

## Algebra

## Student Textbook Skills Program Edition SY 2022-2023

501 Grant St., Suite 1075
Pittsburgh, PA 15219
Phone 888.851.7094
Customer Service Phone 412.690.2444
Fax 412.690.2444
www.carnegielearning.com
Cover Design by Anne Milliron

Copyright © 2021 by Carnegie Learning, Inc. All rights reserved. Carnegie Learning and MATHia are registered marks of Carnegie Learning, Inc. All other company and product names mentioned are used for identification purposes only and may be trademarks of their respective owners. Permission is granted for photocopying rights within licensed sites only. Any other usage or reproduction in any form is prohibited without the expressed consent of the publisher.

ISBN: 978-1-63862-064-8
Student Edition, Volume 1
Printed in the United States of America
123456789 CC 2120191817

## MANIFESTO

## Our Manifesto

WE BELIEVE that quality math education is important for all students, to help them develop into creative problem solvers, critical thinkers, life-long learners, and more capable adults.

WE BELIEVE that math education is about more than memorizing equations or performing on tests-it's about delivering the deep conceptual learning that supports ongoing growth and future development.

WE BELIEVE all students learn math best when teachers believe in them, expect them to participate, and encourage them to own their learning.

WE BELIEVE teachers are fundamental to student success and need powerful, flexible resources and support to build dynamic cultures of collaborative learning.

WE BELIEVE our learning solutions and services can help accomplish this, and that by working together with educators and communities we serve, we guide the way to better math learning.

## LONG + LIVE + MATH

## ACKNOWLEDGMENTS

## Middle School Math Solution Authors

- Sandy Bartle Finocchi, Chief Mathematics Officer
- Amy Jones Lewis, Senior Director of Instructional Design
- Kelly Edenfield, Instructional Designer
- Josh Fisher, Instructional Designer


## Foundation Authors (2010)

- William S. Hadley, Algebra and Proportional Reasoning
- Mary Lou Metz, Data Analysis and Probability
- Mary Lynn Raith, Number and Operations
- Janet Sinopoli, Algebra
- Jaclyn Snyder, Geometry and Measurement


## Vendors

- Lumina Datamatics, Ltd.
- Cenveo Publisher Services, Inc.


## Images

- www.pixabay.com


## Special Thanks

- Alison Huettner for project management and editorial review.
- Jacyln Snyder and Janet Sinopoli for their contributions to the Teacher's Implementation Guide facilitation notes.
- Victoria Fisher for her review of content and contributions to all the ancillary materials.
- Valerie Muller for her contributions and review of content.
- The members of Carnegie Learning Cognitive Scientist Team—Brendon Towle, John Connelly, Bob Hausmann, Chas Murray, and Martina Pavelko-for their insight in learning science and review of content.
- Bob Hausmann for his contributions to the Family Guide.
- John Jorgenson, Chief Marketing Officer, for all his insight and messaging.
- Carnegie Learning Education Services Team for content review and providing customer feedback.
- In Memory of David Dengler, Director of Curriculum Development (deceased), who made substantial contributions to conceptualizing Carnegie Learning's middle school software.


## Texas Math Solution Content Authors

- Tarin Barrow, STEM Instructional Designer
- Sami Briceño, Senior Custom Solution Content Lead
- Brandy King, Custom Solution Content Specialist
- Christine Mooney, Custom Solution Content Specialist


## Texas Math Solution Custom Development Team

- Eddie Altomare
- Katie Barsanti
- Desiree Brown
- Allison Carden
- Courtney Comley
- Jesse Hinojosa
- Karrie Holland
- Bethany Jameson
- Steven Mendoza
- Jennifer Penton
- Jason Ulrich
- Lucy Yu
- Rob Zimmerman


## Special Thanks

- The entire Carnegie Learning Production Team, with extreme gratitude for Sara Kozelnik, Julie Leath, Lindsay Ryan, Angela Cerbone, Lenore MacLeod, Hannah Mumm, Olivia Rangel, Chloe Thompson and Emily Tope, for their patience, attention to detail, and around-the-clock hours that made the production of this textbook possible.
- Thank you to all the Texas educators and education professionals who supported the review process and provided feedback for this resource.

Mathematics is so much more than memorizing rules. It is learning to reason, to make connections, and to make sense of the world. We believe in Learning by Doing ${ }^{\text {TM }}$-you need to actively engage with the content if you are to benefit from it. The lessons were designed to take you from your intuitive understanding of the world and build on your prior experiences to then learn new concepts. My hope is that these instructional materials help you build a deep understanding of math.

Sandy Bartle Finocchi, Chief Mathematics Officer

You have been learning math for a very long time-both in school and in your interactions in the world. You know a lot of math! In this course, there's nothing brand new. It all builds on what you already know. So, as you approach each activity, use all of your knowledge to solve problems, to ask questions, to fix mistakes, and to think creatively.

Amy Jones Lewis, Senior Director of Instructional Design

At Carnegie Learning, we have created an organization whose mission and culture is defined by your success. Our passion is creating products that make sense of the world of mathematics and ignite a passion in you. Our hope is that you will enjoy our resources as much as we enjoyed creating them.

Barry Malkin, CEO

## Module 1: Searching for Patterns

## Topic 1: Quantities and Relationships

1 A Picture Is Worth a Thousand Words
Understanding Quantities and Their Relationships
2 A Sort of Sorts
Analyzing and Sorting Graphs
3 Fof X
Recognizing Functions and Function Families
4 Function Families for 2000, Alex
Recognizing Functions by Characteristics

## Topic 2: Sequences

1 Is There a Pattern Here? Recognizing Patterns and Sequences

2 The Password Is ... Operations! Arithmetic and Geometric Sequences
3 Did You Mean: Recursion?
Determining Recursive and Explicit Expressions from Contexts
43 Pegs, N Discs
Modeling Using Sequences
Topic 3: Linear Regressions
1 Like a Glove
Least Square Regressions
2 Gotta Keep It Correlatin'
Correlation

## Module 2: Exploring Constant Change

## Topic 1: Linear Functions

1 Connecting the Dots
Making Connections Between Arithmetic Sequences and Linear Functions
2 What's the Point?
Point-Slope Form of a Line
3 The Arts Are Alive
Using Linear Equations
4 Fun Functions, Linear Ones
Making Sense of Different Representations of a Linear Function
5 Move It!
Transforming Linear Functions
6 Get A Move On!
Vertical and Horizontal Transformations of Linear Functions
7 Amirite?
Determining Slopes of Perpendicular Lines
8 Making a Connection
Comparing Linear Functions in Different Forms

## Topic 2: Linear Equations and Inequalities

1 Strike a Balance
Solving Linear Equations
2 It's Literally About Literal Equations
Literal Equations
3 Not All Statements Are Made Equal
Modeling Linear Inequalities

## Topic 3: Systems of Equations and Inequalities

1 The County Fair
Using Substitution to Solve Linear Systems
2 Double the Fun
Using Graphing to Solve Systems of Equations
3 The Elimination Round
Using Linear Combinations to Solve a System of Linear Equations
4 Throwing Shade
Graphing Inequalities in Two Variables
5 Working with Constraints
Systems of Linear Inequalities
6 Working the System
Solving Systems of Equations and Inequalities

# Module 3: Investigating Growth and Decay 

## Topic 1: Introduction to Exponential Functions

| 1 | It's a Generational Thing <br> Properties of Powers with Integer Exponents |
| :--- | :--- |
| 2 | Show What You Know <br> Analyzing Properties of Powers |
| 3 | A Constant Ratio <br> Geometric Sequences and Exponential Functions |
| 4 | The Power Within <br> Rational Exponents and Graphs of Exponential Functions |

## Topic 2: Using Exponential Equations

1 Uptown and Downtown
Exponential Equations for Growth and Decay
2 Powers and the Horizontal Line Interpreting Parameters in Context

3 Savings, Tea, and Carbon Dioxide Modeling Using Exponential Functions
4 BAC Is Bad News
Choosing a Function to Model Data

## Module 4: Maximizing and Minimizing

## Topic 1: Introduction to Quadratic Functions

## 1 Up and Down or Down and Up Exploring Quadratic Functions

2 Endless Forms Most Beautiful
Key Characteristics of Quadratic Functions
3 Parabolas in Motion
Quadratic Function Transformations
4 Keep It Moving
Transformations of Quadratic Functions
5 You Lose Some, You Lose Some
Comparing Functions Using Key Characteristics and Average Rate of Change

## Topic 2: Solving Quadratic Equations

1 This Time, with Polynomials
Adding, Subtracting, and Multiplying Polynomials
2 The Great Divide
Polynomial Division
3 Solutions, Plus or Minus
Representing Solutions to Quadratic Equations
4 Transforming Solutions
Solutions to Quadratic Equations in Vertex Form
5 The Missing Link
Factoring and Completing the Square
6 Ladies and Gents, Please Welcome the Quadratic Formula!
The Quadratic Formula
7 Fit This Model
Using Quadratic Functions to Model Data

# End of Course Topic 

## Formative Assessment

1 Health Club Payment PlansPerformance Task
2 Taco FestivalPerformance Task
3 Randy's Raises
Performance Task
4 Undergraduate Tuition
Performance Task
Glossary

Each lesson has the same structure. Key features are noted.

## A Picture Is Worth a Thousand Words

Understanding Quantities and Their Relationships

## Warm Up

Emma bought a new video game. The graph shown describes the number of hours Emma spent playing the game over a period of 7 days.


1. Label the axes.
2. What does the highest point on the graph represent with respect to the scenario? The lowest point?

## Learning Goals

1
each other

- Identify the independent and dependent quantities for a scenario.
- Match a graph with an appropriate scenario.
- Use a reasonable scale for a graph modeling a scenario.
- Identify key characteristics of graphs.
- Describe similarities and differences between pairs of graphs and scenarios.


## Key Terms

- dependent quantity
- independent quantity

You have analyzed graphs of relationships and identified important features such as intercepts and slopes. How can the key characteristics of a graph tell a story?
3. Getting Started Each lesson begins with Getting Started. When working on Getting Started, use what you know about the world, what you have learned previously, or your intuition. The goal is just to get you thinking and ready for what's to come.

| What Comes First? |
| :--- | :--- | :--- |



## 4. Activities

You are going to build a deep understanding of mathematics through a variety of activities in an environment where collaboration and conversations are important and expected.

You will learn how to solve new problems, but you will also learn why those strategies work and how they are connected to other strategies you already know.

Remember:

- It's not just about answer-getting. The process is important.
- Making mistakes are a critical part of learning, so take risks.
- There is often more than one way to solve a problem.

Activities may include real-world problems, sorting activities, worked examples, or analyzing sample student work.

Be prepared to share your solutions and methods with your classmates.
5. Talk the Talk Talk the Talk gives you an opportunity to reflect on the main ideas of the lesson.

- Be honest with yourself.
- Ask questions to clarify anything you don't understand.
- Show what you know!
Don't forget to revisit the question posed on the lesson opening page to gauge your understanding.



## ASSIGNMENT

## Assignment

6
Write
Describe how you can distinguish between an independent quantity and a dependent quantity. Use an example in your description.

LESSON 1: A Picture Is Worth a Thousand Words

Remember

## 7

When one quantity is determined by another in a problem situation, it is said to be the dependent quantity. The quantity it is determined from is called the independent quantity. The independent quantity is represented on the $x$-axis and the dependent quantity is represented on the $y$-axis.

Practice

1. Read each scenario and identify the independent and dependent quantities. Be sure to include the appropriate units of measure. Then analyze each grap and determine which of the provided scenarios it models. For each graph, label the $x$ - and $y$-axis with the appropriate quantity and unit of measure.
a. Endangered Species The Elkwood Aquatic with various reptile st populations. The initi endangered turtles tr past five years.
c. Sales Commission Julian works as a sale monthly salary of $\$ 30$ commission on the ar
e. Commuter Flight A commuter flight be Oregon takes about 4 increases its altitude the flight until it gets then it descends for $t$ flight. The plane asce constant rate of 900


2. Compare the pair of graphs and describe any similarities and differences you notice.



9
Stretch
Read the scenario and identify the independent and dependent quantities. Be sure to include the appropriate units of measure

1. A student performs several experiments in which he swings a pendulum for a 20 -second duration He uses a string that is 27 cm long, and he tests pendulum masses of different sizes, varying from 2 to 12 grams. He records the number of swings each pendulum makes in 20 seconds
2. The student then decides to make a second graph showing the string length (in cm ) as the independent quantity. What changes must the student make to his experiment?

## 10

## Review

1. Solve the equation $-2 x+8=-3 x+14$.
2. Evaluate the expression $x^{2}-3 y+12$ for $x=-2$ and $y=5$.

## 6. Write

 Reflect on your work and clarify your thinking.
## 7. Remember

 Take note of the key concepts from the lesson.
## 8. Practice

 Use the concepts learned in the lesson to solve problems.
## 9. Stretch

 Ready for a challenge?
## 10. Review

 Remember what you've learned by practicing concepts from previous lessons and topics.
## PROBLEM TYPES YOU WILL SEE

## Worked Example

You can represent $a_{n}$ using function notation.

$$
\begin{aligned}
a_{n} & =2+4(n-1) \\
f(n) & =2+4(n-1)
\end{aligned}
$$

Next, rewrite the expression $2+4(n-1)$.

$$
\begin{aligned}
f(n) & =2+4 n-4 & & \text { Distributive Property } \\
& =4 n+2-4 & & \text { Commutative Property } \\
& =4 n-2 & & \text { Combine Like Terms }
\end{aligned}
$$

So, $a_{n}=2+4(n-1)$ written in function notation is $f(n)=4 n-2$.

Maya and Sherry each convert the given formula to degrees Fahrenheit.

$$
\begin{aligned}
& \text { Maya } \\
& \begin{aligned}
C & =\frac{5}{9}(F-32) \\
C & =\frac{5}{9} F-\frac{160}{9} \\
9(C) & =9\left(\frac{5}{9} F-\frac{160}{9}\right) \\
9 C & =5 F-160 \\
9 C+160 & =5 F \\
\frac{9 C}{5}+\frac{160}{5} & =\frac{5 F}{5} \\
\frac{9}{5} C+32 & =F
\end{aligned}
\end{aligned}
$$

Sherry

$$
C=\frac{5}{9}(F-32)
$$

$$
C=\frac{5}{9} F-32
$$

$$
9(C)=9\left(\frac{5}{9} F-32\right)
$$

$$
9 C=5 F-288
$$

$$
9 C+288=5 F
$$

$$
\frac{9 C}{5}+\frac{288}{5}=\frac{5 F}{5}
$$

$$
\frac{9}{5} C+57.6=F
$$

## Thumbs Up

## When you see a

 Thumbs Up icon:- Take your time to read through the correct solution.
- Think about the connections between steps.


## Ask Yourself:

- Why is this method correct?
- Have I used this method before?


## Worked Example

## When you see a Worked Example:

- Take your time to read through it.
- Question your own understanding.
- Think about the connections between steps.


## Ask Yourself:

- What is the main idea?
- How would this work if I changed the numbers?
- Have I used these strategies before?


## Thumbs Down

## When you see a

 Thumbs Down icon:- Take your time to read through the incorrect solution.
- Think about what error was made.


## Ask Yourself:

- Where is the error?
- Why is it an error?
- How can I correct it?

4. Carlos and Mikala do not like working with fractions. Each rewrites the equation so that it does not have fractions. Their work is shown.

## Carlos

$$
\begin{aligned}
F & =\frac{9}{5} c+32 \\
(5) F & =5\left(\frac{9}{5} c+32\right) \\
5 F & =9 C+160 \\
5 F-9 C & =160
\end{aligned}
$$

Mikala

$$
C=\frac{5}{9}(F-32)
$$

$(9) C=(9)\left(\frac{5}{9}(F-32)\right)$
$9 C=5(F-32)$
$9 C=5 F-160$
$9 C-5 F=-160$

Carlos and Mikala got two different equations. Who is correct? Explain your reasoning.

## Who's Correct?

## When you see a Who's Correct icon:

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine if correct or not correct.


## Ask Yourself:

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?


## MATHEMATICAL PROCESS STANDARDS

## Texas Mathematical Process Standards

Effective communication and collaboration are essential skills of a successful learner. With practice, you can develop the habits of mind of a productive mathematical thinker. The "I can" expectations listed below align with the TEKS Mathematical Process Standards and encourage students to develop their mathematical learning and understanding.

## - Apply mathematics to problems arising in everyday life, society, and the workplace.

## I can:

- use the mathematics that I learn to solve real world problems.
- interpret mathematical results in the contexts of a variety of problem situations.
- Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem-solving process and reasonableness of the solution.

I can:

- explain what a problem "means" in my own words.
- create a plan and change it if necessary.
- ask useful questions in an attempt to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.
- Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate; and techniques including mental math, estimation, and number sense as appropriate, to solve problems.

I can:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.
- Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

I can:

- communicate and defend my own mathematical understanding using examples, models, or diagrams.
- use appropriate mathematical vocabulary in communicating mathematical ideas.
- make generalizations based on results.
- apply mathematical ideas to solve problems.
- interpret my results in terms of various problem situations.
- Create and use representations to organize, record, and communicate mathematical ideas.

I can:

- consider the units of measure involved in a problem.
- label diagrams and figures appropriately to clarify the meaning of different representations.
- create an understandable representation of a problem situation.


## Analyze mathematical relationships to connect and communicate mathematical ideas.

I can:

- identify important relationships in a problem situation.
- use what I know to solve new problems.
- analyze and organize information.
- look closely to identify patterns or structure.
- look for general methods and more efficient ways to solve problems.
- Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

I can:

- work carefully and check my work.
- distinguish correct reasoning from reasoning that is flawed.
- use appropriate mathematical vocabulary when I talk with my classmates, my teacher, and others.
- specify the appropriate units of measure when I explain my reasoning.
- calculate accurately and communicate precisely to others.


## ACADEMIC GLOSSARY

There are important terms you will encounter throughout this book. It is important that you have an understanding of these words as you get started on your journey through the mathematical concepts. Knowing what is meant by these terms and using these terms will help you think, reason, and communicate your ideas.

## ANALYZE

## Definition

To study or look closely for patterns. Analyzing can involve examining or breaking a concept down into smaller parts to gain a better understanding of it.

## Ask Yourself

- Do I see any patterns?
- Have I seen something like this before?
- What happens if the shape, representation, or numbers change?


## EXPLAIN YOUR REASONING

## Definition

To give details or describe how to determine an answer or solution. Explaining your reasoning helps justify conclusions.

## Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Does my reasoning make sense?
- How can I justify my answer to others?

Visit the Students \&
Caregivers Portal on the Texas Support
Center at www.
CarnegieLearning. com/texas-help to access the Mathematics Glossary for this course anytime, anywhere.

## Related Phrases

- Examine
- Evaluate
- Determine
- Observe
- Consider
- Investigate
- What do you notice?
- What do you think?
- Sort and match


## Related Phrases

- Show your work
- Explain your calculation
- Justify
- Why or why not?

Related Phrases

- Show
- Sketch
- Draw
- Create
- Plot
- Graph
- Write an equation
- Complete the table

Related Phrases

- Predict
- Approximate
- Expect
- About how much?
- Demonstrate
- Label
- Display
- Compare
- Determine
- Define
- What are the advantages?
- What are the disadvantages?
- What is similar?
- What is different?


## REPRESENT

## Definition

To display information in various ways. Representing mathematics can be done using words, tables, graphs, or symbols.

## Ask Yourself

- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Is my representation accurate?


## ESTIMATE

## Definition

To make an educated guess based on the analysis of given data. Estimating first helps inform reasoning.

## Ask Yourself

- Does my reasoning make sense?
- Is my solution close to my estimation?


## DESCRIBE

## Definition

To represent or give an account of in words. Describing communicates mathematical ideas to others.

## Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?


## Thought Bubbles

Look for these icons as you journey through the textbook. Sometimes they will remind you about things you already learned. Sometimes they will ask you questions to help you think about different strategies. Sometimes they will share fun facts. They are here to help and guide your learning.


## about:

Side notes are included to provide helpful insights as you work.

## Student Lesson Overview Videos

Each lesson has a corresponding lesson overview video(s) for you to use and reference as you are learning. The videos provide an overview of key concepts, strategies, and/or worked examples from the lessons.


## Topic Summary

A Topic Summary is provided at the end of each topic. The Topic Summary lists all key terms of the topic and provides a summary of each lesson. Each lesson summary defines key terms and reviews key concepts, strategies, and/or worked examples.


## Mathematics Glossary

A course-specific mathematics glossary is available to utilize and reference while you are learning. Use the glossary to locate definitions and examples of math key terms.


## absolute maximum

A function has an absolute maximum if there is a point that has a $y$-coordinate that is greater than the $y$-coordinates of every other point on the graph.

## Example

The ordered pair $(4,2)$ is the absolute maximum of the graph of the function $f(x)=-\frac{1}{2} x^{2}+4 x-6$.

absolute minimum
A function has an absolute minimum if there is a point that has a $y$-coordinate that is less than the $y$-coordinates of every other point on the graph.

## Example

The ordered pair $(1,-4)$ is the absolute minimum of the graph of the function $y=\frac{2}{3} x^{2}-\frac{4}{3} x-\frac{10}{3}$.

argument of a function
The argument of a function is the variable on which the function operates.

## Example

In the function $f(x+5)=32$, the argument is $x+5$.

## Module Family and Caregiver Guides

Each module guide will provide a different highlight of the academic glossary, description and examples of TEKS Mathematical Process Standards, and an overview of a different component of our instructional approach known as The Carnegie Learning Way. Also included is a module overview of content, specific key terms, visual representations, and strategies you are learning in each topic of the module.

The purpose of the Family and Caregiver Guides is to bridge student learning in the classroom to student learning at home. Our goal is to empower you and your family to understand the concepts and skills learned in the classroom so that you can review, discuss, and solidify the understanding of these key concepts together. Videos will also be available on the Students \& Caregivers Portal on the Texas Support Center to provide added support.


## Topic Family Guides

Each topic contains a Family Guide that provides an overview of the math of the topic and answers the questions, "Where have we been?" and "Where are we going?" Additional components of the Family Guide are, as follows: new notation or strategy taught in the topic, definitions of a few key terms, connection of math to the real world, related standardized test question sample, or talking points for caregivers to support your learning.

We recognize that learning outside of the classroom is crucial to student success at school. While we don't expect families and caregivers to be math teachers, the Family Guides are designed to assist families and caregivers as they talk to you about what you are learning. Our hope is that both you and your family will read and benefit from these guides.


## Students and Caregivers Portal

Research has proven time and again that family engagement greatly improves a student's likelihood of success in school.

The Students \& Caregivers Portal on the Texas Support Center provides:

- Getting to Know Carnegie Learning video content to provide an introduction to the instructional materials and research.
- Articles and quick tip videos offering strategies for how families and caregivers can support student learning.
- Access to instructional resources to support students and caregivers.
To access new content and resources, visit the Students and Caregivers Portal on the Texas Support Center at https://www.CarnegieLearning.com/texas-help/studentscaregivers/

