Make the Best of It

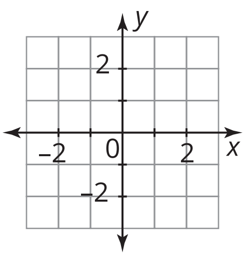
2

Optimization

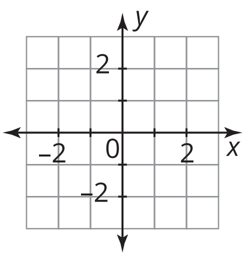
Warm Up

Graph each linear inequality.

1. $y > x - 1$



2. $y \leq -\frac{1}{2}x + 2$



Learning Goals

- Write systems of linear inequalities in two variables.
- Solve systems of two or more linear inequalities in two variables.
- Determine possible solutions in the solution set of systems of two or more linear inequalities in two variables.
- Determine constraints from a problem situation.
- Use the vertices of a solution region to calculate maximum or minimum values.

Key Terms

- solution of a system of inequalities
- linear programming

You have used the intersection of linear equations on a coordinate plane to solve a system of linear equations. How can you use the intersection of linear inequalities on a coordinate plane to determine the greatest or least values that a function can take?

GETTING STARTED

Remember:

A boundary line, determined by the inequality, divides the plane into two half-planes, and the inequality symbol indicates which half-plane contains all the solutions.

She Works Hard for the Money

Recall that you can graph a linear inequality in two variables on a coordinate plane by determining the boundary line and shading the appropriate half-plane.

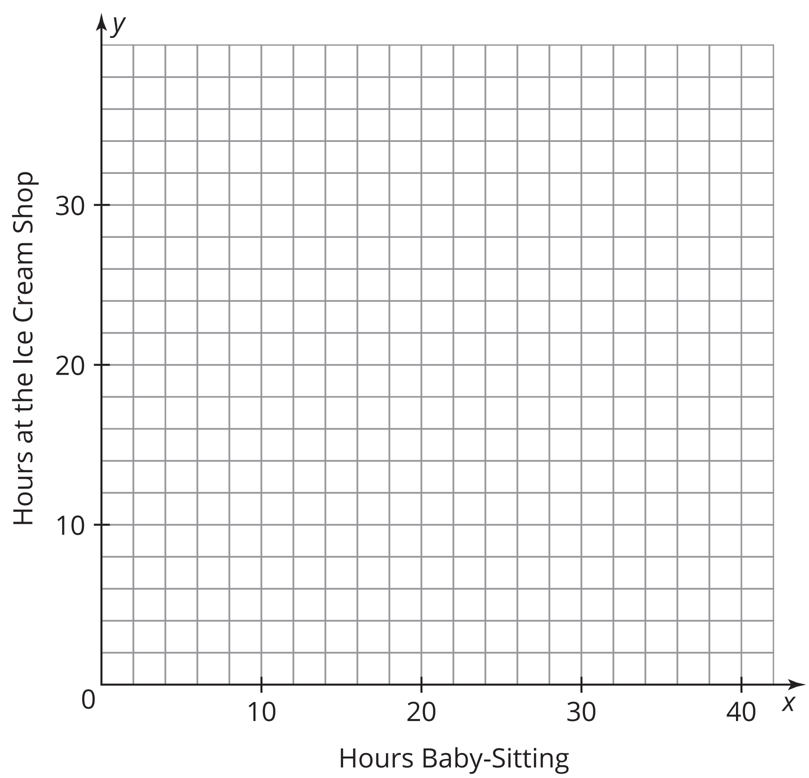
1. Janice works two jobs and is trying to maximize her weekly earnings. Her baby-sitting job pays \$10 per hour, and her job at an ice cream shop pays \$7.50 per hour. Suppose she needs to earn at least \$90 to cover her weekly expenses.

a. Write a linear inequality to represent the total amount of money Janice needs to earn at her jobs. Be sure to define your variables.

Think about:

Will the boundary line be a solid or a dashed line?

b. Graph the linear inequality on the coordinate plane.



c. **Janice's friend Colton suggests that she work 18 hours at her baby-sitting job and 12 hours at the ice cream shop. If Janice follows Colton's advice, will she earn enough to cover her weekly expenses? Explain your reasoning.**

d. **Write this solution as an ordered pair. Where does it fall on the graph?**

e. **Determine another possible solution to this problem situation. Justify your reasoning.**

f. **Why is the graph restricted to the first quadrant?**

g. **Write two inequalities that represent this restriction.**

Think about:

Which values are reasonable for the variables in the problem situation?

ACTIVITY
2.1 Systems of Linear Inequalities

The three inequalities you wrote to represent the possible number of hours Janice can work at her two jobs make up a system of linear inequalities. How does adding another inequality to the system affect the solution?

1. Suppose that Janice can work a maximum of 20 total hours per week.

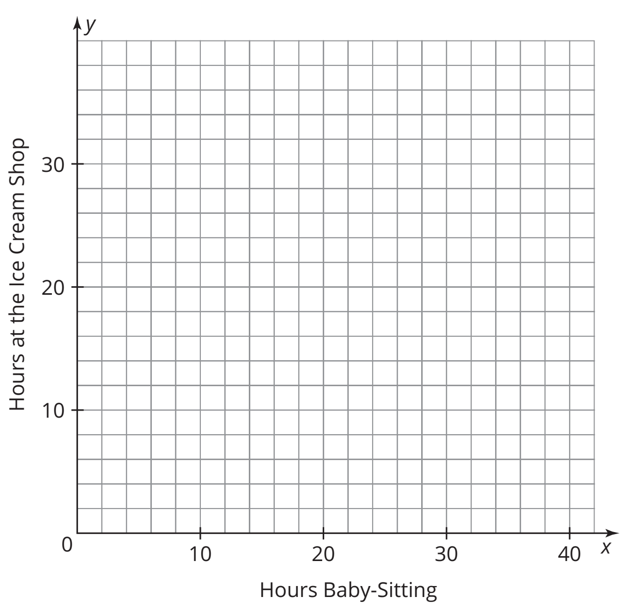
a. Write a linear inequality to represent the total number of hours that Janice could work per week.

Remember:

In a system of linear inequalities, the inequalities are known as constraints because the values of the expressions are "constrained" to lie within a certain region on the graph.

b. Is Colton's suggestion of 18 hours at her baby-sitting job and 12 hours at the ice cream shop still reasonable? Justify your reasoning.

c. Write the four linear inequalities that you have written into a system of inequalities that represents Janice's problem situation.



d. Graph the system of linear inequalities on one coordinate plane.

e. Suppose that Janice is only able to baby-sit for 6 hours. How many hours must she work at the ice cream shop in order to meet her earnings requirement? Justify your reasoning.

f. Write your solution as an ordered pair, and plot it on the graph in part (d). Where does it fall?

g. Erin suggests that Janice also work 10 hours at the ice cream shop. Is this solution reasonable? How does it compare to your answer in part (e)?

h. Write Erin's solution as an ordered pair and plot it on the graph in part (d). Where does it fall?

i. Compare these solutions with the ordered pair (18, 12) suggested by Colton in part (b). What do you notice?

The **solution of a system of inequalities** is the intersection of the solutions to each inequality. Every point in the intersection satisfies all inequalities in the system.

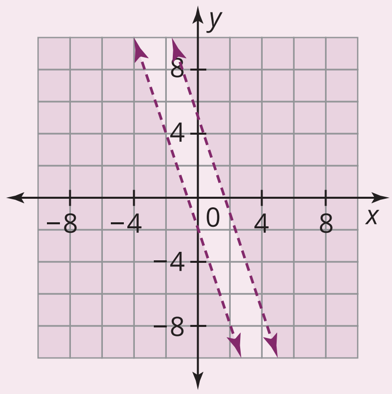
2. Select one ordered pair that represents a point inside the overlapping region. Verify that it is a solution to the system of linear inequalities.

3. Select one ordered pair that represents a point outside the overlapping region. Verify that it is not a solution to the system of linear inequalities.

4. **Serena and Adam were each asked to graph the system of linear inequalities shown.**

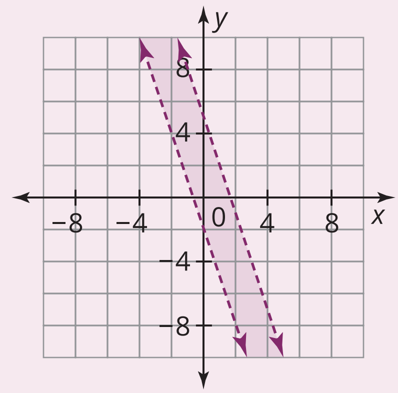
$$\begin{cases} y > -3x + 5 \\ y < -3x - 2 \end{cases}$$

Serena



The two inequalities do not overlap, and therefore the system has no solution.

Adam



The solution to the system of inequalities falls in between the two lines.

a. **Who is correct? Justify your reasoning.**

b. **Select an ordered pair from each region of the graph to verify your response.**

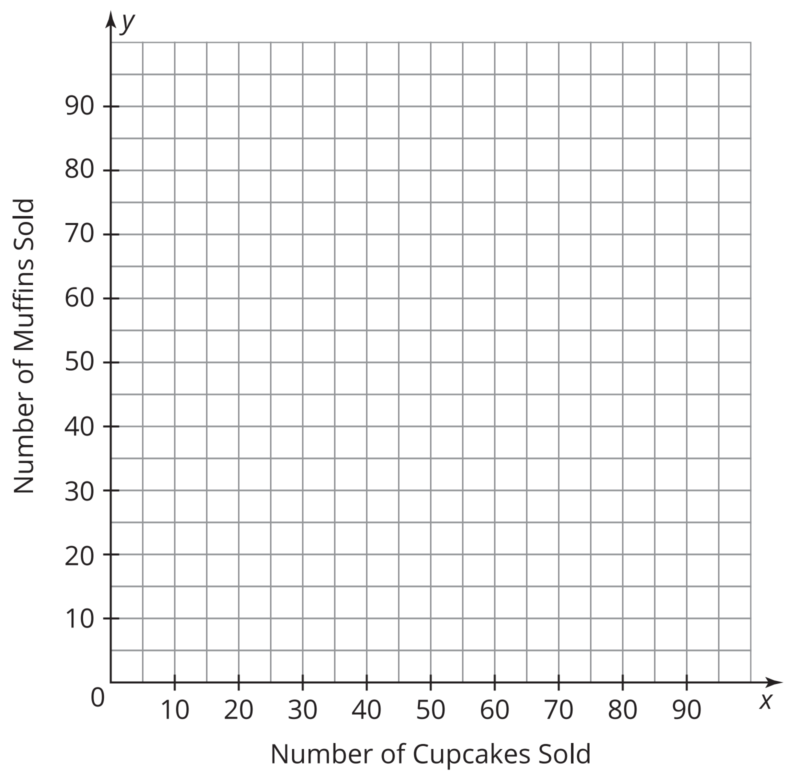
5. **Why are the lines in Question 4 dashed?**

6. **Will two parallel lines always produce a system of linear inequalities with no solution?**

7. **The Whistling Baker is a bakery that specializes in cupcakes and muffins. They sell cupcakes for $2.00 each and muffins for $1.25 each. The Whistling Baker can bake a total of 8 dozen treats each day and needs to make $100 per day.**

   a. **Write a system of linear inequalities to represent this problem situation. Be sure to define your variables.**

   b. **Graph the system of linear inequalities on the coordinate plane.**



   c. **Identify one possible solution. Is this reasonable within the context of the problem situation?**

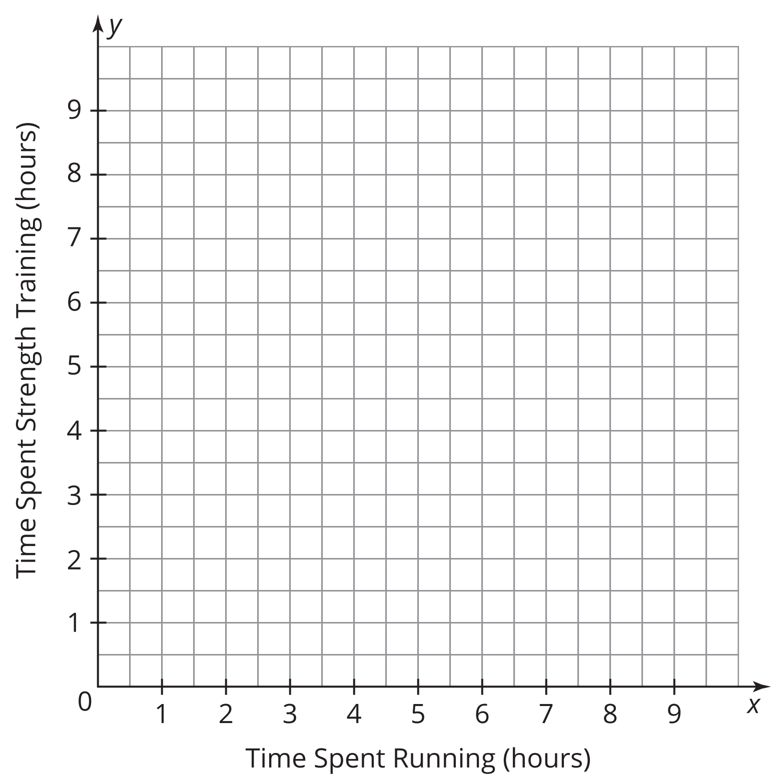
ACTIVITY
2.2

Linear Programming

Brad is a distance runner who wants to design the most effective workout plan to prepare for his next race. To adequately prepare for the race, Brad plans to do a combination of running and strength training. He must run between 3 and 6 hours per week, and wants to devote at least twice as much time to running as to strength training. Brad can spend no more than 8 hours working out each week.

1. Write a system of inequalities to represent the constraints of this problem situation. Be sure to define your unknowns.

2. Graph the system of inequalities on the coordinate plane shown. Shade the region that represents the solution set.



You can use the intersection points to determine the optimal solution to a system of inequalities through a process called *linear programming*.

In **linear programming**, the vertices of the solution region of the system of linear inequalities are substituted into a given equation to determine the maximum or minimum value.

3. Label the vertices of the shaded region.

4. Running burns 600 calories per hour, while strength training burns 250 calories per hour. Write an equation to represent this relationship. Be sure to define your variables.

5. Use the vertices of the solution region to determine how many hours Brad should devote to each type of exercise in order to maximize the number of calories burned.

6. A sponsor wants Brad to advertise their company by wearing its brand name on his workout gear. They will pay him $15 for each hour he wears the brand name while running, and $20 for each hour he wears the brand name while strength training. Determine how many hours Brad should devote to each type of exercise in order to maximize his earnings.

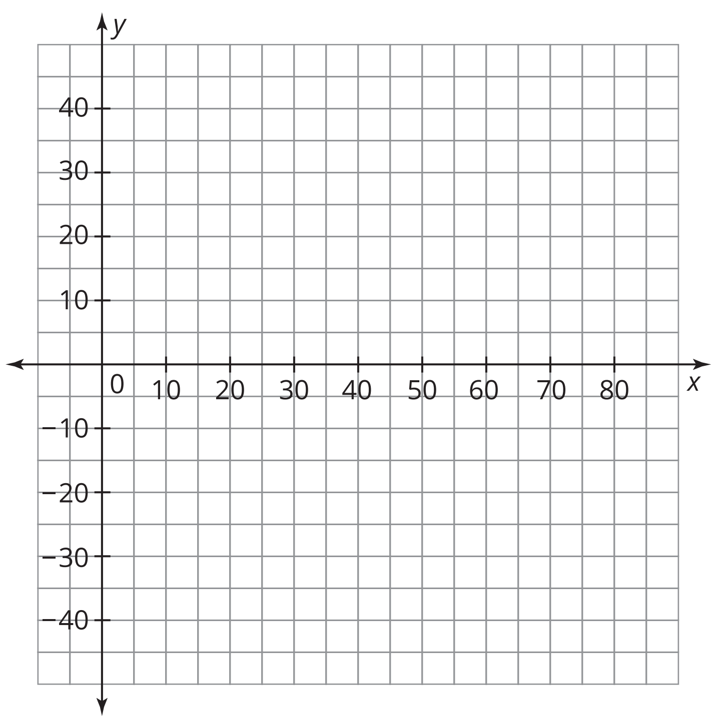
NOTES

TALK the TALK

Making Bling for Cha-Ching

You can write a system of inequalities in two variables to model the constraints in a problem situation and then use linear programming to compute optimal results.

1. **Christy uses glass beads to make bracelets and necklaces. It takes her 30 minutes to make a bracelet and 45 minutes to make a necklace. She works at most 40 hours a week. She wants to make at least 30 bracelets. The profit from a bracelet is $10, and the profit from a necklace is $18. Determine the number of bracelets and necklaces Christy should make to maximize her profit. Show your work.**



2. **Compare the solution of a system of linear inequalities and the solution to an equation calculated by linear programming.**