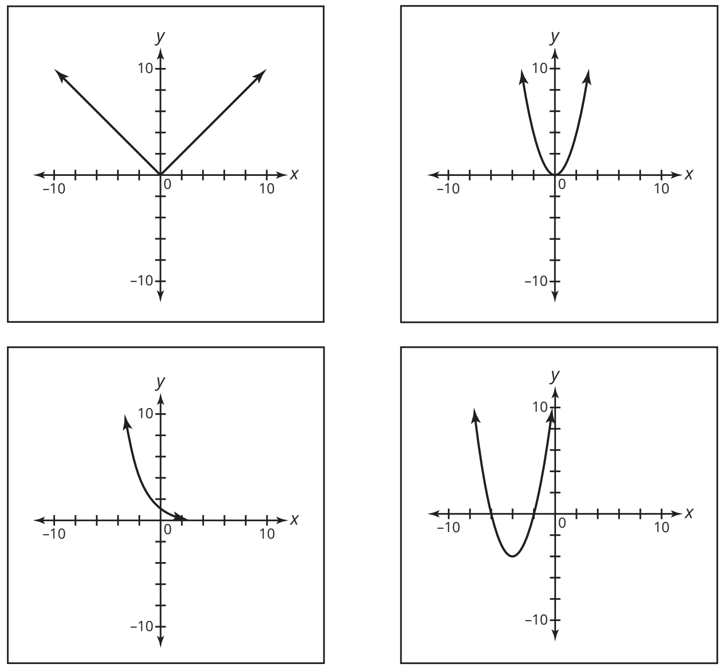
NOTE: This is an example of the math process standard:

Analyze mathematical relationships to connect and communicate mathematical ideas.

- I can analyze and organize information.

Have your student refer to page 2 for more "I can" statements.

Duane grouped these graphs together because each graph goes through only two quadrants.



Explain why Duane's reasoning is not correct.

Can you draw another graph that goes through only two quadrants?

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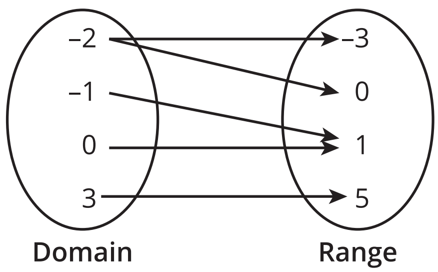
Functions and Relations

A **relation** is a mapping between a set of input values called the **domain** and a set of output values called the **range**. A **function** is a relation between a given set of elements, where each element in the domain is grouped with exactly one element in the range. If each value in the domain has one and only one range value, like Figure 2, then the relation is a function. If any value in the domain has more than one range value, like Figure 1, then the relation is not a function.

Figure 1



The value –2 in the domain has more than one range value. The mapping does not represent a function.

Figure 2

Domain	Range
2	1
6	3
10	5
14	7

Each element in the domain has exactly one element in the range. The table represents a function.

Function Notation

Functions can be represented in a number of ways. An equation representing a function can be written using **function notation**. Function notation is a way of representing functions with algebra. This form allows you to more easily identify the independent and dependent quantities. The function $f(x)$ is read as "f of x" and shows that x is the independent variable.

$$f(x) = 8x + 15$$

name of function

independent variable

The linear equation $y = 8x + 15$ can be written to represent a relationship between the variables x and y. You can write this linear equation as a function with the name f to represent it as a mathematical object that has a specific set of inputs (the domain of the function) and a specific set of outputs (the range of the function).

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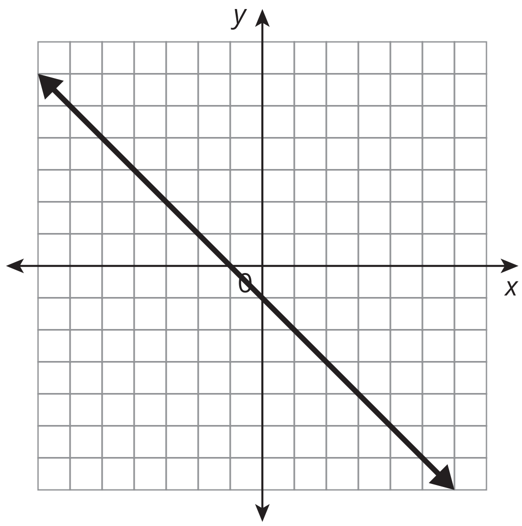
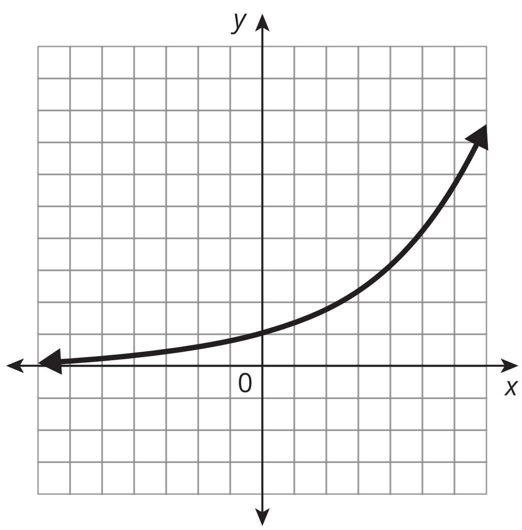
Function Families

A **function family** is a group of functions that share certain properties. Function families have key properties that are common among all functions in the family. Knowing these key properties is useful when sketching a graph of the function.

The family of **linear functions** includes functions of the form $f(x) = ax + b$, where a and b are real numbers. Linear function graphs will be represented by a straight line which can be vertical, horizontal and diagonal.



The family of **exponential functions** includes functions of the form $f(x) = a \cdot b^x + c$, where a, b, and c are real numbers, and b is greater than 0, but not equal to 1. Exponential function graphs will be represented by smooth curves.



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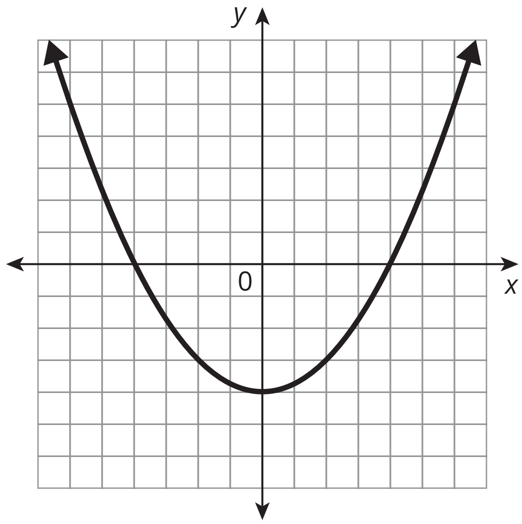
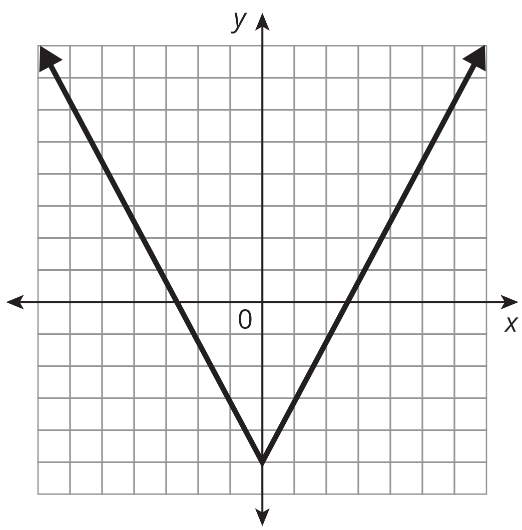
The family of **quadratic functions** includes functions of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers, and a is not equal to 0. Quadratic function graphs will be represented by a parabola as you see below.



The family of **linear absolute value functions** includes functions of the form $f(x) = a|x + b| + c$, where a, b, and c are real numbers, and a is not equal to 0. Linear absolute value function graphs will be represented by V-shapes.



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Topic 2: Sequences

Key Terms	
• sequence • term of a sequence • infinite sequence • finite sequence • arithmetic sequence • common difference	• geometric difference • common ratio • recursive formula • explicit formula • mathematical modeling
A **sequence** is a pattern that has an ordered arrangement of numbers, geometric figures, letters, or other objects. A **term in a sequence** is an individual number, figure, or letter in the sequence.	A **recursive formula** expresses each new term of a sequence based on the term that comes before it in the sequence. The recursive formula for an arithmetic sequence is $a_n = a_{n-1} + d$. The recursive formula for a geometric sequence is $g_n = g_{n-1} \cdot r$. The formula $a_n = a_{n-1} + 2$ is an example of a recursive formula. Each term that comes next is calculated by adding 2 to the previous term. If $a_1 = 1$, then $a_2 = 1 + 2 = 3$.
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Arithmetic Sequence

An **arithmetic sequence** is a sequence of numbers where the difference between any two consecutive terms is a constant. In other words, it is a sequence of numbers where a constant is added to each term to produce the next term. This constant is called the **common difference**. The common difference is typically represented by the variable, *d*.

Consider the sequence shown.

$$11, 9, 7, 5, \ldots$$

The pattern is to add the same negative number, -2, to each term to define the next term.

$$\text{Sequence: } 11 \xrightarrow{\text{add } -2} 9 \xrightarrow{\text{add } -2} 7 \xrightarrow{\text{add } -2} 5, \ldots$$

The sequence is arithmetic and the common difference, *d*, is -2.

Geometric Sequences

A **geometric sequence** is a sequence of numbers in which the ratio between any two consecutive terms is a constant. The constant, which is either an integer or a fraction, is called the **common ratio** and is represented by the variable *r*. For example, in the sequence 27, 9, 3, 1, $\frac{1}{3}$, $\frac{1}{9}$, the pattern is to multiply each term by the same number, $\frac{1}{3}$, to determine the next term. For that reason this sequence is geometric and the common ratio, *r*, is $\frac{1}{3}$.

Geometric Sequence

$$\underset{n\text{th term}}{g_n} = \underset{\text{1st term}}{g_1} \cdot \underset{\text{common ratio}}{r}^{\overbrace{n-1}^{\text{previous term number}}}$$

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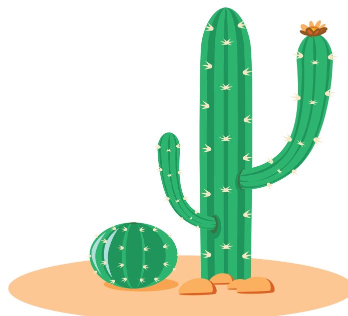
Explicit Formulas

An **explicit formula** for a sequence is a formula for calculating each term of the sequence using the index, which is a term's position in the sequence. The explicit formula to determine the term for any number that you put in the place of n (also known as the "nth term") of an arithmetic sequence is $a_n = a_1 + d(n - 1)$. The explicit formula to determine the nth term of a geometric sequence is $g_n = g_1 \cdot r^{n-1}$.

For example, consider the situation of a cactus that is 3 inches tall and will grow $\frac{1}{4}$ inch every month. The explicit formula for arithmetic sequences can be used to determine how tall the cactus will be in 12 months.

$a_n = a_1 + d(n-1)$ $a_{12} = 3 + \frac{1}{4}(12-1)$ $a_{12} = 3 + \frac{1}{4}(11)$ $a_{12} = 5\frac{3}{4}$		In 12 months, the cactus will be $5\frac{3}{4}$ inches tall.

Mathematical Modeling

A process called **mathematical modeling** involves explaining patterns in the real world based on mathematical ideas. The four basic steps of the mathematical modeling process are Notice and Wonder, Organize and Mathematize, Predict and Analyze, and Test and Interpret.

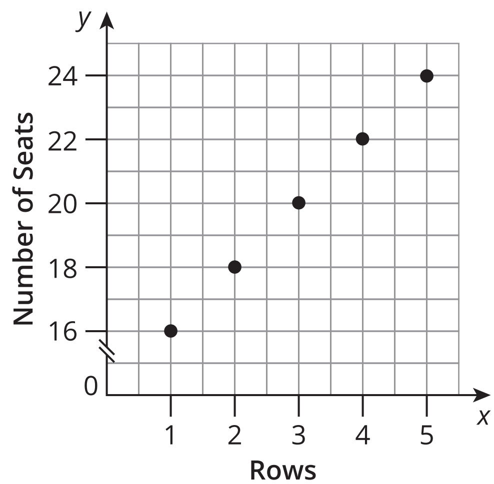
For example, consider a theater that has 25 rows of seats. The first three rows have 16, 18, and 20 seats. The ushers working at this theater need to know how many seats their sections have when they are directing people. The first step of the modeling process, Notice and Wonder, is to gather information, look for patterns, and ask mathematical questions about what you notice. In the example, each row seems to have 2 more seats than the previous row.

The second step of the modeling process, Organize and Mathematize, is to organize the information and write any patterns you see using mathematical notation. A table can be used to represent the given information about the first three rows in the theater. The recursive pattern shown in the table can be expressed as $s_n = s_{n-1} + 2$.

Row	Number of Seats
1	16
2	18
3	20

The third step of the modeling process, Predict and Analyze, is to analyze the mathematical notation and make predictions. The fourth row will have 22 seats and the fifth row will have 24 seats. The pattern can be expressed using the explicit formula $s_n = 16 + 2(n - 1)$.

The fourth and final step of the modeling process, Test and Interpret, is to test and interpret the information. A graph can be constructed for the explicit formula. The graph is discrete, meaning the points are not connected, because rows and seats are integer values, or numbers that do not include fractions. This is because it is not possible to have $\frac{1}{2}$ or $\frac{1}{4}$ of a seat.

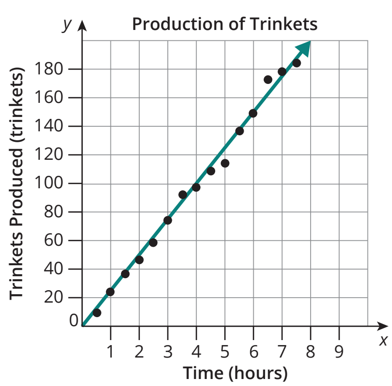


This information can be used to determine that an usher working in row 15 will have 44 seats and an usher in row 16 will have 46 seats.

Topic 3: Linear Regressions

Key Terms		
• Least Squares Method • centroid • regression line • interpolation • extrapolation	• correlation • correlation coefficient • coefficient of determination • causation • necessary condition	• sufficient condition • common response • confounding variable

The **Least Squares Method** is a method that creates a line that is closest to the points of data, known as a regression line, for a scatter plot that has two basic requirements: 1) the line must contain the centroid of the data set, and 2) the sum of the squares of the vertical distances from each given data point is smallest with the line.



The **centroid** is a point whose x-value is the mean of all the x-values of the points on the scatter plot and its y-value is the mean of all the y-values of the points on the scatter plot.

For the data points (1, 3), (1, 7), (2, 6), (3, 5), and (3, 4), the centroid is (2, 5).

mean of all x-values:

$1 + 1 + 2 + 3 + 3 + 2 = 12$

$12 \div 6 = 2$

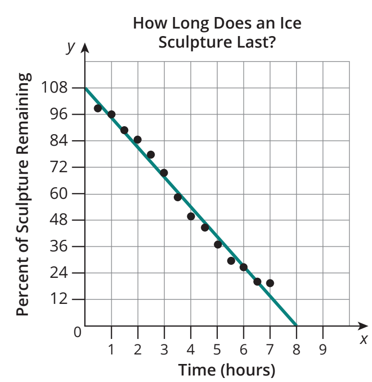
mean of all y-values

$3 + 7 + 6 + 5 + 4 + 5 = 30$

$30 \div 6 = 5$

The **correlation coefficient** is a value between –1 and 1, which shows how close the data are to the graph of the regression equation. The closer the **correlation** coefficient is to –1 or 1, the stronger the relationship is between the two variables. The variable r represents the correlation coefficient.

The correlation coefficient for these data is –0.9935. The value is negative because the graph moves down from left to right. The value is close to –1 because the data are very close to the graph of the equation of the line.



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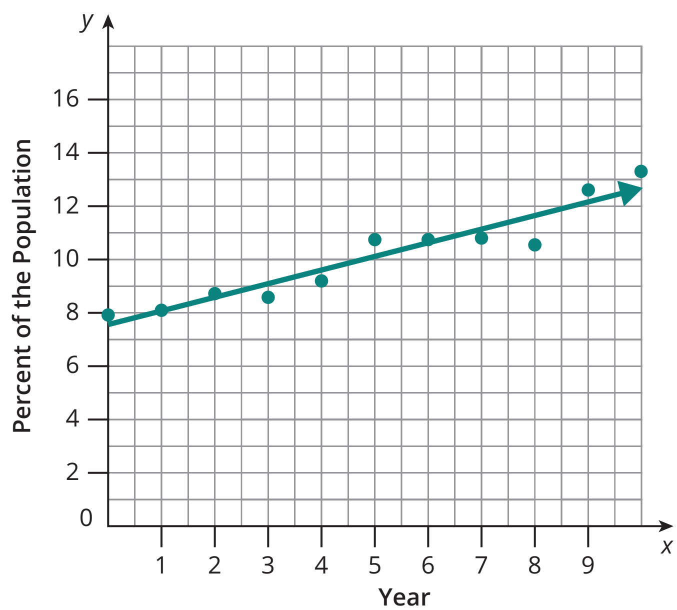
Regression Lines

Real-world data points never fit neatly on a line. But you can model the data points using a line, which represents a linear function. There are an infinite number of lines that can pass through the data points. But there is just one line that models the data with the minimum distances between the data points and the line. The **regression line** has the smallest possible vertical distance from each given data point to the line. By measuring these distances and squaring them, you will see that the squared distances add up to the smallest value with the regression line.



Interpolation and Extrapolation

If there is a linear relationship between the independent and dependent variables, a linear regression can be used to make predictions within the data set. Using a linear regression to make predictions within the data set is called **interpolation**. To make predictions outside the data set is called **extrapolation**.

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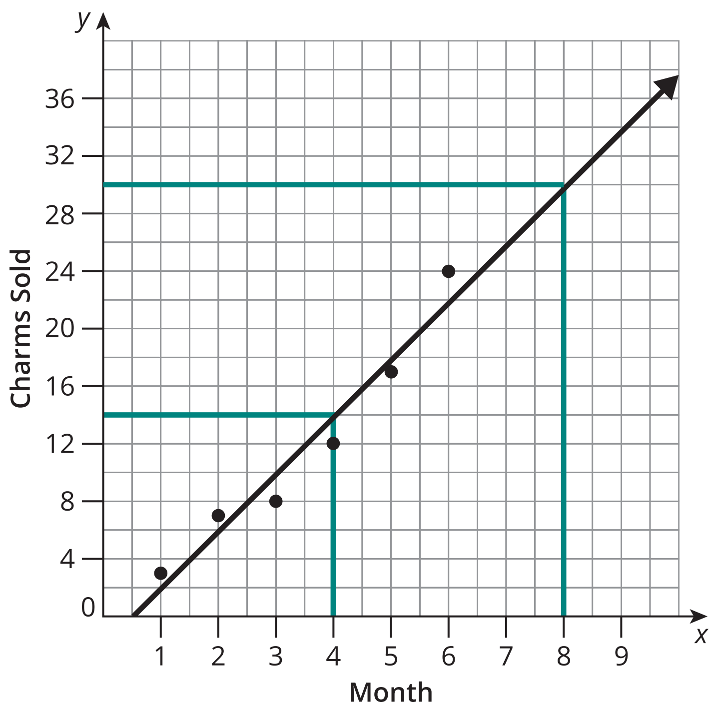
For example, consider the situation of Nina selling charms to her classmates. The table records the sales of her charms over the months since she began selling them.

Month	Charms Sold
1	3
2	7
3	8
4	12
5	17
6	24

The regression line modeling the situation is graphed on the scatter plot shown.



The linear regression equation is $y = 3.97x - 2.07$.

Using the equation to interpolate, Nina should sell about 14 charms in the fourth month.

$$y = 3.97(4) - 2.07$$
$$= 13.81$$

Using the equation to extrapolate, Nina should sell about 30 charms in the eighth month.

$$y = 3.97(8) - 2.07$$
$$= 29.69$$

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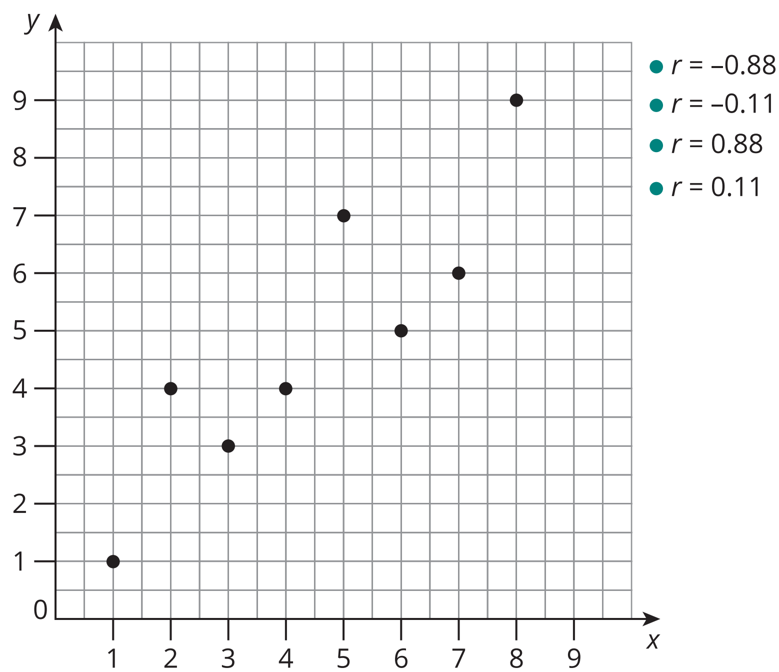
Correlation and Causation

The **coefficient of determination**, r^2, measures how well the regression line fits the data. It represents the difference in percent between the values in the data and the predicted values.



For example, consider the possible *r*-values for a linear regression given for the data graphed in the scatter plot. The points appear higher on the graph as we move from left to right, so the data has a positive correlation. Because of this, *r*-value must be positive. Also, the data are fairly close to forming a straight line, so the *r*-value should be fairly close to $r = 1$. Of the choices, $r = 0.88$ would be the most accurate.

When interpreting the correlation between two variables, you are looking at the connection between the variables. While a connection may exist, that does not mean there is causation between the variables. **Causation** is when one event causes a second event. A correlation is a **necessary condition** for causation, but a correlation is not a **sufficient condition** for causation. In other words, when events are connected it does not always mean that one event is the cause for the other. Correlation may be due to a **common response**, which is when another reason may cause the same result, or a **confounding variable**, which is when other variables are either unknown or unobserved.

For example, consider an experiment conducted by a group of college students that found that more class absences were correlated to rainy days. The group concluded that rain causes students to be sick. However, this correlation does not always mean causation. Rain is neither a necessary condition (because students can get sick on days it does not rain) nor a sufficient condition (because not every student who is absent is necessarily sick) for students being sick.

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TEXAS MATH SOLUTION

Discuss important dates throughout this module such as assessments, assignments, or class events with your student. Use the table to record these dates and reference them as your student progresses through the module.

Important Dates	
Date	Reason

Using the link below, visit the Texas Math Solution Support Center for students and caregivers to access additional resources such as:

- Mathematics Glossaries
- Videos
- Topic Materials
- A Letter to Families and Caregivers

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