



**TEXAS MATH
SOLUTION**

Algebra I

**Teacher's
Implementation Guide**

Skills Program Edition

SY 2022-2023

**Sandy Bartle Finocchi and Amy Jones Lewis
with Josh Fisher, Janet Sinopoli, and Victoria Fisher**



501 Grant St., Suite 1075
Pittsburgh, PA 15219
Phone 888.851.7094
Customer Service Phone 412.690.2444
Fax 412.690.2444

www.carnegielearning.com

Cover Design by Anne Milliron

Copyright © 2021 by Carnegie Learning, Inc. All rights reserved. Carnegie Learning and MATHia are registered marks of Carnegie Learning, Inc. All other company and product names mentioned are used for identification purposes only and may be trademarks of their respective owners. Permission is granted for photocopying rights within licensed sites only. Any other usage or reproduction in any form is prohibited without the expressed consent of the publisher.

ISBN: 978-1-63862-067-9
Teacher's Implementation Guide, Volume 1

Printed in the United States of America
1 2 3 4 5 6 7 8 9 CC 21 20 19 18 17

Our Manifesto

WE BELIEVE that quality math education is important for all students, to help them develop into creative problem solvers, critical thinkers, life-long learners, and more capable adults.

WE BELIEVE that math education is about more than memorizing equations or performing on tests—it's about delivering the deep conceptual learning that supports ongoing growth and future development.

WE BELIEVE all students learn math best when teachers believe in them, expect them to participate, and encourage them to own their learning.

WE BELIEVE teachers are fundamental to student success and need powerful, flexible resources and support to build dynamic cultures of collaborative learning.

WE BELIEVE our learning solutions and services can help accomplish this, and that by working together with educators and communities we serve, we guide the way to better math learning.

LONG + LIVE + MATH



At Carnegie Learning, we choose the path that has been proven most effective by research and classroom experience. We call that path the Carnegie Learning Way. Follow this code to take a look inside.

ACKNOWLEDGMENTS

Middle School Math Solution Authors

- Sandy Bartle Finocchi, Chief Mathematics Officer
- Amy Jones Lewis, Senior Director of Instructional Design
- Kelly Edenfield, Instructional Designer
- Josh Fisher, Instructional Designer

Foundation Authors (2010)

- William S. Hadley, Algebra and Proportional Reasoning
- Mary Lou Metz, Data Analysis and Probability
- Mary Lynn Raith, Number and Operations
- Janet Sinopoli, Algebra
- Jaclyn Snyder, Geometry and Measurement

Vendors

- Lumina Datamatics, Ltd.
- Cengage Publisher Services, Inc.

Images

- www.pixabay.com

Special Thanks

- Alison Huettner for project management and editorial review.
- Jaclyn Snyder and Janet Sinopoli for their contributions to the Teacher's Implementation Guide facilitation notes.
- Victoria Fisher for her review of content and contributions to all the ancillary materials.
- Valerie Muller for her contributions and review of content.
- The members of Carnegie Learning Cognitive Scientist Team—Brendon Towle, John Connelly, Bob Hausmann, Chas Murray, and Martina Pavelko—for their insight in learning science and review of content.
- Bob Hausmann for his contributions to the Family Guide.
- John Jorgenson, Chief Marketing Officer, for all his insight and messaging.
- Carnegie Learning Education Services Team for content review and providing customer feedback.
- In Memory of David Dengler, Director of Curriculum Development (deceased), who made substantial contributions to conceptualizing Carnegie Learning's middle school software.

ACKNOWLEDGMENTS

Texas Math Solution Content Authors

- Tarin Barrow, STEM Instructional Designer
- Sami Briceño, Senior Custom Solution Content Lead
- Brandy King, Custom Solution Content Specialist
- Christine Mooney, Custom Solution Content Specialist

Texas Math Solution Custom Development Team

- Eddie Altomare
- Katie Barsanti
- Desiree Brown
- Allison Carden
- Courtney Comley
- Jesse Hinojosa
- Karrie Holland
- Bethany Jameson
- Steven Mendoza
- Jennifer Penton
- Jason Ulrich
- Lucy Yu
- Rob Zimmerman

Special Thanks

- The entire Carnegie Learning Production Team, with extreme gratitude for Sara Kozelnik, Julie Leath, Lindsay Ryan, Angela Cerbone, Lenore MacLeod, Hannah Mumm, Olivia Rangel, Chloe Thompson and Emily Tope, for their patience, attention to detail, and around-the-clock hours that made the production of this textbook possible.
- Thank you to all the Texas educators and education professionals who supported the review process and provided feedback for this resource.

“ Mathematics is so much more than rules and algorithms. It is learning to reason, to make connections, and to make sense of the world. We believe in Learning by Doing™—students need to actively engage with the content if they are to benefit from it. Your classroom environment will determine what type of discourse, questioning, and sharing will take place. Students deserve a safe place to talk, to make mistakes, and to build deep understanding of mathematics. My hope is that these instructional materials help you shift the mathematical authority in your class to your students. Be mindful to facilitate conversations that enhance trust and reduce fear. ”

Sandy Bartle Finocchi, Chief Mathematics Officer

“ My hope is that you know that your students are capable of thinking like mathematicians. This book is designed to give them the opportunity to struggle with challenging tasks, to talk about math with their classmates, and to make and fix mistakes. I hope that you use this book to build this capacity in your students—to ask the necessary questions to uncover what students already know and connect it to what they are learning, to encourage creative thinking, and to give just enough support to keep students on the right path. ”

Amy Jones Lewis, Senior Director of Instructional Design

“ At Carnegie Learning, we have created an organization whose mission and culture is defined by student success. Our passion is creating products that make sense of the world of mathematics and ignite a passion in students. Our hope is that students will enjoy our resources as much as we enjoyed creating them. ”

Barry Malkin, CEO

At Carnegie Learning, we choose the path proven most effective by research and classroom experience. We call that path the **Carnegie Learning Way**.

Our Instructional Approach

Carnegie Learning’s instructional approach is based upon the collective knowledge of our researchers, instructional designers, cognitive learning scientists, and master practitioners. It is based on a scientific understanding of how people learn and a real-world understanding of how to apply the science to the classroom. At its core, our instructional approach is based on three simple yet critical components:



ENGAGE

Activate student thinking by tapping into prior knowledge and real-world experiences.

Provide an introduction that generates curiosity and plants the seeds for deeper learning.



DEVELOP

Build a deep understanding of mathematics through a variety of activities.

Students encounter real-world problems, sorting activities, worked examples, and peer analysis—in an environment where collaboration, conversations, and questioning are routine practices.



DEMONSTRATE

Reflect on and evaluate what was learned.

Ongoing formative assessment underlies the entire learning experience, driving real-time adjustments, next steps, insights, and measurements.



Our Research

Carnegie Learning has been deeply immersed in research ever since it was founded by cognitive and computer scientists from Carnegie Mellon University. Our research extends far beyond our own walls, playing an active role in the constantly evolving field of cognitive and learning science. Our internal researchers collaborate with a variety of independent research organizations, tirelessly working to understand more about how people learn, and how learning is best facilitated. We supplement this information with feedback and data from our own products, teachers,

and students, to continuously evaluate and elevate our instructional approach and its delivery.

Our Support

We're all in. In addition to our instructional resources, implementing Carnegie Learning in your classroom means you get access to an entire ecosystem of ongoing classroom support, including:

- **Professional Learning:** Our team of Master Math Practitioners is always there for you, from implementation to math academies to a variety of other options to help you hone your teaching practice.
- **Texas Support Center:** We've customized a Support Center just for you and your students. The Texas Support Center provides articles and videos to help you implement the Texas Math Solution, from the basics to get you started to more targeted support to guide you as you scaffold instruction for all learners in your classroom. Visit www.CarnegieLearning.com/texas-help to explore online and to access content that you can also share with your students and their caregivers.
- **MyCL:** This is the central hub that gives you access to all of the products and resources that you and your students will need. Visit MyCL at www.CarnegieLearning.com/login.
- **LONG + LIVE + MATH:** When you join this community of like-minded math educators, suddenly you're not alone. You're part of a collective, with access to special content, events, meetups, book clubs, and more. Because it's a community, it's constantly evolving! Visit www.longlivemath.com to get started.

Scan this code to visit the Texas Support Center and look for references throughout the Front Matter to learn more about the robust resources you will find in the Support Center.



Our Blend of Learning

The Texas Math Solution delivers instructional resources that make learning math attainable for all students. Learning Together and Learning Individually resources work in parallel to engage students with various learning experiences they need to understand the mathematics at each grade level.

For **Learning Together**, the student textbook is a consumable resource that empowers students to become creators of their mathematical knowledge. This resource is designed to support teachers in facilitating active learning so that students feel confident in sharing ideas, listening to each other, and learning together.

Over the course of a year, based on the recommended pacing, teachers will spend approximately 60% of their instructional time teaching whole-class activities as students learn together.

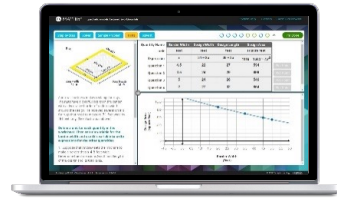
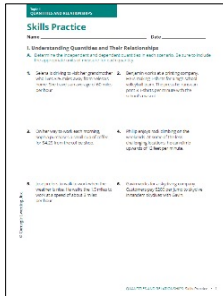
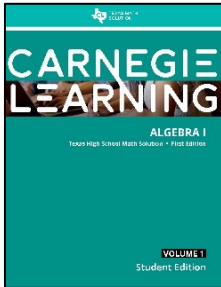
For **Learning Individually**, the Skills Practice provides students the opportunity to engage with problems that target each lesson's skills, concepts, and applications. This resource is designed to target discrete skills for development and mastery, therefore, scaffolding and extension opportunities are provided in the problem sets.

An additional Learning Individually resource is MATHia®, an intelligent software that provides just-in-time support and tracks student progress against fine-grained skills to deliver the right content they need to become proficient with the mathematics.

Over the course of the year, based on the recommended pacing, teachers will spend approximately 40% of their instructional time monitoring students as they work and learn individually.

Learning Together

Learning Individually



TEXTBOOK

I am a record of student thinking, reasoning, and problem solving. My lessons allow students to build new knowledge based upon prior knowledge and experiences, apply math to real-world situations, and learn together in a collaborative classroom.

My purpose is to create mathematical thinkers who are active learners that participate in class.

SKILLS PRACTICE

I am targeted practice of each lesson's skills, mathematical concepts, and applications for each topic in the student textbook.

My purpose is to provide additional problem sets for teachers to assign as needed for additional practice or remediation.

MATHia

I am designed to empower students to learn individually at their own pace with sophisticated AI technology that personalizes their learning experiences, while giving teachers real-time insights to monitor student progress.

My purpose is to coach students alongside teachers as students learn, practice, do, and look forward.



Visit the Texas Support Center for additional information on the Learning Individually resources.

TABLE OF CONTENTS

Module 1: Searching for Patterns

Topic 1: Quantities and Relationships

- 1 A Picture Is Worth a Thousand Words
Understanding Quantities and Their Relationships
- 2 A Sort of Sorts
Analyzing and Sorting Graphs
- 3 F of X
Recognizing Functions and Function Families
- 4 Function Families for 2000, Alex
Recognizing Functions by Characteristics

Topic 2: Sequences

- 1 Is There a Pattern Here?
Recognizing Patterns and Sequences
- 2 The Password Is ... Operations!
Arithmetic and Geometric Sequences
- 3 Did You Mean: *Recursion*?
Determining Recursive and Explicit Expressions from Contexts
- 4 3 Pegs, N Discs
Modeling Using Sequences

Topic 3: Linear Regressions

- 1 Like a Glove
Least Square Regressions
- 2 Gotta Keep It Correlatin'
Correlation

Module 2: Exploring Constant Change

Topic 1: Linear Functions

- 1 Connecting the Dots
Making Connections Between Arithmetic Sequences and Linear Functions
- 2 What's the Point?
Point-Slope Form of a Line
- 3 The Arts Are Alive
Using Linear Equations
- 4 Fun Functions, Linear Ones
Making Sense of Different Representations of a Linear Function
- 5 Move It!
Transforming Linear Functions
- 6 Get A Move On!
Vertical and Horizontal Transformations of Linear Functions
- 7 Amirite?
Determining Slopes of Perpendicular Lines
- 8 Making a Connection
Comparing Linear Functions in Different Forms

Topic 2: Linear Equations and Inequalities

- 1 Strike a Balance
Solving Linear Equations
- 2 It's Literally About Literal Equations
Literal Equations
- 3 Not All Statements Are Made Equal
Modeling Linear Inequalities

Topic 3: Systems of Equations and Inequalities

- 1 The County Fair
Using Substitution to Solve Linear Systems
- 2 Double the Fun
Using Graphing to Solve Systems of Equations
- 3 The Elimination Round
Using Linear Combinations to Solve a System of Linear Equations
- 4 Throwing Shade
Graphing Inequalities in Two Variables
- 5 Working with Constraints
Systems of Linear Inequalities
- 6 Working the System
Solving Systems of Equations and Inequalities

Module 3: Investigating Growth and Decay

Topic 1: Introduction to Exponential Functions

- 1 It's a Generational Thing
Properties of Powers with Integer Exponents
- 2 Show What You Know
Analyzing Properties of Powers
- 3 A Constant Ratio
Geometric Sequences and Exponential Functions
- 4 The Power Within
Rational Exponents and Graphs of Exponential Functions

Topic 2: Using Exponential Equations

- 1 Uptown and Downtown
Exponential Equations for Growth and Decay
- 2 Powers and the Horizontal Line
Interpreting Parameters in Context
- 3 Savings, Tea, and Carbon Dioxide
Modeling Using Exponential Functions
- 4 BAC Is Bad News
Choosing a Function to Model Data

Module 4: Maximizing and Minimizing

Topic 1: Introduction to Quadratic Functions

- 1 Up and Down or Down and Up
Exploring Quadratic Functions
- 2 Endless Forms Most Beautiful
Key Characteristics of Quadratic Functions
- 3 Parabolas in Motion
Quadratic Function Transformations
- 4 Keep It Moving
Transformations of Quadratic Functions
- 5 You Lose Some, You Lose Some
Comparing Functions Using Key Characteristics and Average Rate of Change

Topic 2: Solving Quadratic Equations

- 1 This Time, With Polynomials
Adding, Subtracting, and Multiplying Polynomials
- 2 The Great Divide
Polynomial Division
- 3 Solutions, Plus or Minus
Representing Solutions to Quadratic Equations
- 4 Transforming Solutions
Solutions to Quadratic Equations in Vertex Form
- 5 The Missing Link
Factoring and Completing the Square
- 6 Ladies and Gents, Please Welcome the Quadratic Formula!
The Quadratic Formula
- 7 Fit This Model
Using Quadratic Functions to Model Data

End of Course Topic

Formative Assessment

- 1 Health Club Payment Plans
Performance Task
- 2 Taco Festival
Performance Task
- 3 Randy's Raises
Performance Task
- 4 Undergraduate Tuition
Performance Task

Glossary

In a word, every single piece of Carnegie Learning's Texas Math Solution is intentional. Our instructional designers work alongside our master math practitioners, cognitive scientists, and researchers to intentionally design, draft, debate, test, and revise every piece, incorporating the latest in learning science.

Intentional Mathematics Design

Carnegie Learning's Texas Math Solution is thoroughly and thoughtfully designed to ensure students build the foundation they'll need to experience ongoing growth in mathematics.

Mathematical Coherence

The arc of mathematics develops coherently, building understanding by linking together within and across grades, so students can learn concepts more deeply and apply what they've learned to more complex problems going forward.

Mathematical Process Standards

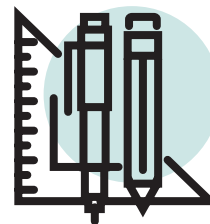
Carnegie Learning is organized around the Mathematical Process Standards to encourage experimentation, creativity, and false starts, which is critical if we expect students to tackle difficult problems in the real world, and persevere when they struggle.

Multiple Representations

Carnegie Learning recognizes the importance of connecting multiple representations of mathematical concepts. Lessons present content visually, algebraically, numerically, and verbally.

Transfer

Carnegie Learning focuses on developing transfer. Doing A and moving on isn't the goal; being able to do A and then do B, C, and D, transferring what you know from A, is the goal.



Texas Math Solution Overview

The instructional materials in the Carnegie Learning Texas Math Solution cover functions, figures, and data sets, from their fundamental concepts to the connections between them. We think about these interrelated ideas in a holistic way to integrate students' understanding with their developing habits of mind.

WHAT ARE THE CARNEGIE LEARNING TEXAS MATH SOLUTION GUIDING PRINCIPLES?

The Texas Math Solution has been strongly influenced by scientific research into the learning process and student motivations for academic success. Its guiding principles are active learning, discourse through collaboration, and personalized learning.

Our classroom activities emphasize active learning and making sense of the mathematics, and we ask deep questions that require students to thoroughly understand the material.

Active Learning

The research makes it clear that students need to actively engage with content in order to benefit from it. Studies show that as instruction moves up the scale from entirely passive to fully interactive, learning becomes more robust. All of the activities we provide for the classroom encourage students to be thoughtful about their work, to consider hypotheses and conclusions from different perspectives, and to build a deep understanding of mathematics. The format of the student text, as a consumable workbook, supports active instruction.

Discourse through Collaborative Learning

Effective collaboration encourages students to articulate their thinking, resulting in self-explanation. Reviewing other students' approaches and receiving feedback on their own provides further metacognitive feedback. Collaborative problem-solving encourages an interactive instructional model, and we have looked to research to provide practical guidance for making collaboration work. The collaborative activities within our lessons are designed to promote active dialogue centered on structured activities.

Personalized Learning

One of the ways to build intrinsic motivation is to relate activities to students' existing interests. Research has proven that problems that capture student interests are more likely to be taken seriously. In the textbook, problems often begin with the students' intuitive understanding of the world and build to an abstract concept, rather than the other way around.

HOW IS THE CONTENT DEVELOPED IN A MATHEMATICALLY COHERENT WAY?

Throughout the high school math courses of the Texas Math Solution, students examine and investigate functions, figures, and data sets. Within each category, we strive to extend and connect students' experience in middle school around the critical mathematical ideas of transformation, equivalence and congruence, and proportionality and similarity.

Functions • Figures • Data Sets

Transformation
Equivalence &
Congruence
Proportionality &
Similarity

Transformation

Transforming functions and figures builds from an understanding of the fundamental behaviors of translations, rotations, reflections, and dilations. These behaviors apply in the same ways to different function types in algebra and to geometric figures on the plane. Understanding the structure of transformations leads to connections across multiple domains in multiple courses.

Equivalence & Congruence

Equivalence is approached in two ways. First, understanding equivalence using multiple relationships of the same function or data set reveals different properties or key characteristics. Second, understanding equivalence in terms of expressions allows students to compose and decompose equations, make sense of solutions, and solve problems. Congruence is treated similarly: understanding congruence using rigid motions highlights key characteristics that are true for both figures, which leads to establishing triangle congruence criteria, an important underpinning for formal proof. The concept of equivalence is extended to the analysis of data, where students learn the critical skill of representing data in equivalent but differently useful ways, enabling them to make analyses and decisions.

Proportionality & Similarity

Developing proportional reasoning is a life-long journey that begins in middle school: from ratios and proportions to understanding how linear functions relate to sequences with common differences and how exponential functions relate to sequences with common ratios. Exploring dilations and the relationships that hold true in similar figures develops spatial reasoning. Analyzing similarity in right triangles extends to right triangle trigonometry, connecting the algebra and geometry domains.

HOW IS THE MATHEMATICS CONTENT DELIVERED TO PROMOTE PRODUCTIVE MATHEMATICAL PROCESSES?

Students deserve math learning that develops them into creative problem solvers, critical thinkers, life-long learners, and more capable adults, while teachers deserve instructional resources that will support them in bringing learning to life. There are three organizing principles that guide these resources.

Seeing Connections

Activities make use of models—e.g., real-world situations, graphs, diagrams, and worked examples—to help students see and make connections between different topics. In each lesson, learning is linked to prior knowledge and experiences so that students build their new understanding on the firm foundation of what they already know. We help students move from concrete representations and an intuitive understanding of the world to more abstract representations and procedures. Activities thus focus on real-world situations to demonstrate the usefulness of mathematics.

Exploring Structure

Questions are phrased in a way that promotes analysis, develops higher-order-thinking skills, and encourages the seeking of mathematical relationships. Students inspect a given function, figure, or data set, and in each case, they are asked to discern a pattern or structure. We want students to become fluent in seeing how the structure of each representation—verbal, graphic, numerical, and algebraic—reveals properties of the function it defines. We want students to become fluent at composing and decomposing expressions, equations, and data sets. We want them to see how the structure of transformations applies to all function types and rigid motions. As students gain proficiency in manipulating structure, they become capable of comparing, contrasting, composing, decomposing, transforming, solving, representing, clarifying, and defining the characteristics of functions, figures, and data sets.

Reflecting and Communicating

A student-centered approach focuses on students thinking about and discussing mathematics as active participants in their own learning. Through articulating their thinking in conversations with a partner, in a group, or as a class, students integrate each piece of new knowledge into their existing cognitive structure. They use new insights to build new connections. Through collaborative activities and the examination of peer work—both within their groups and from examples provided in the lessons—students give and receive feedback, which leads to verifying, clarifying, and/or improving the strategy.

CONTENT AND ALIGNMENT

Algebra I Content at a Glance

This Year at a Glance highlights the sequence of topics and the number of blended instructional days (1 day is a 45-minute instructional session) allocated for Algebra I in the Texas Math Solution. The suggested pacing information includes time for assessments, providing you with an instructional map that covers 180 days of the school year. As you set out at the beginning of the year, we encourage you to still modify this plan as necessary to meet the range of needs for your students.

Texas Algebra I: Year at a Glance

*1 Day Pacing = 45-minute Session

Module	Topic	Pacing	TEKS
Process Standards are embedded in every module: A.1A, A.1B, A.1C, A.1D, A.1E, A.1F, A.1G			
1 Searching for Patterns	1: Quantities and Relationships	13	A.2A, A.3C, A.6A, A.7A, A.9A, A.9D, A.12A
	2: Sequences	14	A.9A, A.9D, A.12A, A.12C, A.12D
	3: Linear Regressions	7	A.3C, A.4A, A.4B, A.4C, A.12A
		34	
2 Exploring Constant Change	1: Linear Functions	28	A.2A, A.2B, A.2C, A.2D, A.2E, A.2F, A.2G, A.3A, A.3B, A.3C, A.3E, A.3F, A.12A, A.12B A.12D
	2: Linear Equations and Inequalities	9	A.2C, A.5A, A.5B, A.12E
	3: Systems of Equations and Inequalities	22	A.2A, A.2C, A.2H, A.2I, A.3D, A.3F, A.3G, A.3H, A.5C
		59	
3 Investigating Growth and Decay	1: Introduction to Exponential Functions	16	A.9B, A.9C, A.9D, A.11A, A.11B, A.12B, A.12D
	2: Using Exponential Equations	13	A.3B, A.3C, A.9A, A.9B, A.9C, A.9D, A.9E, A.11B, A.12B
		29	
4 Maximizing and Minimizing	1: Introduction to Quadratic Functions	20	A.6A, A.6B, A.6C, A.7A, A.7C
	2: Solving Quadratic Equations	25	A.6A, A.7A, A.7B, A.7C, A.8A, A.8B, A.10A, A.10B, A.10C, A.10D, A.10E, A.10F, A.11A
		45	
End of Course Formative Assessment	Performance Tasks	13	A.2B, A.2C, A.2I, A.3B, A.3C, A.3F, A.4A, A.4C, A.5C, A.6C, A.7A, A.8B, A.9C, A.9D, A.9E
		13	
Total Days:		180	

CONNECTING CONTENT AND PRACTICE

Each lesson of the Texas Math Solution has the same structure. This consistency allows both you and your students to track your progress through each lesson. Key features of each lesson are noted.

Lesson Structure

ENGAGE

LESSON STRUCTURE

Each lesson has the same structure. Key features are noted.

1

A Picture Is Worth a Thousand Words

Understanding Quantities and Their Relationships

Warm Up
Emma bought a new video game. The graph shown describes the number of hours Emma spent playing the game over a period of 7 days.

1. Label the axes.
2. What does the highest point on the graph represent with respect to the scenario? The lowest point?

Learning Goals 1

- Understand quantities and their relationships with each other.
- Identify the independent and dependent quantities for a scenario.
- Match a graph with an appropriate scenario.
- Use a reasonable scale for a graph modeling a scenario.
- Identify key characteristics of graphs.
- Describe similarities and differences between pairs of graphs and scenarios.

Key Terms

- dependent quantity
- independent quantity

2 You have analyzed graphs of relationships and identified important features such as intercepts and slopes. How can the key characteristics of a graph tell a story?

LESSON 1: A Picture Is Worth a Thousand Words • 1

1. Learning Goals

Learning goals are stated for each lesson to help you take ownership of the learning objectives.

2. Connection

Each lesson begins with a statement connecting what you have learned with a question to ponder.

Return to this question at the end of this lesson to gauge your understanding.

Establishing Mathematical Goals to Focus Learning

Create a classroom climate of collaboration and establish the learning process as a partnership between you and students.

Communicate continuously with students about the learning goals of the lesson to encourage self-monitoring of their learning.

Visit the Texas Support Center for additional guidance on how to foster a classroom environment that promotes collaboration and communication.



Activating Student Thinking

Your students enter each class with varying degrees of experience and mathematical success. The focus of the Getting Started is to tap into prior knowledge and real-world experiences, to generate curiosity, and to plant seeds for deeper learning.

Pay particular attention to the strategies students use, for these strategies reveal underlying thought processes and present opportunities for connections as students proceed through the lesson.

Supporting Emergent Bilingual Students

Visit the Texas Support Center for facilitation strategies to support students at varying levels of language proficiency as they complete the Getting Started activities in each lesson.

3. Getting Started

Each lesson begins with Getting Started. When working on Getting Started, use what you know about the world, what you have learned previously, or your intuition. The goal is just to get you thinking and ready for what's to come.

When one quantity depends on another in a problem situation, it is said to be the **dependent quantity**. The quantity it depends upon is called the **independent quantity**.

3 GETTING STARTED

What Comes First?

Have you ever planned a party? You may have purchased ice, gone grocery shopping, selected music, made food, or even cleaned in preparation. Many times, these tasks depend on another task being done first. For instance, you wouldn't make food before grocery shopping, now would you?

Consider the two quantities that are changing in each relationship.

- the number of movie tickets purchased and the total cost
- the number of eggs used and the number of cakes baked
- the number of students in attendance at school and the number of lunches served
- the number of hours driven and the number of miles to a vacation destination
- the number of minutes a swimming pool is filled with water and the number of gallons of water in the swimming pool

1. Circle the independent quantity and underline the dependent quantity in each relationship.

2. Describe how you can determine which quantity is independent and which quantity is dependent in any problem situation.

2 • TOPIC 1: Quantities and Relationships

FM-14 • Lesson Structure



Mathematics is the science of patterns. So, we encourage students throughout this course to notice, test, and interpret patterns in a variety of ways—to put their “mental tentacles” to work in every lesson, every activity. Our hope is that this book encourages you to do the same for your students, and create an environment in your math classroom where productive and persistent learners develop and thrive.



Josh Fisher, Instructional Designer



DEVELOP

Aligning Teaching to Learning

Students learn when they are actively engaged in a task: reasoning about the math, writing their solutions, justifying their strategies, and sharing their knowledge with peers.

Support productive struggle by allowing students time to engage with and persevere through the mathematics.

Support student-to-student discourse as well as whole-class conversations that elicit and use evidence of student thinking.

4 **ACTIVITY 1.1** **Connecting Scenarios and Their Graphs**

While a person can describe the monthly cost to operate a business, or talk about a marathon pace a runner ran to break a world record, graphs on a coordinate plane enable people to see the data. Graphs relay information about data in a visual way.

You can use lines or smooth curves to represent relationships between points on a graph. In some problem situations, all the points on the line will make sense. So to you to values sh

This activ end of the

1. Read deeper For ea quant mean

Music Cl Jermaine notice. He becoming wants. Jer

• inde

• dep

ACTIVITY 1.2 **Comparing and Contrasting Graphs**

Now that you have matched a graph with the appropriate problem situation, let's go back and examine all the graphs.

1. What similarities do you notice in the graphs?

2. What differences do you notice in the graphs?

3. How did you label the independent and dependent quantities in each graph?

4. Analyze each graph from left to right. Describe any graphical characteristics you notice.

Think about:
Look closely when analyzing the graphs. What do you see?

LESSON 1: A Picture Is Worth a Thousand Words • 7

4. Activities

You are going to build a deep understanding of mathematics through a variety of activities in an environment where collaboration and conversations are important and expected.

You will learn how to solve new problems, but you will also learn why those strategies work and how they are connected to other strategies you already know.

Remember:

- It's not just about answer-getting. The process is important.
- Making mistakes are a critical part of learning, so take risks.
- There is often more than one way to solve a problem.

Activities may include real-world problems, sorting activities, worked examples, or analyzing sample student work.

Be prepared to share your solutions and methods with your classmates.

Lesson Structure • FM-15

Supporting Emergent Bilingual Students

Visit the Texas Support Center for facilitation strategies to support students at varying levels of language proficiency as they engage in mathematical discourse throughout each lesson.



DEMONSTRATE

Ongoing Formative Assessment Drives Instruction

For students to take responsibility for their own learning, they need to be encouraged to self-assess. Students can use the Talk the Talk to monitor their own progress towards mastering the learning goals. Listen and review their answers and explanations and provide feedback to help them improve their understanding.

As you plan the next lesson, consider the connections you can make to build off the strengths or fill any gaps identified from this formative assessment.

5. Talk the Talk

Talk the Talk gives you an opportunity to reflect on the main ideas of the lesson.

- Be honest with yourself.
- Ask questions to clarify anything you don't understand.
- Show what you know!

Don't forget to revisit the question posed on the lesson opening page to gauge your understanding.

5 TALK the TALK

A Writer and a Mathematician

1. Write a scenario and sketch a graph to describe a possible trip to school.

Scenario _____

Graph

2. Describe the meaning of the points, or smooth curve, represented by your graph.

3. Compare your scenario and sketch with your classmates' scenarios and sketches. What similarities do you notice? What differences do you notice?

LESSON 1: A Picture Is Worth a Thousand Words • 9

FM-16 • Lesson Structure

Student Lesson Overview Videos

Each lesson has a corresponding lesson overview video(s) for students to utilize and reference to support their learning. The videos provide an overview of key concepts, strategies, and/or worked examples from the lessons.

Assignment

An intentionally designed Assignment follows each lesson.

ASSIGNMENT

Assignment

LESSON 1: A Picture Is Worth a Thousand Words

6 Write
Describe how you can distinguish between an independent quantity and a dependent quantity. Use an example in your description.

7 Remember
When one quantity is determined by another in a problem situation, it is said to be the dependent quantity. The quantity it is determined from is called the independent quantity. The independent quantity is represented on the x-axis and the dependent quantity is represented on the y-axis.

8 Practice
1. Read each scenario and identify the independent and dependent quantities. Be sure to include the appropriate units of measure. Then analyze each graph and determine which of the provided scenarios it models. For each graph, label the x- and y-axis with the appropriate quantity and unit of measure.

a. **Endangered Species**
The Elkwood Aquatic with various reptile populations. The initial endangered turtles that past five years.

b. **Sales Commission**
Julian works as a sales monthly salary of \$30 commission on the

c. **Commuter Flight**
A commuter flight between Oregon takes about 4 increases its altitude the flight until it gets then it descends for flight. The plane ascends constant rate of 9000

A.

D.

E.

2. Compare the pair of graphs and describe any similarities and differences you notice.

9 Stretch
Read the scenario and identify the independent and dependent quantities. Be sure to include the appropriate units of measure.

1. A student performs several experiments in which he swings a pendulum for a 20-second duration. He uses a string that is 27 cm long, and he tests pendulum masses of different sizes, varying from 2 to 12 grams. He records the number of swings each pendulum makes in 20 seconds.

2. The student then decides to make a second graph showing the string length (in cm) as the independent quantity. What changes must the student make to his experiment?

10 Review

1. Solve the equation $-2x + 8 = -3x + 14$.

2. Evaluate the expression $x^2 - 3y + 12$ for $x = -2$ and $y = 5$.

2 • TOPIC 1: Quantities and Relationships

6. Write
Reflect on your work and clarify your thinking.

7. Remember
Take note of the key concepts from the lesson.

8. Practice
Use the concepts learned in the lesson to solve problems.

9. Stretch
Ready for a challenge?

10. Review
Remember what you've learned by practicing concepts from previous lessons and topics.

There is one Assignment per lesson. Lessons often span multiple days. Be thoughtful about which portion of the Assignment students can complete based on that day's progress.

The **Stretch** section is not necessarily appropriate for all learners. Assign this to students who are ready for more advanced concepts.

The **Review** section provides spaced practice of concepts from the previous lesson and topic and of the fluency skills important for the course.

Topic Summary

A Topic Summary is provided for students at the end of each topic. The Topic Summary lists all key terms of the topic and provides a summary of each lesson. Each lesson summary defines key terms and reviews key concepts, strategies, and/or worked examples.

Quantities and Relationships Summary

KEY TERMS

- dependent quantity
- independent quantity
- relation
- domain
- range
- function
- function notation
- Vertical Line Test
- discrete graph
- continuous graph
- increasing function
- decreasing function
- constant function
- function family
- linear functions
- exponential functions
- absolute maximum
- absolute minimum
- quadratic functions
- linear absolute value functions
- x-intercept
- y-intercept

LESSON 1

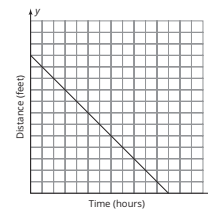
A Picture Is Worth a Thousand Words

Many problem situations are related to each other, it is said to be dependent upon is called

Graphs relay information in a line or smooth curve graphed on the horizontal x-axis. Graphs can be s

For example, consider the graph which models the situation where Pedro is hiking in a canyon. At the start of his hike, he was at 3500 feet. During the first 20 minutes of the hike, he descended 500 feet at a constant rate. Then he rested for half an hour before continuing the hike at the same rate.

Time is the independent quantity and distance is the dependent quantity.



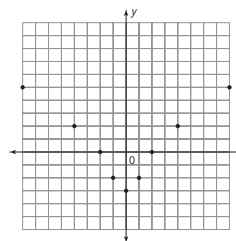
LESSON 2

A Sort of Sorts

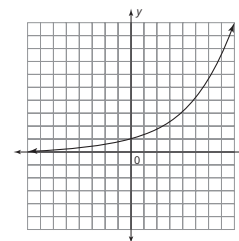
Looking for patterns can help when sorting and comparing graphs. Some graphs show vertical symmetry (if a vertical line were drawn through the middle of the graph, the image is the same on both sides). Other possible patterns to look for include: only goes through two quadrants, always increasing from left to right, always decreasing from left to right, straight lines, smooth curves, the graph goes through the origin, the graph forms a U shape, the graph forms a V shape.

For example, Graph A has vertical symmetry. Graph B is a smooth curve that increases from left to right.

Graph A



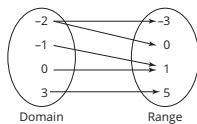
Graph B



A **relation** is the mapping between a set of input values called the **domain** and a set of output values called the **range**.

A **function** is a relation between a given set of elements, such that for each element in the domain there exists exactly one element in the range. If each value in the domain has one and only one range value, then the relation is a function. If any value in the domain has more than one range value, then the relation is not a function.

The value -2 in the domain has more than one range value. The mapping does not represent a function.



Each element in the domain has exactly one element in the range. The table represents a function.

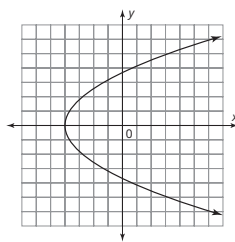
Domain	Range
2	1
6	3
10	5
14	7

Functions can be represented in a number of ways. An equation representing a function can be written using **function notation**. Function notation is a way of representing functions algebraically. This form allows you to more efficiently identify the independent and dependent quantities. The function $f(x)$ is read as "f of x" and indicates that x is the independent variable.

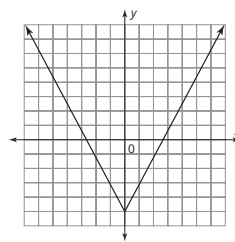
For example, consider the situation in which U.S. Shirts charges \$8 per shirt plus a one-time charge of \$15 to set up a T-shirt design. The equation that models the situation, $y = 8x + 15$, where x represents the number of shirts ordered and y represents the total cost of the order, can be written in function notation as $f(x) = 8x + 15$. The cost, defined by f , is a function of x , defined as the number of shirts ordered.

The **Vertical Line Test** is a visual method used to determine whether a relation represented as a graph is a function. To apply the Vertical Line Test, consider all of the vertical lines that could be drawn on the graph of a relation. If any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function. The Vertical Line Test applies to both discrete and continuous graphs. A **discrete graph** is a graph of isolated points. A **continuous graph** is a graph of points that are connected by a line or smooth curve with no breaks in the graph.

A line drawn vertically through the graph touches more than one point. The graph does not represent a function.



A line drawn vertically through the graph only touches one point. The graph represents a function.



A function is described as increasing when both the independent and dependent variables are increasing. If a function increases across the entire domain, then the function is called an **increasing function**. A function is described as decreasing when the dependent variable decreases as the independent variable increases. If a function decreases across the entire domain, then the function is called a **decreasing function**. If the dependent variable of a function does not change or remains constant over the entire domain, then the function is called a **constant function**.

A **function family** is a group of functions that share certain characteristics.

Problem Types You Will See

Lessons include a variety of problem types to engage students in reasoning about the math.

Worked Examples

Research shows students learn best when they are actively engaged with a task. Many students need a model to know how to engage effectively with Worked Examples. Students need to be able to question their understanding, make connections with the steps, and ultimately self-explain the progression of the steps and the final outcome. Worked Examples provide a means for students to view each step taken to solve the example problem. The questions that follow are designed to serve as a model for self-questioning and self-explanations. They represent and mimic an internal dialogue about the mathematics and the strategies. This approach doesn't allow students to skip over the example without interacting with it, thinking about it, and responding to the questions. This approach will help students develop the desired habits of mind for being conscientious about the importance of steps and their order.

PROBLEM TYPES YOU WILL SEE

Worked Example

You can represent a_n using function notation.

$$a_n = 2 + 4(n - 1)$$
$$f(n) = 2 + 4(n - 1)$$


Next, rewrite the expression $2 + 4(n - 1)$.

$$f(n) = 2 + 4n - 4 \quad \text{Distributive Property}$$
$$= 4n + 2 - 4 \quad \text{Commutative Property}$$
$$= 4n - 2 \quad \text{Combine Like Terms}$$


So, $a_n = 2 + 4(n - 1)$ written in function notation is $f(n) = 4n - 2$.

Maya and Sherry each convert the given formula to degrees Fahrenheit.

Maya


$$C = \frac{5}{9}(F - 32)$$
$$C = \frac{5}{9}F - \frac{160}{9}$$
$$9(C) = 9\left(\frac{5}{9}F - \frac{160}{9}\right)$$
$$9C = 5F - 160$$
$$9C + 160 = 5F$$
$$\frac{9C}{5} + \frac{160}{5} = \frac{5F}{5}$$
$$\frac{9}{5}C + 32 = F$$

Sherry


$$C = \frac{5}{9}(F - 32)$$
$$C = \frac{5}{9}F - 32$$
$$9(C) = 9\left(\frac{5}{9}F - 32\right)$$
$$9C = 5F - 288$$
$$9C + 288 = 5F$$
$$\frac{9C}{5} + \frac{288}{5} = \frac{5F}{5}$$
$$\frac{9}{5}C + 57.6 = F$$

FM-18 • Problem Types You Will See

Worked Example

When you see a Worked Example:

- Take your time to read through it.
- Question your own understanding.
- Think about the connections between steps.

Ask Yourself:

- What is the main idea?
- How would this work if I changed the numbers?
- Have I used these strategies before?

Thumbs Up

When you see a Thumbs Up icon:

- Take your time to read through the correct solution.
- Think about the connections between steps.

Ask Yourself:

- Why is this method correct?
- Have I used this method before?

Thumbs Down

When you see a Thumbs Down icon:

- Take your time to read through the incorrect solution.
- Think about what error was made.

Ask Yourself:

- Where is the error?
- Why is it an error?
- How can I correct it?

Thumbs Up/Thumbs Down

Thumbs Up problems provide a framework that allows students the opportunity to analyze viable methods and problem-solving strategies. Questions are presented to help students think deeper about the various strategies, and to focus on an analysis of correct responses. Research shows that only providing positive examples does not eliminate some of the things students may think; it is also efficient to show negative examples. From the Thumbs Down incorrect responses, students learn to determine where the error in calculation is, why the method is an error, and also how to correct the method to correctly calculate the solution.

4. Carlos and Mikala do not like working with fractions. Each rewrites the equation so that it does not have fractions. Their work is shown.

Carlos

$$F = \frac{9}{5}C + 32$$

$$(5)F = 5\left(\frac{9}{5}C + 32\right)$$

$$5F = 9C + 160$$

$$5F - 9C = 160$$

Mikala

$$C = \frac{5}{9}(F - 32)$$

$$(9)C = (9)\left(\frac{5}{9}(F - 32)\right)$$

$$9C = 5(F - 32)$$

$$9C = 5F - 160$$

$$9C - 5F = -160$$

Carlos and Mikala got two different equations. Who is correct? Explain your reasoning.



Who's Correct?

When you see a Who's Correct icon:

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine if correct or not correct.

Ask Yourself:

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

Who's Correct?

Who's Correct? problems are an advanced form of correct vs. incorrect responses. In this problem type, students are not given who is correct. Students have to think more deeply about what the strategies really mean and whether the solutions makes sense. Students will determine what is correct and what is incorrect, and then explain their reasoning. These types of problems will help students analyze their own work for errors and correctness.

Promoting Self-Reflection

Thought Bubbles

The thought bubbles embedded throughout the Texas Math Solution promote productive reflection by reminding students to stop and think. This feature is used in a variety of ways: it may remind students to recall a previous mathematical concept, help students develop expertise to think through problems, and occasionally, present a fun fact.

Thought Bubbles

Look for these icons as you journey through the textbook. Sometimes they will remind you about things you already learned. Sometimes they will ask you questions to help you think about different strategies. Sometimes they will share fun facts. They are here to help and guide your learning.



Side notes are included to provide helpful insights as you work.



A mathematician is an artist who works with patterns. I think the beauty of mathematics lies in the new connections you can make to express the patterns around you, no matter your age. The art is in the process, not the outcome. When we can get students to see the beauty of the mathematics, and equip them with the tools to express themselves mathematically, then we can truly create critical thinkers.



Victoria Fisher, Instructional Designer

Mathematical Process Standards

MATHEMATICAL PROCESS STANDARDS

Texas Mathematical Process Standards

Effective communication and collaboration are essential skills of a successful learner. With practice, you can develop the habits of mind of a productive mathematical thinker. The “I can” expectations listed below align with the TEKS Mathematical Process Standards and encourage students to develop their mathematical learning and understanding.

► Apply mathematics to problems arising in everyday life, society, and the workplace.

I can:

- use the mathematics that I learn to solve real world problems.
- interpret mathematical results in the contexts of a variety of problem situations.

► Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem-solving process and reasonableness of the solution.

I can:

- explain what a problem “means” in my own words.
- create a plan and change it if necessary.
- ask useful questions in an attempt to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

FM-20 • Mathematical Process Standards

Note

Each lesson provides opportunities for students to think, reason, and communicate their mathematical understanding. However, it is your responsibility as a teacher to recognize these opportunities and incorporate these practices into your daily rituals. Expertise is a long-term goal, and students must be encouraged to apply these practices to new content throughout their school career.

Supporting Students to Use Mathematical Tools

Visit the Texas Support Center for strategies to support students as they use mathematical tools, including formula charts and reference sheets.



Note

When you are facilitating each lesson, listen carefully and value diversity of thought, redirect students' questions with guiding questions, provide additional support with those struggling with a task, and hold students accountable for an end product. When students share their work, make your expectations clear, require that students defend and talk about their solutions, and monitor student progress by checking for understanding.

There is one more page of mathematical process standards that is not provided here, but is available in the Student Textbook Front Matter.

► **Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate; and techniques including mental math, estimation, and number sense as appropriate, to solve problems.**

I can:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.

► **Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.**

I can:

- communicate and defend my own mathematical understanding using examples, models, or diagrams.
- use appropriate mathematical vocabulary in communicating mathematical ideas.
- make generalizations based on results.
- apply mathematical ideas to solve problems.
- interpret my results in terms of various problem situations.

► **Create and use representations to organize, record, and communicate mathematical ideas.**

I can:

- consider the units of measure involved in a problem.
- label diagrams and figures appropriately to clarify the meaning of different representations.
- create an understandable representation of a problem situation.

Mathematical Process Standards • FM-21



Supporting ALL Learners

Visit the Texas Support Center for facilitation strategies to support ALL students as they engage in the Mathematical Process Standards.

Academic Glossary

ACADEMIC GLOSSARY

There are important terms you will encounter throughout this book. It is important that you have an understanding of these words as you get started on your journey through the mathematical concepts. Knowing what is meant by these terms and using these terms will help you think, reason, and communicate your ideas.

Visit the Students & Caregivers Portal on the Texas Support Center at www.CarnegieLearning.com/texas-help to access the Mathematics Glossary for this course anytime, anywhere.



ANALYZE

Definition

To study or look closely for patterns. Analyzing can involve examining or breaking a concept down into smaller parts to gain a better understanding of it.

Ask Yourself

- Do I see any patterns?
- Have I seen something like this before?
- What happens if the shape, representation, or numbers change?

Related Phrases

- Examine
- Evaluate
- Determine
- Observe
- Consider
- Investigate
- What do you notice?
- What do you think?
- Sort and match

EXPLAIN YOUR REASONING

Definition

To give details or describe how to determine an answer or solution. Explaining your reasoning helps justify conclusions.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Does my reasoning make sense?
- How can I justify my answer to others?

Related Phrases

- Show your work
- Explain your calculation
- Justify
- Why or why not?

Academic Glossary • FM-23

Language Expectations

It is critical for students to possess an understanding of the language of their text. Students must learn to read for different purposes and write about what they are learning. Encourage students to become familiar with the key words and the questions they can ask themselves when they encounter these words.

It is our recommendation to be explicit about your expectations of language used and the way students write responses throughout the text. Encourage students to answer questions with complete sentences. Complete sentences help students reflect on how they arrived at a solution, make connections between topics, and consider what a solution means both mathematically as well as in context.

Supporting Students at Varying Levels of Language Proficiency

Visit the Texas Support Center for guidance on how to leverage the Academic Glossary to support students at varying levels of language proficiency.



Ask Yourself

The Ask Yourself questions help students develop the proficiency to explain to themselves the meaning of problems.

Real-World Context

Real-world contexts confirm concrete examples of mathematics. The scenarios in the lessons help students recognize and understand that quantitative relationships seen in the real world are no different that quantitative relationships in mathematics. Some problems begin with a real-world context to remind students that the quantitative relationships they already use can be formalized mathematically. Other problems will use real-world situations as an application of mathematical concepts.

Related Phrases

- Show
- Sketch
- Draw
- Create
- Plot
- Graph
- Write an equation
- Complete the table

REPRESENT

Definition

To display information in various ways. Representing mathematics can be done using words, tables, graphs, or symbols.

Ask Yourself

- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Is my representation accurate?

Related Phrases

- Predict
- Approximate
- Expect
- About how much?

ESTIMATE

Definition

To make an educated guess based on the analysis of given data. Estimating first helps inform reasoning.

Ask Yourself

- Does my reasoning make sense?
- Is my solution close to my estimation?

Related Phrases

- Demonstrate
- Label
- Display
- Compare
- Determine
- Define
- What are the advantages?
- What are the disadvantages?
- What is similar?
- What is different?

DESCRIBE

Definition

To represent or give an account of in words. Describing communicates mathematical ideas to others.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?

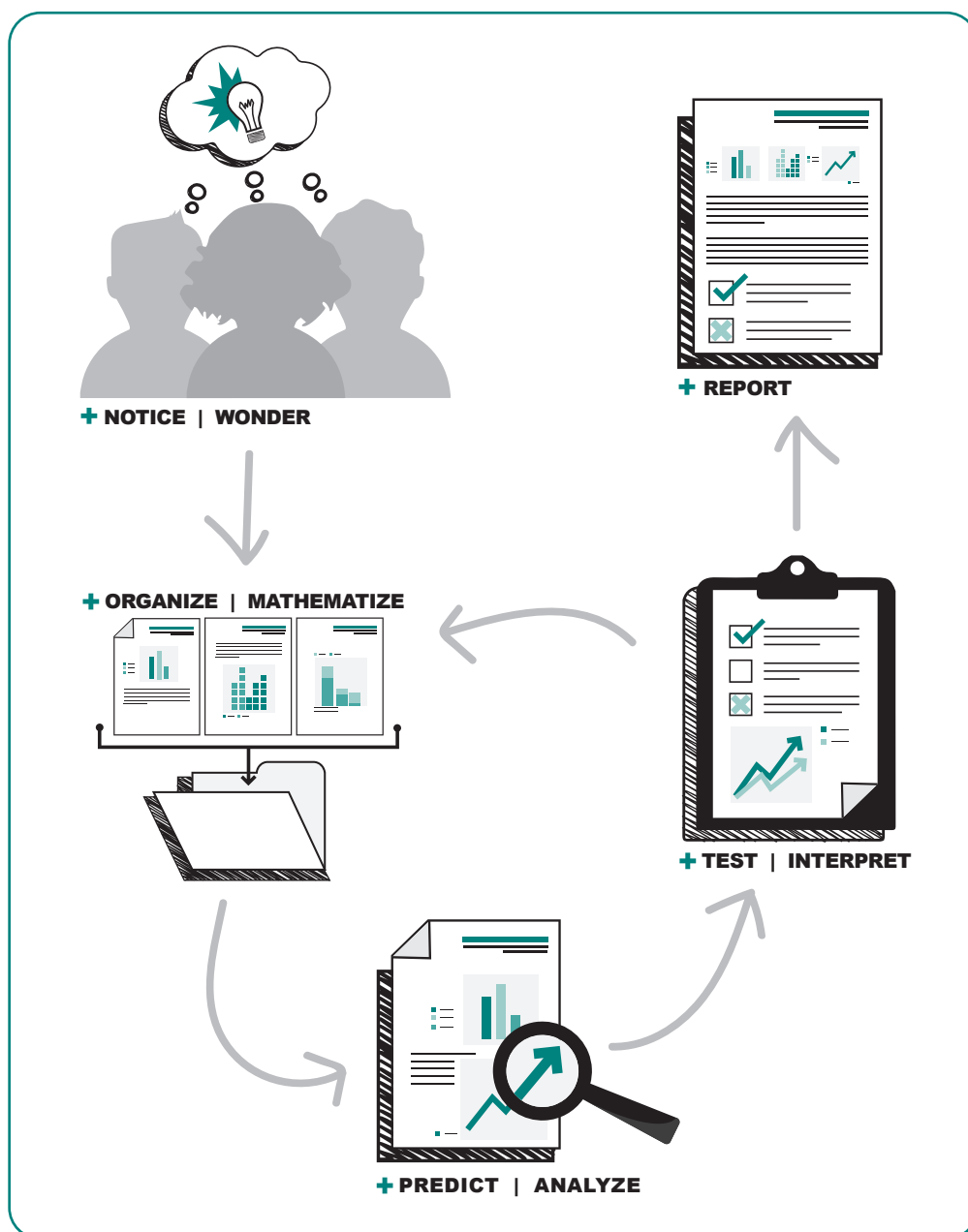
FM-24 • Academic Glossary

Mathematics Glossary

A course-specific mathematics glossary is available for students to utilize and reference during their learning. Definitions and examples of key terms are provided in the glossary.

The Modeling Process

Modeling is the process of choosing appropriate mathematical tools to analyze and understand real-world phenomena and to make decisions accordingly. The Modeling Process provides a structure to help students become better problem solvers. In the textbook, students will encounter activities that explicitly guide them through the four steps of the Modeling Process. As they progress through high school mathematics, they should start to use this process intuitively.



Notice and Wonder

Gather information, notice patterns, and formulate mathematical questions about what you notice.

Organize and Mathematize

Organize your information and represent it using mathematical notation.

Predict and Analyze

Extend the patterns created, complete operations, make predictions, and analyze the mathematical results.

Test and Interpret

Interpret your results and test your mathematical predictions in the real world. Make adjustments as necessary.

FACILITATING STUDENT LEARNING

Teacher's Implementation Guide

The Teacher's Implementation Guide (TIG) is designed to fully support a wide range of teachers implementing our materials: from first-year teachers to 30-year veterans and from first-time Carnegie Learning users to master practitioners.

One goal in developing the TIG was to make our instructional design apparent to the users.

The lessons of each topic were written to be accessible to the full range of learners. With every instructional decision you make, keep in mind your mathematical objectives for the topic and module and the course. Plan each lesson by thinking about how you will create access for your particular group of students, maintain access and pace throughout the lesson, and assess their understanding along the way. We recommend that you do the math in each topic before implementing the activities with your specific group of students.

WHAT MAKES THIS TIG USEFUL?

Effective Lesson Design

Each lesson has a consistent structure for teachers and students to follow. The learning experiences are engaging and effective for students.

Pacing

Each course is designed to be taught in a 180-day school year. Pacing suggestions are provided for each lesson. Each day in the pacing guide is equivalent to a 45-minute instructional session.

Instructional Supports

Guiding questions are provided for teachers to use as they're circulating the room, as well as differentiation strategies, common student misconceptions, and student look-fors.

Clearly Defined Mathematics

The content and instructional goals are clearly described at the module, topic, lesson, and activity levels.

The TIG is critical to understanding how the mathematics that students encounter should be realized in the classroom. The TIG describes the depth of understanding that students need to develop for each standard and a pathway for all learners to be successful. It provides differentiation strategies to support students who struggle, to extend certain activities for students who are advanced in their understanding of the content, and to support emergent bilingual students.

Visit the Texas Support Center at www.CarnegieLearning.com/texas-help for additional resources to support you anytime, anywhere.



Module and Topic Overviews

“Teachers must first develop their ideas about where the curriculum program is going mathematically (curriculum vision) before deciding whether the curriculum materials will help them reach that mathematical goal (curriculum trust)” (Drake & Sherin, 2009, p. 325).

You are responsible for teaching the essential concepts associated with a particular course. You need to understand how activities within lessons build to achieve understanding within topics, and how topics build to achieve understanding throughout the module and course. In the Texas Math Solution, Carnegie Learning seeks to establish a shared curriculum vision with you.

Module 1 Overview
Searching for Patterns

“Critical to algebraic thinking is the ability to identify situations in which important mathematical relationships exist. Thinking: A Guide for Teachers”

Why is the Searching for Patterns?

Students have been searching for patterns in math class since kindergarten. They recognize patterns in a pattern beyond a given list. This module is designed to help students understand of function families, including linear, exponential, quadratic, and absolute value functions.

Throughout the module, students explore relationships between quantities. In this topic, students explore relationships presented as graphs and equations. They investigate their differences. Once they recognize patterns and equations, they search for their corresponding functions. In the second topic, students explore sequences of numbers that while all sequences arithmetic sequences, some geometric sequences. Finally, students can be used to model

Quantities and Relationships
Topic 1 Overview

How is Quantities and Relationships organized?

In *Quantities and Relationships*, students encounter different scenarios representing the functions they will study throughout the course. The intent is merely to introduce these new functions, providing an overview but not a deep understanding at this point. The topic is designed to help students recognize that different function families have different key characteristics. In later study—both in this course and in future courses—they will formalize their understanding of the defining characteristics of each type of function.

Students begin by analyzing real-world scenarios. These scenarios move beyond the linear relationships familiar from middle school to include various nonlinear functions. Students connect the scenarios to corresponding graphs. They examine the graphical behavior of different function types by exploring a wide variety of graphs. Students search for patterns in the graphs’ shape and structure, and then sort them according to defined characteristics.

Students review the definition of *function*, *domain*, and *range*. Building on their knowledge from middle school, they formalize their representations of functions by writing equations in function notation. They use graphical behavior and the structure of the corresponding equations to classify each function according to its function family. Finally, with a more thorough understanding of the key characteristics of graphs of functions, students return to the scenarios from the first lesson and define each in terms of function family and graphical behavior.

At the end of *Quantities and Relationships*, students create a graphic organizer with the equation representation and graphical behavior of each function family studied in this course: linear, exponential, quadratic, and linear absolute value.

What is the entry point for students?

Throughout middle school, students explored relationships between quantities. In grade 6, students defined independent and dependent variables and used them to write equations and create tables and graphs for various relationships. In grade 8, students defined a function and used linear functions to model the relationship between two quantities. Students have written linear functions in slope-intercept form and should be able to identify the slope and *y*-intercept in the equation. *Quantities and Relationships* introduces formal function notation as a natural extension of the informal expression evaluation mastered in grades 6 through 8. In grade 8, students learned that a graph of a function is the set of ordered pairs consisting of an input and the corresponding output. They characterized graphs as functions using the terms *increasing*, *decreasing*, *constant*, *discrete*, *continuous*, *linear*, and *nonlinear*. In *Quantities and Relationships*, students build on these characteristics to define new function families.

TOPIC 1: Quantities and Relationships • 1

Module Overview

Each module begins with an overview that describes the reasoning behind the name, the mathematics being developed, the connections to prior learning, the connections to future learning.

Topic Overview

A Topic Overview describes how the topic is organized, the entry point for students, how a student will demonstrate understanding, why the mathematics is important, how the activities promote expertise in the mathematical process standards, what materials are needed, examples of new tools and notations, and more detailed information to help with pacing.

Facilitation Notes

For each lesson, you are provided with detailed facilitation notes to fully support your planning process. This valuable resource provides point-of-use support that serves as your primary resource for planning, guiding, and facilitating student learning.

1. Materials

Materials required for the lesson are identified.

2. Lesson Overview

The Lesson Overview sets the purpose and describes the overarching mathematics of the lesson, explaining how the activities build and how the concepts are developed.

3. TEKS Addressed

The focus TEKS for each lesson are listed. Carnegie Learning recognizes that some lessons could list several TEKS based on the skills needed to complete the activities, however, the TEKS listed are what the lesson is focused on developing or mastering.

4. ELPS Addressed

The English Language Proficiency Standards for each lesson are listed. As you plan, consider these ELPS and determine the instructional strategies that you will use to meet these ELPS.

1

Is There a Pattern Here?

Recognizing Patterns and Sequences

MATERIALS ①
None

② **Lesson Overview**
Students begin by exploring various patterns in Pascal's triangle. *Sequence* and *term of a sequence* are defined. Given ten geometric patterns or contexts, students write a numeric sequence to represent each problem. They are guided to represent each sequence as a table of values and conclude that all sequences are functions. Students then organize the sequences in a table, state whether each sequence is increasing or decreasing, and describe the sequence using a starting value and operation. They determine that all sequences have a domain that includes only positive integers. *Infinite sequence* and *finite sequence* are defined and included as another characteristic for students to consider as they write sequences.

③ **Algebra I**
Exponential Functions and Equations
(9) **The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:**
(A) determine the domain and range of exponential functions of the form $f(x) = ab^x$ and represent the domain and range using inequalities.

Number and Algebraic Methods
(12) **The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:**
(A) decide whether relations represented verbally, tabularly, graphically, and symbolically define a function.
(D) write a formula for the n^{th} term of arithmetic and geometric sequences, given the value of several of their terms.

④ **ELPS**
1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G

LESSON 1: Is There a Pattern Here? • 1

reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:

(D) graph exponential functions that model growth and decay and identify key features, including y-intercept and asymptote, in mathematical and real-world problems.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G

5 Essential Ideas

- There are two quantities that change in problem situations.
- When one quantity depends on another, it is said to be the dependent quantity. The quantity that the dependent quantity depends upon is called the independent quantity.
- The independent quantity is used to label the x -axis. The dependent quantity is used to label the y -axis.
- The domain includes the values that make sense for the independent quantity. The range includes the values that make sense for the dependent quantity.
- Graphs can be used to model problem situations.

6 Lesson Structure and Pacing: 1 Day 7

Day 1

Engage

Getting Started: What Comes First?

Students read descriptions of relationships between two quantities and identify which is independent and which is dependent.

Develop

Activity 1.1: Connecting Scenarios and Their Graphs

Students are presented with six different scenarios. For each scenario, they identify the independent and dependent quantities and match a graph. Students then scale the axes and determine the domain and range for each scenario.

Day 2

Activity 1.2: Comparing and Contrasting Graphs

Students make basic observations about the similarities and differences in the graphs from the previous activity. They then look more deeply at pairs of scenarios along with their graphs to focus on key characteristics, such as intercepts, increasing and decreasing intervals, and maximum and minimum points.

Demonstrate

Talk the Talk: A Writer and a Mathematician

Students create a scenario based upon a possible trip to school. They then sketch a graph to model their scenario. Students share their work with classmates and note similarities and differences.

2 • TOPIC 1: Quantities and Relationships

5. Essential Ideas

These statements are derived from the standards and state the concepts students will develop.

6. Lesson Structure

This section highlights how the parts of the lesson fit within the instructional design: Engage, Develop, and Demonstrate. A summary of each activity included.

7. Pacing

Lessons often span more than one 45-minute instructional session. Suggested pacing is provided for each lesson so that the entire course can be completed in a 180-day school year.

8. Facilitation Notes by Activity

A detailed set of guidelines walks the teacher through implementing the Getting Started, Activities, and Talk the Talk portions of the lesson. These guidelines include an activity overview, grouping strategies, guiding questions, possible student misconceptions, differentiation strategies, student look-fors, and an activity summary.

9. Activity Overview

Each set of Facilitation Notes begins with an overview that highlights how students will actively engage with the task to achieve the learning goals.

10. Differentiation Strategies

To scaffold instruction, suggestions are provided on additional scaffolding or alternative methods of instruction to ensure all students fully engage in the lesson.

Getting Started: What Comes First?

ENGAGE

8

Facilitation Notes

In this activity, students read descriptions of relationships between two quantities and identify which is the independent and which is the dependent.

Ask a student to read the introduction before Question 1 aloud. Review the definitions of dependent quantity and independent quantity as a class.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

As students work, look for

Strategies and phrases they use to determine which quantity depends on the other.

Questions to ask

- Which quantity for...
- Which quantity dep...
- What information it purchased or the t...
- Does the number of the tickets, or does of movie tickets pu...
- What information it the number of cake...
- Does the number of the number of lunch...
- Does the time drive destination, or doe determine the time...
- Does the number of depend on the nur gallons of water de pool is being filled...

Differentiation strate

To scaffold support wh independent, provide a For example, ask stude is the cost of two movi tickets?" Use a follow u the other, or which val

Misconception

Students may confuse the independent variable with the dependent variable. For example, they could think the number of movie tickets is determined by the total cost of the tickets (if the cost of three tickets is \$22.50, then each ticket must have been \$7.50). Just because the value of one variable can be determined using the value of a second variable, this does not signify dependence or independence.

Summary

There are two quantities that change in problem situations. When one quantity depends on another, it is said to be the dependent quantity. The quantity that the dependent quantity depends upon is called the independent quantity.

DEVELOP

Activity 1.1

Connecting Scenarios and Their Graphs



9

Facilitation Notes

In this activity, students are presented with eight different scenarios. For each scenario, they identify the independent and dependent quantities and match a graph. Students then scale the axes and determine the domain and range for each scenario.

Ask a student to read the introduction before Question 1 aloud. As a class, discuss the directions to this task because it has several parts and includes cutting out and gluing graphs next to their scenario descriptions.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

As students work, look for

- Characteristics of the graphs that students use to connect them to the scenarios.
- Clues students use in the scenarios to determine the scale.

10

Differentiation strategy

As an alternative grouping method, use the jigsaw strategy for scaling the axes for each scenario. This strategy is meant to save time while providing a brief recall of scaling, but the sharing part is necessary so that students can use the information to determine the domain and range for each problem.

Note

Differentiation strategies are provided that will ensure all students acquire the knowledge of the activity. These strategies provide flexibility within the lesson to allow for varying student acquisition and demonstration of learning. These strategies provide suggestions to benefit the full range of learners.

ENGAGE**Getting Started: Name That Function!****11****Facilitation Notes**

In this activity, students are given one or two characteristics of a graph and determine whether the function could be a member of the linear, exponential, quadratic, and/or linear absolute value function family.

Differentiation strategy

To scaffold support, prior to beginning the lesson, review the four function families as a class: linear, exponential, quadratic, and linear absolute.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- What does the graph of a smooth curve look like? Not a smooth curve?

Sum

Graph
func
smoo
func
or de
value
it is de

4 • TOPIC 1: Quantities and Relations

Activity 4.1
Categorizing Scenarios into Their Function Families**DEVELOP****Facilitation Notes**

In this activity, students revisit the scenarios and their graphs from the first lesson of the topic to complete a table naming the function family associated with each scenario, identifying the domain, and describing the graphical behavior as increasing, decreasing, constant or both increasing and decreasing.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- What is a function?
- How can you tell from a scenario whether or not it represents a function?
- How can you tell from the scenario that Graph B represents an exponential rather than half of the graph of a quadratic function?
- How many of the scenarios are associated with an exponential function? How did you tell them apart?
- How can you tell from the scenario that it is associated with an absolute value function?
- How can you tell from the scenario whether the domain should be continuous or discrete?
- How many of the scenarios contain an absolute maximum? Absolute minimum?
- How many of the scenarios can be described as increasing? As decreasing?

Misconception

Students may think the graphs are incorrect because they all are continuous, while some of the scenarios have domains that are discrete. Discuss the fact that the graphs relate to functions that are mathematical models of the scenarios; the scenarios require an interpretation of the necessary components of the mathematical model.

Summary

A scenario and its graph provide the necessary characteristics to determine the function family to which it belongs.

LESSON 4: Function Families for 2000, Alex • 5

11. Grouping Strategies

Suggestions appear to help chunk each activity into manageable pieces and establish the cadence of the lesson.

Learning is social. Whether students work in pairs or in groups, the critical element is that they are engaged in discussion. Carnegie Learning believes, and research supports, that student-to-student discourse is a motivating factor; it increases student learning and supports ongoing formative assessment. Additionally, it provides students with opportunities to have mathematical authority.

Working collaboratively can, when done well, encourage students to articulate their thinking (resulting in self-explanation) and also provides metacognitive feedback (by reviewing other students' approaches and receiving feedback on their own).

The student discussion is then transported to a classroom discussion facilitated by the teacher to guarantee all necessary mathematics is addressed, once again, with the same benefits of discussion.

Note: Alternative Grouping Strategies

Differentiation strategies that provide other grouping strategies, such as whole class participation and the jigsaw method, are sometimes recommended for specific activities. These are listed as Differentiation Strategies.

More information about grouping strategies is available online in the Texas Support Center at **www.CarnegieLearning.com/texas-help**.

12. Summary

The summary brings the activity to closure. This statement encapsulates the big mathematical ideas of the particular activity.

13. Differentiation Strategies

To assist all students, instructional strategies are provided that benefit the full range of learners.

14. Questions to Ask

The overarching questioning strategies throughout each lesson promote analysis and higher-order thinking skills beyond simple yes or no responses.

These questions can be used to gather information, probe thinking, make the mathematics explicit, and encourage reflection and justification as students are working together or when they are sharing responses as a class. These questions are an embedded formative assessment strategy to provide feedback as students are actively engaged in learning.

- Why is the *Baton Twirling* graph a smooth curve, while the *Jelly Bean Challenge* graph includes straight lines?

12

Summary

Key characteristics of graphs, such as intercepts, increasing and decreasing intervals, and maximum and minimum points are used to interpret scenarios and differentiate graphs.

DEMONSTRATE

Talk the Talk: A Writer and a Mathematician

Facilitation Notes

In this activity, students create a scenario based upon a possible trip to school. They then sketch a graph to model their scenario. They share their work with classmates and note similarities and differences.

Have stu
and 2. S

Ques

• V

• V

• V

• F

• V

• V

• V

• C

• I

• D

• H

• D

• I

• I

• V

• V

• V

• T

• Y

• D

• T

• Y

• D

• T

• Y

• D

• T

• Y

• D

• T

• Y

• D

• T

• Y

• D

• T

• Y

• D

• T

• Y

• D

• T

• Y

• D

• T

• Y

• D

• T

• Y

• D

• T

• Y

• D

• T

• Y

• D

• T

• Y

• D

• T

• Y

8 • TOPIC 1: Quantities and Relations

ENGAGE

Getting Started: Name That Function!

Facilitation Notes

In this activity, students are given one or two characteristics of a graph and determine whether the function could be a member of the linear, exponential, quadratic, and/or linear absolute value function family.

13

Differentiation strategy

To scaffold support, prior to beginning the lesson, review the four function families as a class: linear, exponential, quadratic, and linear absolute.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

14

Questions to ask

- What does the graph of a smooth curve look like? Not a smooth curve?
- What does the graph of a function that increases over the entire domain look like?
- What does the graph of a function that decreases over the entire domain look like?
- What does the graph of a function that has an absolute maximum look like?
- Can the graph of a function have more than one maximum? Absolute maximum?
- What does a graph that has symmetry look like?

Summary

Graphs described as straight lines may be associated with a linear function or linear absolute value function, while those described as smooth curves may be associated with an exponential or quadratic function. The graph of a linear or exponential function is either increasing or decreasing, while the graph of a quadratic function or a linear absolute value function has an interval where it is increasing and an interval where it is decreasing.

4 • TOPIC 1: Quantities and Relationships

15

Misconception

Students may confuse the independent variable with the dependent variable. For example, they could think the number of movie tickets is determined by the total cost of the tickets (if the cost of three tickets is \$22.50, then each ticket must have been \$7.50). Just because the value of one variable can be determined using the value of a second variable, this does not signify dependence or independence.

Summary

There are two quantities that change in problem situations. When one quantity depends on another, it is said to be the dependent quantity. The quantity that the dependent quantity depends upon is called the independent quantity.

DEVELOP

Activity 1.1

Connections

Facilitation

In this activity, students match a scenario and range.

Ask a student to discuss cutting.

Have students share responses.

As students work, look for

Differences

As an activity, provide students each a...

16

Summary

Graphs can be used to model scenarios. Knowing the independent and dependent variables, as well as the domain and range, is helpful in making connections between the scenario and its graph.

Activity 1.2
Comparing and Contrasting Graphs



Facilitation Notes

In this activity, students make basic observations about the similarities and differences in the graphs from the previous activity. They then look more deeply at pairs of scenarios along with their graphs to focus on key characteristics, such as intercepts, increasing and decreasing intervals, and maximum and minimum points.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

As students work, look for

- Mathematical terms used to describe similarities and differences in the graphs.
- Instances where students would benefit from an increased mathematical vocabulary to describe graphical characteristics.

Questions to ask

- Is the independent quantity always located on the same axis? Which axis?
- Is the dependent quantity always located on the same axis? Which axis?
- Which graphs contain straight lines? Curves?
- Which graphs could be described as increasing from left to right?
- Which graphs could be described as decreasing from left to right?
- Could any graphs be considered both increasing and decreasing?
- Is it possible for a graph to be both increasing and decreasing?
- Can the curves on the graph be described as smooth curves? Are all curves considered smooth curves?
- Which graphs have a maximum value? A minimum value?
- Do both the *Something's Fishy* and *It's Magic* graphs decrease?
- Why is the *Something's Fishy* graph a straight line, while the *It's Magic* graph is a smooth curve?
- Do both the *Baton Twirling* and *Jelly Bean Challenge* graphs increase and decrease?

15. Misconceptions

Common student misconceptions are provided in places where students may overgeneralize mathematical relationships or have confusion over the vocabulary used. Suggestions are provided to address the given misconception.

16. As Students Work, Look For

These notes provide specific language, strategies, and/or errors to look and listen for as you circulate and monitor students working in pairs or groups. You can incorporate these ideas when students share their responses with the class.

Note

Talk the Talk helps you to assess student learning and to make decisions about helpful connections you need to make in future lessons.

17. Differentiation Strategies

To extend an activity for students who are ready to advance beyond the scope of the activity, additional challenges are provided.

18. White Space

The white space in each margin is intentional. Use this space to make additional planning notes or to reflect on the implementation of the lesson.

Misconception

Students may confuse the independent variable with the dependent variable. For example, they could think the number of movie tickets is determined by the total cost of the tickets (if the cost of three tickets is \$22.50, then each ticket must have been \$7.50). Just because the value of one variable can be determined using the value of a second variable, this does not signify dependence or independence.

Summary

There are two quantities that change in problem situations. When one quantity depends on another, it is said to be the dependent quantity. The quantity that the dependent quantity depends upon is called the independent quantity.

DEVELOP

Activity 1.1

Connecting Scenarios and Their Graphs



Facilitation Notes

In this activity, students are presented with eight different scenarios. For each scenario, they identify the independent and dependent quantities and match a graph. Students then scale the axes and determine the domain and range for each scenario.

18

Ask a student to read the introduction before Question 1 aloud. As a class, discuss the directions to this task because it has several parts and includes cutting out and gluing graphs next to their scenario descriptions.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

As students work, look for

- Characteristics of the graphs that students use to connect them to the scenarios.
- Clues students use in the scenarios to determine the scale.

17

Differentiation strategy

As an alternative grouping method, use the jigsaw strategy for scaling the axes for each scenario. This strategy is meant to save time while providing a brief recall of scaling, but the sharing part is necessary so that students can use the information to determine the domain and range for each problem.



Position yourself to take full advantage of the richness of the mathematics addressed in the textbook. The Facilitation Notes provide guidance to reach each student from their current level of understanding to advance to the next stage. Place yourself in the position of the student by experiencing the textbook activities prior to class. Realize your role in the classroom—empower your students! Step back and let them do the math with confidence in their role as learner and your role as facilitator of learning.



Janet Sinopoli, Instructional Designer

Supporting Emergent Bilingual Students

Emergent bilingual students often face multiple challenges in the mathematics classroom beyond language development skills, including a lack of confidence, peer-to-peer understanding, and building solid conceptual mastery. The Carnegie Learning Texas Math Solution seeks to support Emergent Bilingual Students as they develop skills in both mathematics and language.

Throughout instruction, ELL tips are placed for teachers at point-of-use on the mini-lesson page in the TIG. They provide additional modifications to support this special population.

These tips:

- Inform teachers of potential learning obstacles specific to the lesson.
- Provide engaging activities for learning and assessment.
- Reinforce newly acquired mathematical language to gain an increasing level of comprehension of English.
- Introduce students to language needed to understand a specific context.

Students internalize new content language by using and reusing it in meaningful ways in a variety of different speaking activities that build concept and language attainment.

For More Support

Visit the Texas Support Center for many more resources to support you and your students who are emergent bilingual students.



Answers

- 1a. exponential function or quadratic function
- 1b. linear function or linear absolute value function
- 1c. linear function or exponential function
- 1d. quadratic function or linear absolute value function
- 2a. quadratic function
- 2b. linear function
- 2c. exponential function
- 2d. linear absolute value function

GETTING STARTED

Name That Function!

You have sorted graphs according to their function family. Now, consider which function families have the given characteristics.

Function Families
linear
exponential
quadratic
linear absolute value

1. Which function families can be described by the characteristic provided? Choose from the given list.

- a. The graph is a smooth curve.
- b. The graph is made up of one or more straight lines.
- c. The graph increases or decreases over the entire domain.
- d. The graph has an absolute maximum or minimum.

2. One or more characteristics have been added to the graphical description of each function. Name the possible function families.

- a. The graph has an absolute minimum or absolute maximum and is a smooth curve.
- b. The graph either increases or decreases over the entire domain and is a straight line.
- c. The graph is a smooth curve, and either increases or decreases over the entire domain.
- d. The graph has either an absolute minimum or an absolute maximum, has symmetry, and is made up of 2 straight lines.

Each function family has certain graphical behaviors, with some behaviors common among different function families. Notice, the more specific characteristics that are given, the more specifically you can name that function!

2 • TOPIC 1: Quantities and Relationships

ELL Tip

Review the difference between behaviors of functions and characteristics of functions. Discuss as a class the characteristics of a person compared to the behaviors of a person. Help students make the connection that characteristics are usually nouns and behaviors are usually verbs. Create a list of words that are characteristics of a function and a list of words that can be described as behaviors of a function. Have students add to the list as they come across the different functions.

Assessments

Both formative and summative assessments are an integral part of information gathering. Formative assessment tools are provided throughout each lesson, providing you with ongoing feedback of student performance and encouraging students to monitor their own progress. Ongoing formative assessment underlies the entire learning experience, driving real-time adjustments, next steps, insights, and measurements.

End of topic summative assessments are provided to measure student performance on a clearly denoted set of standards. For certain topics that extend longer than four instructional weeks, a mid-topic summative assessment is also provided.

Enhanced End of Topic Assessment

There are three problem type sections per assessment. Multiple-choice questions, open-response questions, and griddable response questions prepare students for enhanced standardized tests.

The answer key provides teachers with the TEKS aligned to each question, as well as sample answers for open-response and griddable response questions.

Page 1: Multiple-Choice Questions

Topic 1
QUANTITIES AND RELATIONSHIPS

Enhanced End of Topic Assessment

Name _____ Date _____

Part A: Multiple-Choice Questions

TEKS A.9D

1. Which characteristics best describe the graph?

- a. is a function
is exponential
is discrete
is increasing
- ★ b. is a function
is exponential
is continuous
is decreasing
- c. is a function
is quadratic
is continuous
is both increasing and decreasing
- d. is a function
is linear
is discrete
is constant

© Carnegie Learning, Inc.

Page 2: Open-Response Questions

Topic 1
QUANTITIES AND RELATIONSHIPS

Part B: Open-Response Questions

TEKS A.2A, A.3C

6. Arlene begins a test with a possible total of 80 points. She loses 2 points for every question she answers incorrectly. The graph represents this situation.

- a. Identify the domain and range.
Domain: $0 \leq x \leq 40$
Range: $0 \leq y \leq 80$
- b. Identify the maximum point.
(0, 80); If Arlene answers all questions correctly, she will score 80 points.
- c. Is the graph increasing or decreasing?
The graph is decreasing.

© Carnegie Learning, Inc.

Page 3: Griddable Response Questions

Topic 1
QUANTITIES AND RELATIONSHIPS

Part C: Griddable Response Questions

Record your answers and fill in the bubbles.

TEKS A.7A

14. The graph of the quadratic function f is shown on the grid. The coordinates of the x -intercepts, y -intercept, and vertex are integers.

What is the maximum value of f ?

9

© Carnegie Learning, Inc.

Sample griddable response:

+	0	0	0	0	0	0	0	0	9
-	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0

QUANTITIES AND RELATIONSHIPS: Enhanced End of Topic Assessment • 9



End of Course Topic

The End of Course Topic is the final topic of the course which includes a collection of problem-based performance tasks that are aligned with selected priority math standards of the course. This final topic provides students an additional opportunity to demonstrate their ability to make sense of multi-step, real-world problems, communicate their thinking, represent solutions, and justify their reasoning on content aligned with these selected math standards.

Performance Tasks

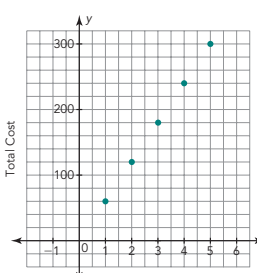
Each performance task is a formative assessment tool that allows students to demonstrate their learning of the selected course content. At the end of each task, a section titled “Your Work Should Include” lists the categories and the corresponding max scoring points from the grading rubric.

1 PERFORMANCE TASK

Health Club Payment Plans

Your friends D’juan, Cho, and Lina are interested in joining a health club. After searching online, they gathered the following information about the cost of three different health clubs.

Fit Club



Energym

Months	Total Cost
1	\$195
2	\$240
3	\$285
4	\$330
5	\$375

Fitness Zone

Fitness Zone has an initial fee of \$250 and charges \$35 per month.

Your friends are interested in joining a health club for different lengths of time.

- D’juan wants to join for just 6 months before he leaves for college.
- Cho wants to join a club for the length of the school year — 10 months.
- Lina wants to commit to a health club for an entire year.

• Which health club should each friend join so that they spend the least amount of money on their health club membership as possible? Explain your reasoning.

Your work should include:

- Linear equations for each health club (3 points)
- Explanation of the meaning of the slope and y-intercept for each club (3 points)
- Written advice for D’juan, Cho, and Lina, including an explanation (9 points)

Grading Rubric

The grading rubric is for students and teachers to set clear expectations for how each completed performance task will be evaluated. Students should use the rubric to guide their work and self-monitor their progress. Teachers should use the rubric to evaluate and provide feedback for the completed performance task.

RUBRIC: 15 TOTAL POINTS

	0 points	1 point	2 points	3 points
Equations	No equations correct.	Only one equation is correct.	Two equations are correct.	All three equations are correct.
Explanations of Slope and y-Intercept	No explanations are correct.	Explanation for one equation is correct.	Explanation for two equations are correct.	Explanation for all three equations are correct.
Advice for D'juan	No advice given.	Advice is given, but with no mathematical basis in the equations for each club.	Advice is given based on mathematics, but includes some incorrect calculations.	Advice is complete and correct.
Advice for Cho	No advice given.	Advice is given, but with no mathematical basis in the equations for each club.	Advice is given based on mathematics, but includes some incorrect calculations.	Advice is complete and correct.
Advice for Lina	No advice given.	Advice is given, but with no mathematical basis in the equations for each club.	Advice is given based on mathematics, but includes some incorrect calculations.	Advice is complete and correct.

Teacher's Implementation Guide

The Teacher's Implementation Guide for the End of Course Topic contains a performance task overview, list of aligned TEKS and ELPS, essential ideas, facilitation notes which describe how to pace the two-day performance task, sample answer, and grading rubric.

1

Performance Task

Health Club Payment Plans

Performance Task Overview

Students are provided with three different health club payment plan representations: one graph, one table and one verbal description. Students must define variables and write a linear equation in the form $y = mx + b$ to represent each health club's payment plan. They explain what the slope and y-intercept of each equation means in terms of the problem situation. Then, based on the provided information of three friends looking to join a health club for different lengths of time, students provide advice to each friend on which health club they should join for the better deal.

Algebra I

Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:

(B) write linear equations in two variables in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$, given one point and the slope and given two points.

(C) write linear equations in two variables given a table of values, a graph, and a verbal description.

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:

(B) calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in context of mathematical and real-world problems.

ELPS

1.A.1.B, 1.C, 1.E, 1.G, 2.B, 2.D, 2.E, 2.G, 2.H, 2.I, 3.A, 3.B, 3.D, 3.E, 3.F, 3.G, 3.H, 3.I, 3.J, 4.E, 4.F, 4.G, 4.H, 4.J, 5.B, 5.C, 5.D, 5.F, 5.G

END-OF-COURSE TOPIC: Performance Task 1 • 1

SAMPLE ANSWER

Let x = number of months and y = total cost.

Fit Club: $y = 60x$

Energym: $y = 45x + 150$

Fitness Zone: $y = 35x + 250$

In each equation, the slope represents the cost per month, and the y-intercept represents the initial fee.

D'juan should join Fit Club. The cost for 6 months is \$360 at Fit Club, instead of \$420 at Energym or \$450 at Fitness Zone.

Cho can join whichever club she chooses, as the cost will be the same (\$600) at all three locations.

Lina should join Fitness Zone. The cost for 12 months is \$670 at Fitness Zone, instead of \$600 at Energym or \$720 at Fit Club.

Similar to the other topics in this course, the End of Course Topic also has a Topic Family Guide for students and caregivers, and a Topic Overview for teachers. The End of Course Topic does not include an end of topic assessment since each performance task is a formative assessment.

Carnegie Learning recognizes that it is the classroom teachers who make the material come alive for students, transforming the way math is taught. Implementation requires integrating learning together and learning individually.

Prepare for Learning Together

The most important first step you can take in preparing to teach with these instructional materials is to become comfortable with the mathematics.

- Read through the Module 1 Overview and the Topic 1 Overview.
- Do the math of the first Topic, and consider the facilitation notes.
- Prepare team-building activities to intentionally create a student-centered environment.

PREPARE
YOURSELF

Prepare for Learning Individually

Plan how you will utilize Skills Practice as a Learning Individually resource. Then, determine how you will introduce Skills Practice to students. Explain to them the benefits of working individually and why practice is important.

- Read through Module 1 Topic 1 Skills Practice.
- Determine which problem sets align with the activities in the corresponding student lessons.
- Based on student performance in the lesson, be prepared to assign the class, small groups of students, or individual students different problem sets to practice skills to develop mastery.

Plan how you will introduce students to MATHia. Explain to them the benefits of working individually and why practice is important.

- Test out the computers or tablets that your students will be using.
- Verify your classes have been set up in Teacher's Toolkit with correct MATHia content assigned. Or manually set up your classes in Teacher's Toolkit if applicable.
- Use the Content Browser in Teacher's Toolkit to explore the content students are assigned.
- Be prepared to demonstrate how students will access and log into MATHia.

PREPARE YOUR CLASSROOM

Prepare the Environment

The classroom is often considered the third teacher. Consider how to create a learning environment that engages students and fosters a sense of ownership. The use of space in your classroom should be flexible and encourage open sharing of ideas.

- Consider how your students are going to use the consumable book. It is the student's record of their learning. Many teachers have students move an entire topic to a three-ring binder as opposed to carrying the entire book.
- Arrange your desks so students can talk and collaborate with each other.
- Prepare a toolkit for groups to use as they work together and share their reasoning (read the materials list in each Topic Overview).
- Consider where you will display student work, both complete and in-progress.
- Create a word wall of key terms used in the textbook.

PREPARE YOUR STUDENTS

Prepare the Learners

If you expect students to work well together, they need to understand what it means to collaborate and how it will benefit them. It is important to establish classroom guidelines and structure groups to create a community of learners.

- Facilitate team-building activities and encourage students to learn each others' names.
- Set clear expectations for how the class will interact:
 - ✦ Their text is a record of their learning and is to be used as a reference for any assignments or tests you give.
 - ✦ They will be doing the thinking, talking, and writing in your classroom.
 - ✦ They will be working and sharing their strategies and reasoning with their peers.
 - ✦ Mistakes and struggles are normal and necessary.

PREPARE FAMILIES AND CAREGIVERS

Prepare the Support

- Prepare a letter to send home on the first day. Visit the Texas Support Center for a sample letter.
- Encourage families and caregivers to read the introduction of the textbook.
- Ensure that families and caregivers receive the module Family and Caregiver Guide at the start of each module. They should also receive the topic Family Guide at the start of the first topic and each subsequent topic.
- Consider a Family Math Night some time within the first few weeks of the school year.
- Encourage families and caregivers to explore the Students & Caregivers Portal on the Texas Support Center at www.CarnegieLearning.com/texas-help/students-caregivers.

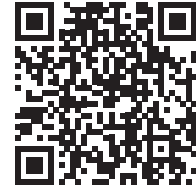
Students and Caregivers Portal

Research has proven time and again that family engagement greatly improves a student's likelihood of success in school.



The Students & Caregivers Portal on the Texas Support Center provides:

- Getting to Know Carnegie Learning video content to provide an introduction to the instructional materials and research.
- Articles and quick tip videos offering strategies for how families and caregivers can support student learning. **Visit the Texas Support Center regularly to access new content and resources for students and caregivers as they learn mathematics in a variety of environments outside of the classroom.**



MODULE FAMILY AND CAREGIVER GUIDES

Each module has a Family and Caregiver Guide available through the Students & Caregivers Portal on the Texas Support Center. Each module guide of the course will provide a different highlight of the academic glossary, description and examples of TEKS Mathematical Process Standards, and an overview of a different component of our instructional approach known as The Carnegie Learning Way. Also included is a module overview of content, specific key terms, visual representations, and strategies students are learning in each topic of the module.

The purpose of the Family and Caregiver Guide is to bridge student learning in the classroom to student learning at home. The goal is to empower families to understand the concepts and skills learned in the classroom so that families can review, discuss, and solidify the understanding of these key concepts together. Videos will also be available on the Students & Caregivers Portal to provide added support.

The collage displays three pages from the Family and Caregiver Guides. The first page (Algebra 1) includes a 'How to support your student' section with a 'Searching for Patterns' activity, an 'Academic Glossary' table, and a 'Mapping' diagram. The second page (Geometry 1) features a 'Develop' section with a diagram of a triangle and a 'Demonstrate' section. The third page (Algebra 2) shows a 'Find Relationships' section with a graph of a parabola and a table of key terms.

TOPIC FAMILY GUIDES

Each topic contains a Family Guide that provides an overview of the mathematics of the topic, how that math is connected to what students already know, and how that knowledge will be used in future learning. It also incorporates an illustration of math from the real world, a sample standardized test question, talking points, and a few of the key terms that students will learn.

We recognize that learning outside of the classroom is crucial to students' success at school. While we don't expect families and caregivers to be math teachers, the Family Guides are designed to assist caregivers as they talk to their students about what they are learning. Our hope is that both the students and their caregivers will read and benefit from the guides.

Carnegie Learning Family Guide Algebra I

Module 1: Searching for Patterns

TOPIC 1: QUANTITIES AND RELATIONSHIPS

In this topic, students explore a variety of different functions. The intent is merely to introduce these new functions, providing an overview but not a deep understanding at this point. The topic is designed to help students recognize that different function families have different key characteristics. In later study in this course, they will formalize their understanding of the defining characteristics of each type of function.

Where have we been?

In previous grades, students defined a function and used linear functions to model the relationship between two quantities. They have written linear functions in slope-intercept form and should be able to identify the slope and y -intercept in the equation. Students have also characterized graphs as functions using the terms *increasing*, *decreasing*, *constant*, *discrete*, *continuous*, *linear*, and *nonlinear*.

Where are we going?

The study of functions is a main focus of high school mathematics. This topic builds the foundation for future, more in-depth study by familiarizing students with the concept of a

Function Notation


The linear equation $y = 8x + 15$ can be written as $f(x) = 8x + 15$. You can write this linear equation as a mathematical object that has a specific set of outputs (the range of the function).

name of function

The input of the function, x , is represented by a whole collection of values.

Functions Are Everywhere. Google It.

Every time you open a web page, you are calling hundreds, if not thousands, of functions. At the time of this writing, there were 88 functions mentioned in the background on the homepage of a popular search engine, which contains just a name and a search box.



Functions that programmers write are very similar to the functions students study in mathematics. They take inputs and produce outputs. And they are often written in the same way too—with a function name and an input variable in parentheses, like $f(x)$. Search functions take in search terms as inputs and output hundreds of thousands or millions of results. Mathematical functions can only output one result for each input.

Talking Points

Functions are an important topic to know about for college admissions tests. Here is a sample question:

For the function $f(x) = 2x^2 - 3x$, what is the value of $f(-5)$?

To solve this, students need to know that the input -5 is substituted for x in the equation:

$$\begin{aligned} f(-5) &= 2(-5)^2 - 3(-5) \\ &= 2(25) + 15 \\ &= 50 + 15 \\ &= 65 \end{aligned}$$

The point $(-5, 65)$ is on the graph of the function.

Key Terms

increasing function
If a function increases across the entire domain, then the function is called an increasing function.

decreasing function
If a function decreases across the entire domain, then the function is called a decreasing function.

function family
A function family is a group of functions that all share some characteristics.

x-intercept
The x -intercept is the point where a graph crosses the x -axis.

y-intercept
The y -intercept is the point where a graph crosses the y -axis.

2 • TOPIC 1: Quantities and Relationships

YOU MIGHT BE WONDERING . . .

Why do we believe in our brand of blended: Learning Together and Learning Individually?

There has been lots of research on the benefits of learning collaboratively. Independent practice is necessary for students to become fluent and automatic in a skill. A balance of these two pieces provides students with the opportunity to develop a deep conceptual understanding through collaboration with their peers, while demonstrating their understanding independently.

Why don't we have a Worked Example at the start of every lesson?

In all aspects of the Texas Math Solution, we provide worked examples. Sweller and Cooper (1985) argue that worked examples are educationally efficient because they reduce working memory load. Ward and Sweller (1990) found that alternating between problem solving and viewing worked examples led to the best learning. Students often read worked examples with the intent to confirm that they understand the individual steps. However, the educational value of the worked example often lies in thinking about how the steps connect to each other and how particular steps might be added, omitted, or changed, depending on context.

Where are the colorful graphics to get students' attention?

Our instructional materials have little extraneous material; we do not use illustrations unless they are essential to helping students understand the material. This approach follows from research showing that “seductive details” used to spice up the presentation of material often have a negative effect on student learning (Mayer et al., 2001; Harp & Meyer, 1998). Students may not know which elements of an instructional presentation are essential and which are intended simply to provide visual interest. So, we focus on the essential material. While we strive to make our educational materials attractive and engaging to students, research shows that only engagement based on the mathematical content leads to learning.

Why is the book so big?

The student textbook contains all of the resources students need to complete the Learning Together component of the course. Students are to actively engage in this textbook, topic-by-topic, creating a record of their learning as they go. There is room to record answers, take notes, draw diagrams, and fix mistakes. Visit the Texas Support Center at <https://www.CarnegieLearning.com/texas-help/> for tips on managing your textbooks.

CUSTOMER SUPPORT

The Carnegie Learning Texas Support Team is available to help with any issue at help@carnegielearning.com.

**Monday-Friday
8:00 am-8:00 pm CST**
via email, phone, or
live chat

Our expert team provides support for installations, networking, and technical issues, and can also help with general questions related to pedagogy, classroom management, content, and curricula.

Notes



If you have questions, reach out to us for support. Our team of master practitioners have been where you are. We made mistakes and we learned from them. We want to help you. We have many professional development options. Whether we come to your school for a workshop, join you in your classroom for modeling or coaching, or you join us online for a webinar or an entire course, our goal is to make sure you feel supported and prepared to use the tasks you'll find in this book to their fullest!



Kasey Bratcher, Senior VP of Professional Learning