5

Play Ball!

Absolute Value Equations and Inequalities

Warm Up

Evaluate each expression.

1. $|9 + (-4)|$

2. $|-1 - 5|$

3. $|4 \cdot (-6)|$

4. $|0 \div (-2)|$

Learning Goals

- Understand and solve absolute value equations.
- Solve and graph linear absolute value inequalities on number lines.
- Graph absolute value functions and use the graph to determine solutions.

Key Terms

- linear absolute value equation
- linear absolute value inequality
- equivalent compound inequality

You know what the graphs of absolute value functions look like. How can you use what you know about graphs and linear equations to solve absolute value equations and inequalities?

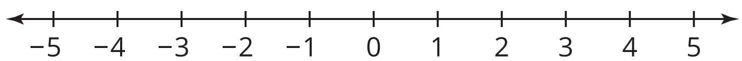
GETTING STARTED

Opposites Attract? Absolutely!

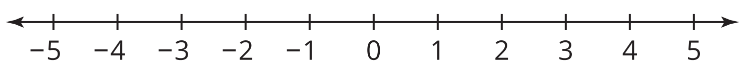
You can solve many absolute value equations using inspection.

1. Graph the solution set of each equation on the number line given.

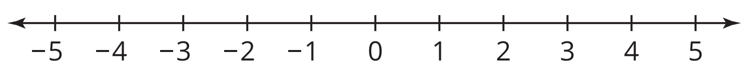
a. $|x| = 5$



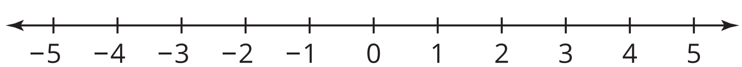
b. $|x| = 2$



c. $|x| = -3$

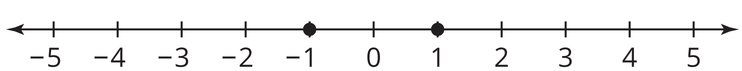


d. $|x| = 0$

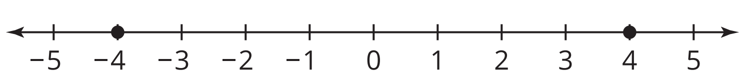


2. Write the absolute value equation for each solution set graphed.

a.



b.

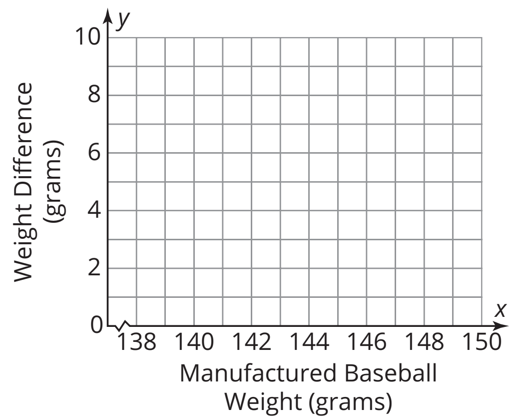


ACTIVITY **5.1**

Creating an Absolute Value Function from a Situation

The official rules of baseball state that all baseballs used during professional games must be within a specified range of weights. The baseball manufacturer sets the target weight of the balls at 145.045 grams on its machines.

1. Sketch a graph that models the relationship between a manufactured baseball's weight, *x*, and its distance from the target weight, *y*. Explain how you constructed your sketch. Then write an absolute value equation to represent the situation and the graph.



2. The specified weight allows for a difference of 3.295 grams in the actual weight of a ball and the target weight. Since the weight must be within a distance of 3.295 grams from the target weight, $y = 3.295$.

a. Graph the equation $y = 3.295$ on the coordinate plane in Question 1.

b. What two equations can you write, without absolute values, to show the least acceptable weight and the greatest acceptable weight of a baseball? Explain your reasoning.

c. Use the graph to write the solutions to the equations you wrote in part (b). Show your work.

Ask yourself:

How is the function transformed from the basic function $f(x) = |x|$?

ACTIVITY 5.2

Solving Absolute Value Equations

The two equations you wrote can be represented by the **linear absolute value equation** $|w - 145.045| = 3.295$. To solve any absolute value equation, recall the definition of absolute value.

Worked Example

Consider this linear absolute value equation.

$$|a| = 6$$

There are two points that are 6 units away from zero on the number line: one to the right of zero, and one to the left of zero.

$$+(a) = 6 \quad \text{or} \quad -(a) = 6$$

$$a = 6 \quad \text{or} \quad a = -6$$

Now consider the case where $a = x - 1$.

$$|x - 1| = 6$$

If you know that $|a| = 6$ can be written as two separate equations, you can rewrite any absolute value equation.

$$+(a) = 6 \quad \text{or} \quad -(a) = 6$$
$$+(x - 1) = 6 \quad \text{or} \quad -(x - 1) = 6$$

1. How do you know the expressions $+(a)$ and $-(a)$ represent opposite distances?

2. Martina and Bob continued to solve the linear absolute value equation $|x - 1| = 6$ in different ways. Compare their strategies and then determine the solutions to the equation.

Martina

$(x - 1) = 6$ or $(x - 1) = -6$

Bob

$x - 1 = 6$ or $-x + 1 = 6$

3. **Solve each linear absolute value equation. Show your work.**

a. $|x + 7| = 3$

b. $|x - 9| = 12$

c. $|3x + 7| = -8$

d. $|2x + 3| = 0$

Ask yourself:

Before you solve each equation, think about the number of solutions each equation may have. You may be able to save yourself some work—and time!

4. **Artie, Donald, Cho, and Steve each solved the equation $|x| - 4 = 5$.**

Artie

$|x| - 4 = 5$

$x - 4 = 5$ $\quad$ $-(x) - 4 = 5$

$x = 9$ $\quad$ $-x = 9$

$x = -9$

Donald

$|x| - 4 = 5$

$|x| = 9$

$x = 9$ $\quad$ $-(x) = 9$

$x = -9$

Cho

$|x| - 4 = 5$

$(x) - 4 = 5$ $\quad$ $-[(x) - 4] = 5$

$x - 4 = 5$ $\quad$ $-x + 4 = 5$

$x = 9$ $\quad$ $-x = 1$

$x = -1$

Steve

$|x| - 4 = 5$

$(x) - 4 = 5$ $\quad$ $-(x) - 4 = -5$

$x = 9$ $\quad$ $-x - 4 = -5$

$-x = -1$

$x = 1$

a. **Explain how Cho and Steve incorrectly rewrote the absolute value equation as two separate equations.**

b. **Explain the difference in the strategies that Artie and Donald used.**

Think about:

Consider isolating the absolute value part of the equation before you rewrite it as two equations.

5. Solve each linear absolute value equation.

a. $|x| + 16 = 32$

b. $23 = |x - 8| + 6$

c. $3|x - 2| = 12$

d. $35 = 5|x + 6| - 10$

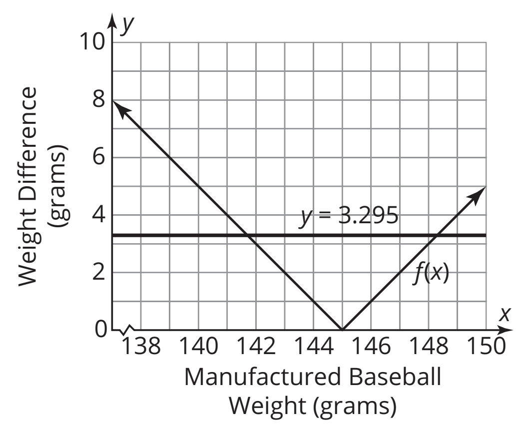
ACTIVITY

5.3 Absolute Value Inequalities

You determined the linear absolute value equation $|w - 145.045| = 3.295$ to identify the most and least a baseball could weigh and still be within the specifications. The manufacturer wants to determine all of the acceptable weights that the baseball could be and still fit within the specifications. You can write a **linear absolute value inequality** to represent this problem situation.

1. Write a linear absolute value inequality to represent all baseball weights that are within the specifications.

2. Use the graph to determine whether the weight of each given baseball is acceptable. Substitute each value in the inequality to verify your answer.



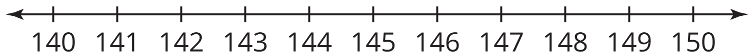
a. 147 grams

b. 140.8 grams

c. 148.34 grams

d. 141.75 grams

3. Use the graph on the coordinate plane to graph the inequality on the number line showing all the acceptable weights. Explain the process you used.



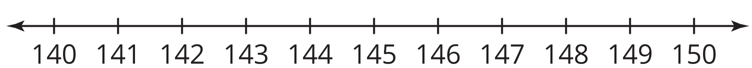
4. Complete the inequality to describe all the acceptable weights, where *w* is the baseball's weight.

$\underline{\hspace{3cm}} \leq w \leq \underline{\hspace{3cm}}$

5. Raymond has the job of disposing of all baseballs that are not within the acceptable weight limits.

a. Write an absolute value inequality to represent the weights of baseballs that Raymond can dispose of.

b. Graph the inequality on the number line. Explain the process you used.



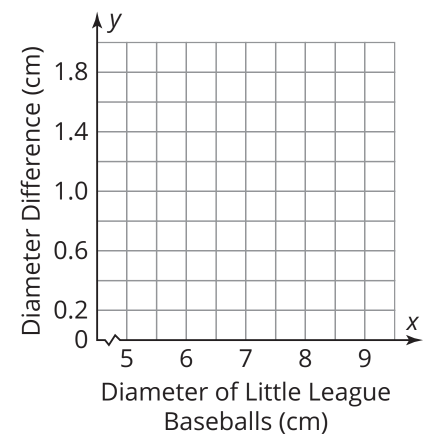
ACTIVITY
5.4

Solving Problems with Absolute Value Functions

In Little League Baseball, the diameter of the ball is slightly smaller than that of a professional baseball.

1. For Little League baseballs, the manufacturer sets the target diameter to be 7.47 centimeters. The specified diameter allows for a difference of 1.27 centimeters.

a. Sketch the graph of the linear absolute value function, *f*(*d*), on the coordinate plane.



b. Use your graph to estimate the diameters of all the Little League baseballs that fit within the specifications. Explain how you determined your answer.

c. Algebraically determine the diameters of all the baseballs that fit within the specification. Write your answer as an inequality.

2. The manufacturer knows that the closer the diameter of the baseball is to the target, the more likely it is to be sold. The manufacturer decides to keep only the baseballs that are less than 0.75 centimeter from the target diameter.

a. Algebraically determine which baseballs will not fall within the new specified limits and will not be kept. Write your answer as an inequality.

b. How can you use your graph to determine whether you are correct?

ACTIVITY 5.5 Absolute Value and Compound Inequalities

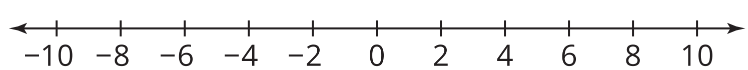
Absolute value inequalities can take four different forms, as shown in the table. To solve a linear absolute value inequality, you can first write it as an **equivalent compound inequality**.

Notice that the equivalent compound inequalities do not contain absolute values.

Absolute Value Inequality	Equivalent Compound Inequality
$\lvert ax + b \rvert < c$	$-c < ax + b < c$
$\lvert ax + b \rvert \leq c$	$-c \leq ax + b \leq c$
$\lvert ax + b \rvert > c$	$ax + b < -c$ or $ax + b > c$
$\lvert ax + b \rvert \geq c$	$ax + b \leq -c$ or $ax + b \geq c$

1. Solve the linear absolute value inequality by rewriting it as an equivalent compound inequality. Then graph your solution on the number line.

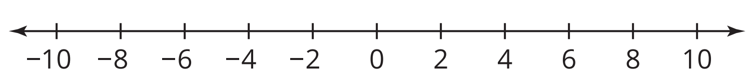
a. $\mathbf{\lvert x + 3 \rvert < 4}$



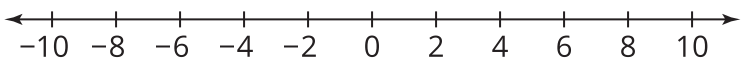
Remember:

As a final step, don't forget to check your solution.

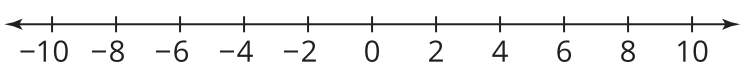
b. $\mathbf{6 \leq \lvert 2x - 4 \rvert}$



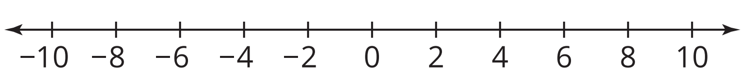
c. $|-5x + 8| + 2 < 25$



d. $|x + 5| > -1$



e. $|x + 5| < -1$



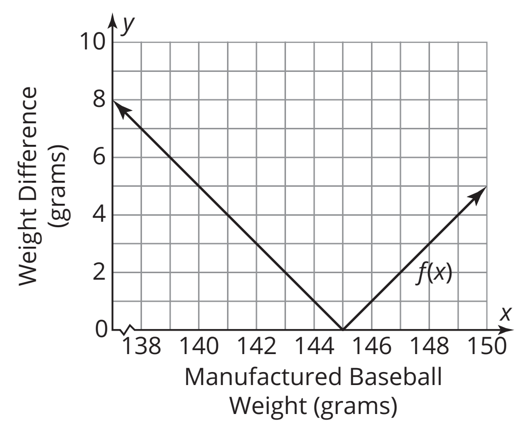
NOTES

TALK the TALK

Seeing Double

Consider the situation from the first activity: a baseball manufacturer sets the target weight of the baseballs at 145.045 grams. The specified weight allows for a certain distance, *y*, between the actual weight and the target weight.

1. Suppose this distance between the target weight and the actual weight is cut in half. Describe how this represents a transformation of the original function. Sketch a graph of the new function and write the new equation.



2. Describe why you can rewrite an absolute value equation as two separate equations.