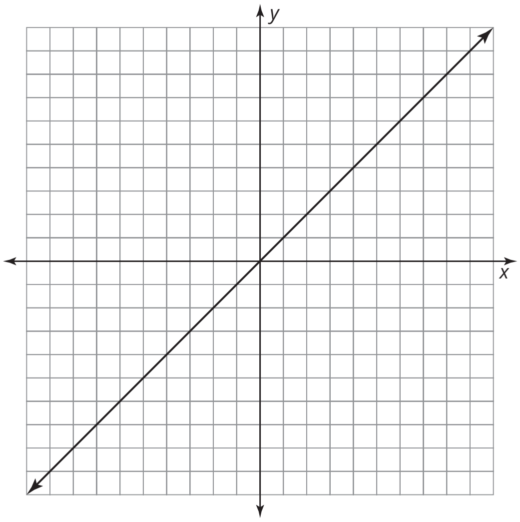
4

Putting the V in Absolute Value

Defining Absolute Value Functions and Transformations

Warm Up

The graph of $f(x) = x$ is shown. Graph each transformation.



1. $g(x) = f(x) + 5$
2. $h(x) = 2 \cdot f(x) - 3$
3. $j(x) = \frac{1}{2} \cdot f(x) - 1$

Learning Goals

- Experiment with transformations of absolute value functions using technology.
- Graph absolute value functions and transformations of absolute value functions.
- Determine the effect of replacing the basic absolute value function $f(x) = |x|$ with $f(x) + D$, $Af(x)$, $f(Bx)$, and $f(x - C)$ for different values of A, B, C, and D.
- Distinguish between function transformations that occur outside the function and inside the argument of the function.

Key Terms

- absolute value
- reflection
- line of reflection
- argument of a function

You know how to transform linear functions. How can you define absolute value functions and show transformations of this function type?

GETTING STARTED

Distance Is Always Positive

Absolute value is indicated with vertical bars: $|-4|$ is read as "the absolute value of -4."

The **absolute value** of a number is its distance from zero on the number line.

1. Follow your teacher's instructions to model each absolute value expression on the *x*-axis of a classroom coordinate plane. Rewrite each expression without the absolute value symbol.

a. $|-2|$

b. $|2|$

c. $|1 - 2|$

d. $|-3 - (-5)|$

e. $|-2 \cdot 3|$

f. $|0 \cdot 4|$

g. $\left|\frac{12}{-3}\right|$

h. $|8 \div (-4)|$

2. Write your observations about the absolute value expressions you and your classmates modeled on the number line.

3. Provide counterexamples to show why Sonja's statement is incorrect.

Sonja

Absolute values are always positive. So, $|a| = -a$ is not possible.

ACTIVITY

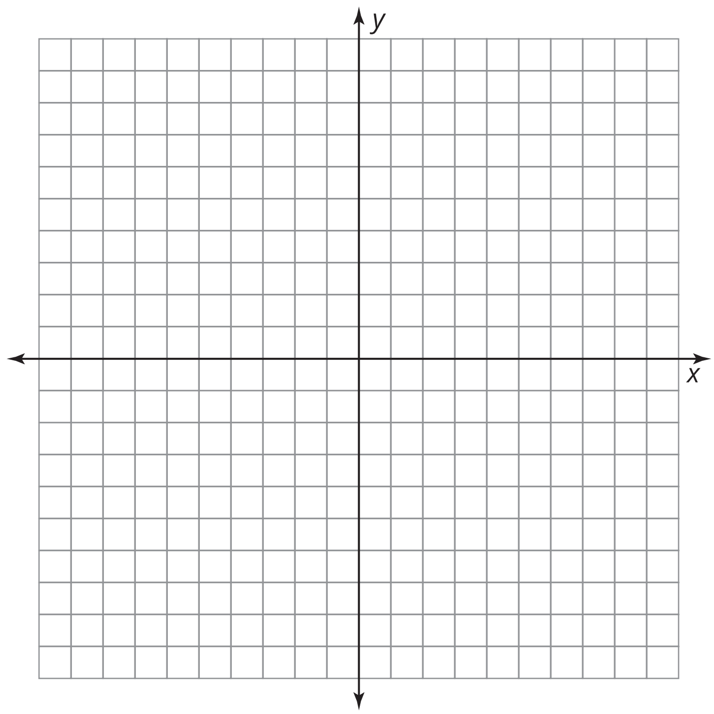
4.1

Graphs of Absolute Value Functions

Follow your teacher's instructions to model the function $f(x) = x$ on the classroom coordinate plane with your classmates.

1. Record the coordinates of the plotted points for $f(x) = x$ in the table.

x	y	
	$f(x) = x$	$f(x) = \|x\|$
−9		
−6		
−4		
−1		
0		
3		
5		
8		



2. Change all the plotted points to model the function $f(x) = |x|$. In the table, record the coordinates of the new points for $f(x) = |x|$.

3. Describe how the points move from the graph of $f(x) = x$ to the graph of $f(x) = |x|$.

4. Graph the function $f(x) = |x|$. Describe the characteristics of the function that you notice.

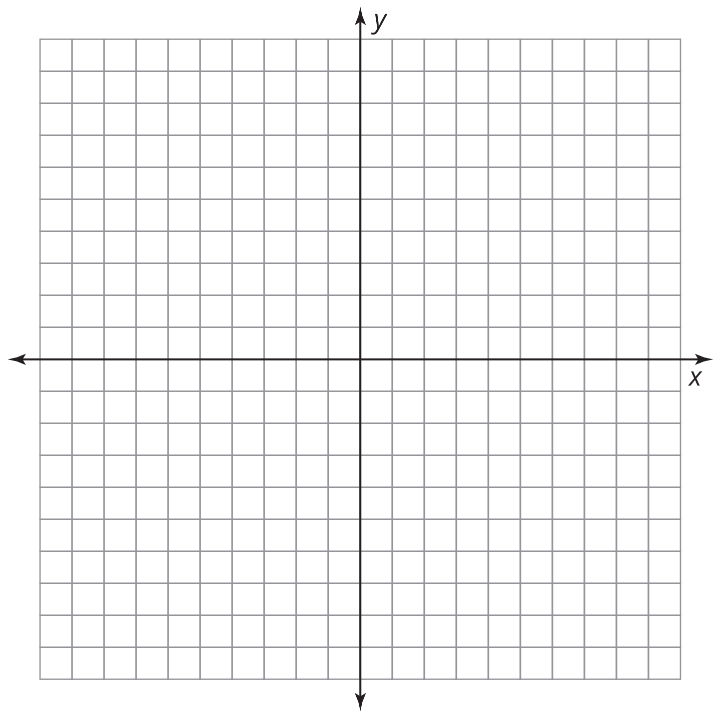
Think about:

What are the domain and range? Is there a maximum value or a minimum value?

Next, consider the function $f(x) = -x$. Model this function on the classroom coordinate plane with your classmates.

5. Record the coordinates of the plotted points for $f(x) = -x$ in the table.

x	y	
	$f(x) = -x$	$f(x) = \|-x\|$
−9		
−6		
−4		
−1		
0		
3		
5		
8		



6. Change all the plotted points to model the function $f(x) = |-x|$. In the table, record the coordinates of the new points for $f(x) = |-x|$.

7. Describe how the points move from the graph of $f(x) = -x$ to the graph of $f(x) = |-x|$.

Remember:

Use a straightedge to be precise when you graph.

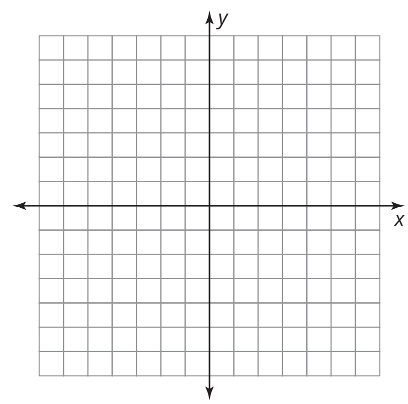
8. Graph the function $f(x) = |-x|$. Compare this function with the function $f(x) = |x|$.

ACTIVITY
4.2 Transformations Outside the Function

Consider the three absolute value functions shown.

$g(x) = |x|$ $\quad$ $c(x) = |x| + 3$ $\quad$ $d(x) = |x| - 3$

1. Use technology to graph each function. Then, sketch and label the graph of each function.



2. Write the functions $c(x)$ and $d(x)$ in terms of the basic function $g(x)$. Then describe the transformations of each function.

3. Describe the similarities and differences between the three graphs. How do these similarities and differences relate to the equations of the functions $g(x)$, $c(x)$, and $d(x)$?

Recall that a function $t(x)$ of the form $t(x) = f(x) + D$ is a vertical translation of the function $f(x)$. The value $|D|$ describes how many units up or down the graph of the original function is translated.

4. Describe each graph in relation to the basic function $g(x) = |x|$. Then use coordinate notation to represent the vertical translation.

a. $f(x) = g(x) + D$ when $D > 0$

b. $f(x) = g(x) + D$ when $D < 0$

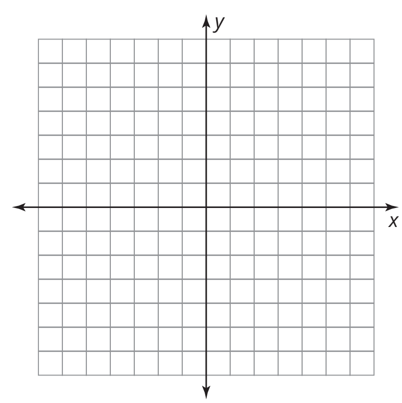
c. Each point (x, y) on the graph of $g(x)$ becomes the point ______________ on $f(x)$.

Consider these absolute value functions.

$$g(x) = |x| \qquad k(x) = \frac{1}{2}|x|$$

$$j(x) = 2|x| \qquad p(x) = -|x|$$

5. Use technology to graph each function. Then, sketch and label the graph of each function.



6. Write the functions $j(x)$, $k(x)$, and $p(x)$ in terms of the basic function $g(x)$. Then describe the transformations of each function.

NOTES

Recall that a function $t(x)$ of the form $t(x) = A \cdot f(x)$ is a vertical dilation of the function $f(x)$. The A-value describes the vertical dilation of the graph of the original function.

7. Describe each graph in relation to the basic function $g(x) = |x|$. Then use coordinate notation to represent the vertical dilation.

a. $f(x) = A \cdot g(x)$ when $A > 1$

b. $f(x) = A \cdot g(x)$ when $A < 0$

c. $f(x) = A \cdot g(x)$ when $0 < A < 1$

d. Each point (x, y) on the graph of $g(x)$ becomes the point ______________ on $f(x)$.

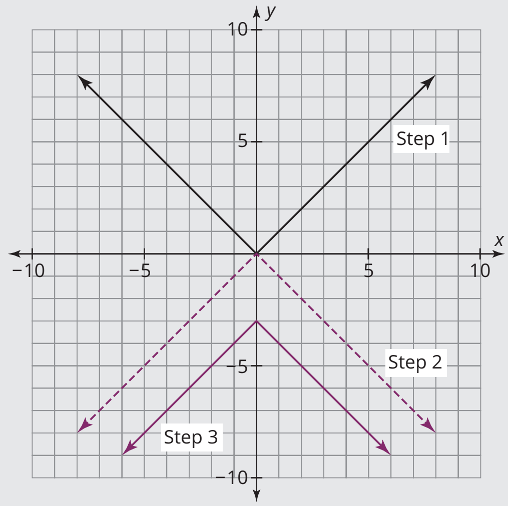
A **reflection** of a graph is the mirror image of the graph about a line of reflection.

A **line of reflection** is the line that the graph is reflected across. A horizontal line of reflection affects the y-coordinates.

You know that changing the A-value of a function to its opposite reflects the function across a horizontal line. But the *line of reflection* for the function might be different depending on how you write the transformation and the order the transformations are applied.

8. **Josh and Vicki each sketched a graph of the function $b(x) = -|x| - 3$ using different strategies. Write the step-by-step reasoning used by each student.**

Josh

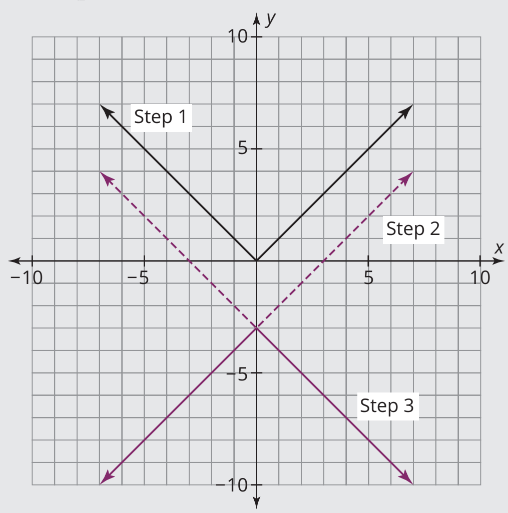


Step 1:

Step 2:

Step 3:

Vicki



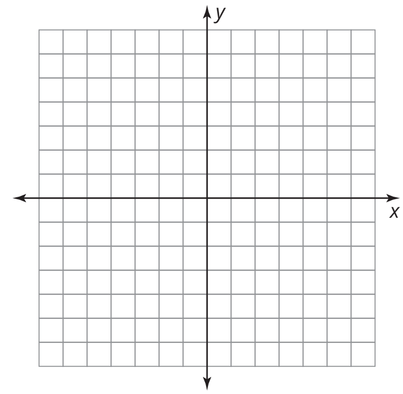
Step 1:

Step 2:

Step 3:

9. **Explain how changing the order of the transformations affects the line of reflection.**

Given the function $f(x) = |x|$. Use the coordinate plane shown to answer Questions 10 through 14.



10. Consider the function $a(x) = 2f(x) + 1$.

a. Use coordinate notation to describe how each point (x, y) on the graph of $f(x)$ becomes a point on the graph of $a(x)$.

b. Graph and label $a(x)$ on the coordinate plane shown.

11. Consider the function $b(x) = -2f(x) + 1$.

a. Use coordinate notation to describe how each point (x, y) on the graph of $f(x)$ becomes a point on the graph of $b(x)$.

b. Graph and label $b(x)$ on the same coordinate plane shown.

12. Describe the graph of $b(x)$ in terms of $a(x)$.

13. Consider the function $-a(x)$.

a. Use coordinate notation to describe how each point (x, y) on the graph of $a(x)$ becomes a point on the graph of $-a(x)$.

b. Graph and label $-a(x)$ on the coordinate plane shown.

14. Describe the graph of $-a(x)$ in terms of $a(x)$.

15. How do the *A*-value and *D*-value affect the minimum and maximum values of the function?

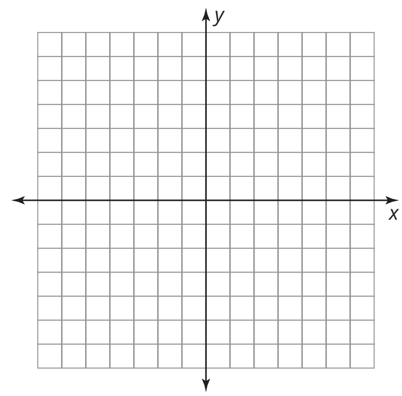
ACTIVITY **4.3**

Transformations Inside the Function

Consider these absolute value functions.

$g(x) = |x|$ $\qquad$ $m(x) = |x - 2|$ $\qquad$ $n(x) = |x + 2|$

1. Use technology to graph each function. Then, sketch and label the graph of each function. Describe how $m(x)$ and $n(x)$ relate to $g(x)$.



Remember:

The expression $x + C$ is the same as $x - (-C)$.

A function $t(x)$ of the form $t(x) = f(x - C)$ is a horizontal translation of the function $f(x)$. The value $|C|$ describes the number of units the graph of $f(x)$ is translated right or left. If $C > 0$, the graph is translated to the right. If $C < 0$, the graph is translated to the left.

2. Write the functions $m(x)$ and $n(x)$ in terms of the basic function $g(x)$. Describe how changing the C-value in the functions $m(x)$ and $n(x)$ horizontally translated the function $g(x)$.

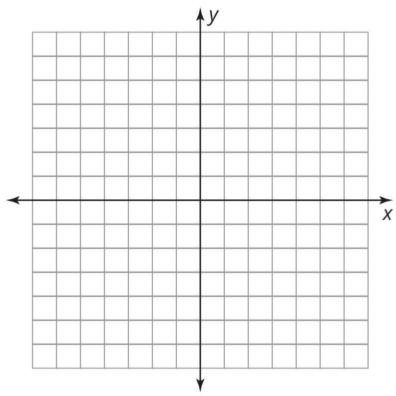
3. Use coordinate notation to show how each point (x, y) on the graph of $g(x)$ becomes a point on a graph that has been horizontally translated.

Consider these absolute value functions.

$$g(x) = |x| \qquad k(x) = \left|\tfrac{1}{2}x\right|$$

$$j(x) = |2x| \qquad p(x) = |-x|$$

4. Use technology to graph each function. Then, sketch and label the graph of each function.



5. Write the functions $j(x)$, $k(x)$, and $p(x)$ in terms of the basic function $g(x)$. Then describe the transformations of each function.

Think about:

How does changing the *B*-value compare to changing the *A*-value?

Recall that a function $t(x)$ of the form $t(x) = f(B \cdot x)$ is a horizontal dilation of the function $f(x)$. The *B*-value describes the horizontal dilation of the graph of the original function.

6. Describe each graph in relation to the basic function $g(x) = |x|$. Then use coordinate notation to represent the horizontal dilation.

a. $f(x) = g(B \cdot x)$ when $B > 1$

b. $f(x) = g(B \cdot x)$ when $B < 0$

c. $f(x) = g(B \cdot x)$ when $0 < B < 1$

d. Each point (x, y) on the graph of $g(x)$ becomes the point ________________ on $f(x)$.

ACTIVITY 4.4 Combining Transformations of Absolute Value Functions

> The **argument of a function** is the expression inside the parentheses.
>
> For $y = f(x - C)$ the expression $x - C$ is the argument of the function.

When a function is transformed by changing the *A*- or *D*-values or both, these changes are said to occur "outside the function." These values affect the output to a function, *y*. When the *B*- or *C*-values are changed, this changes the *argument of the function*. A change to the argument of a function is said to happen "inside the function." These values affect the input to a function, *x*.

outside the function

$$g(x) = A \cdot f(B(x - C)) + D$$

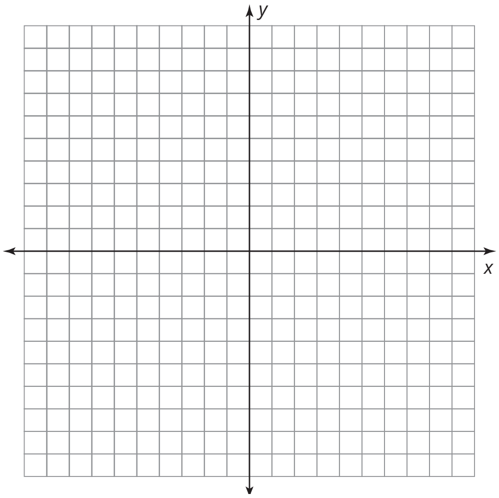
inside the function

1. Use coordinate notation to describe how each point (x, y) on the graph of $f(x)$ becomes a point on the graph of $g(x)$.

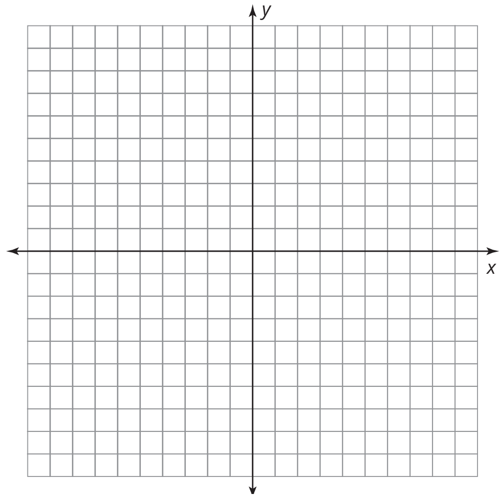
The ordered pair $(x, |x|)$ describes any point on the graph of the basic absolute value function $f(x) = |x|$. For a transformation of the function, any point on the graph of the new function can be written as $(\frac{1}{B}x + C, A|(B(x - C))| + D)$.

2. Given the basic absolute value function $f(x) = |x|$. Consider each transformation. Describe how the transformations affected $f(x)$. Then use coordinate notation to describe how each point (x, y) on the graph of $f(x)$ becomes a point on the graph of the transformed function. Finally, sketch a graph of each new function.

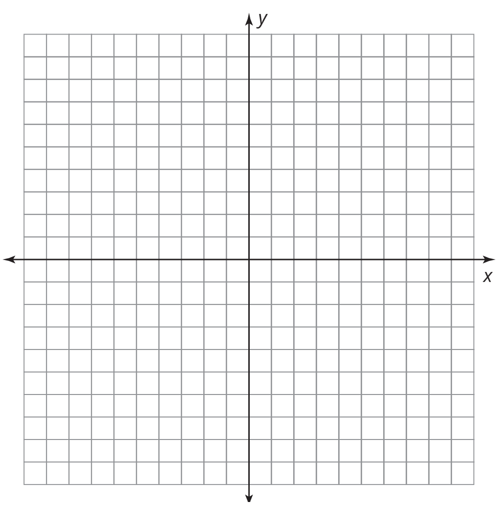
a. $m(x) = 2f(x - 1)$



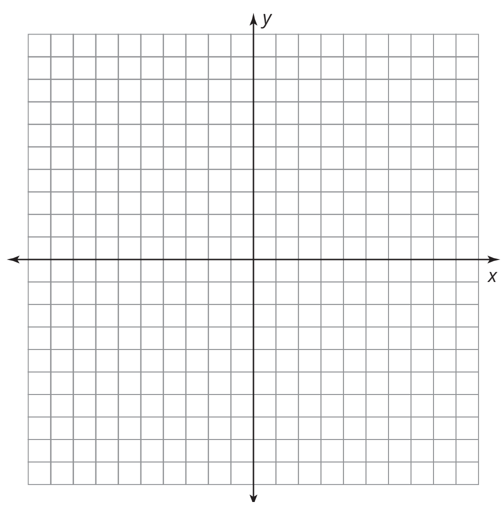
b. $r(x) = \frac{1}{2}f(x + 2) - 2$



c. $w(x) = 2f(x + 3) + 1$



d. $v(x) = -2f(x + 3) + 1$



3. Graph $-w(x)$ on the same coordinate plane as $w(x)$ in Question 2 part (c). Describe the similarities and differences between the graph of $v(x)$ and the graph of $-w(x)$.

ACTIVITY **4.5**

Writing Equations in Transformation Form

1. Consider the function, $f(x) = |x|$. Write the function in transformation function form in terms of the transformations described, then write an equivalent equation.

Transformation	Transformation Function Form	Equation
a. Reflection across the x-axis		
b. Horizontal translation of 2 units to the left and a vertical translation of 3 units up		
c. Vertical stretch of 2 units and a reflection across the line $y = 0$		
d. Vertical dilation of 2 units and a reflection across the line $y = 3$		
e. Horizontal translation of 3 units to the right, a vertical translation down 2 units, and a vertical dilation of $\frac{1}{2}$		
f. Vertical compression by a factor of 4		
g. Vertical stretch by a factor of 4		
h. Horizontal compression by a factor of 3		

TALK the TALK

A, B, C, and D

The function $f(x) = A|x - C| + D$ is graphed with varying values for A, C, and D.

1. Match the given values of A, C, and D with the graph of the function with corresponding values. Explain your reasoning.

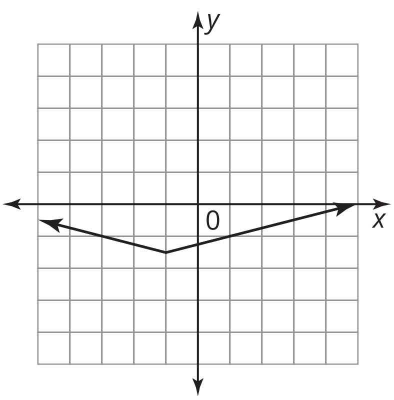
a. $A = 1$, $C = 0$, and $D > 0$

b. $A = 1$, $C = 0$, and $D < 0$

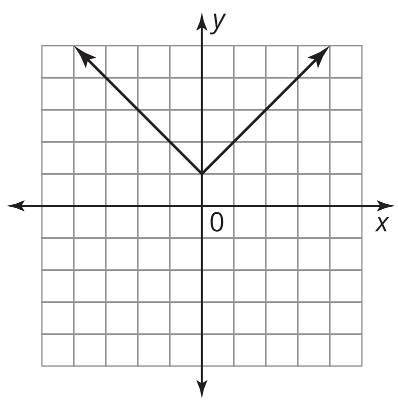
c. $A > 1$, $C > 0$, and $D > 0$

d. $0 < A < 1$, $C < 0$, and $D < 0$

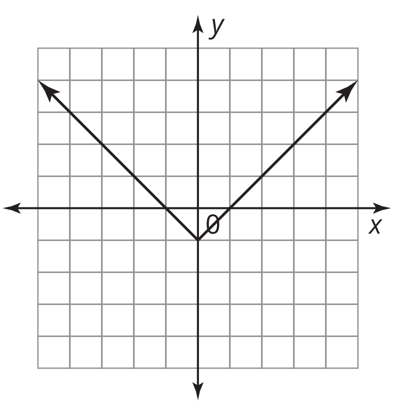
Graph A



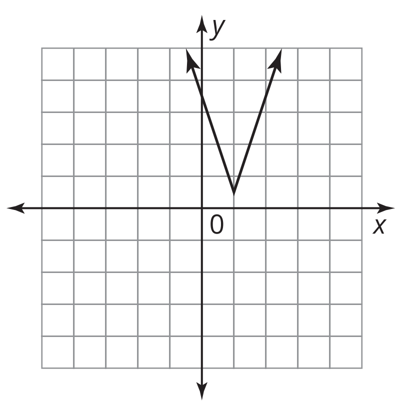
Graph B



Graph C



Graph D



NOTES

2. Complete the table by describing the graph of each function as a transformation of the basic function $f(x) = |x|$.

Function Form	Equation Information	Description of Transformation
$f(x) = \|x\| + D$	$D < 0$	
	$D > 0$	
$f(x) = A\|x\|$	$A < 0$	
	$0 < A < 1$	
	$A > 1$	
$f(x) = \|x - C\|$	$C < 0$	
	$C > 0$	
$f(x) = \|Bx\|$	$B < 0$	
	$0 < B < 1$	
	$B > 1$	

NOTES

3. **Determine whether each statement is true or false. If the statement is false, rewrite the statement as true.**

   a. **In the transformation function form $g(x) = Af(B(x - C)) + D$, the A-value vertically stretches or compresses $f(x)$, the C-value translates $f(x)$ horizontally, the B-value horizontally stretches or compresses $f(x)$, and the D-value translates the function $f(x)$ vertically.**

   b. **Key characteristics of the basic absolute value function include a domain and range of real numbers.**

   c. **The domain of absolute value functions is not affected by translations or dilations.**

   d. **Vertical translations do not affect the range of absolute value functions.**

   e. **Horizontal translations do not affect the range of absolute value functions.**

   f. **Vertical dilations do not affect the range of absolute value functions.**