5

Play Ball!

Absolute Value Equations and Inequalities

MATERIALS

None

Lesson Overview

Students begin this lesson by graphing the solution sets of simple absolute value equations on number lines and writing simple absolute value equations given their number line graphs. They then investigate absolute value functions using a real-world context. First, students write an absolute value equation to represent the context and solve it graphically. They then learn through a worked example and student work how to solve absolute value equations and practice this skill. Students revisit the real-world context; however, this time they write an absolute value inequality and solve it graphically. Students are provided compound inequalities that are equivalent to absolute value inequalities and they use these relationships to solve and graph absolute value inequalities.

Algebra 2
Cubic, Cube Root, Absolute Value and Rational Functions, Equations, and Inequalities

(6) The student applies mathematical processes to understand that cubic, cube root, absolute value and rational functions, equations, and inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

(D) formulate absolute value linear equations.

(E) solve absolute value linear equations.

(F) solve absolute value linear inequalities.

ELPS

1.A, 1.D, 1.E, 1.G, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.G, 4.K, 5.E

Essential Ideas

- Linear absolute value equations have 0, 1, or 2 solutions. The solution set for linear absolute value inequalities may contain all real numbers, a subset of the real numbers represented by a compound inequality, or no solutions.
- Linear absolute value inequalities can be rewritten as equivalent compound inequalities.
- Linear absolute value equations and inequalities can be used to represent real-world situations.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Opposites Attract? Absolutely!

Given simple absolute value equations in the form $|x| = c$, where c is some constant, students graph each solution set on a number line. Students then write absolute value equations given a solution set graphed on a number line.

Develop

Activity 5.1: Creating an Absolute Value Function from a Situation

Given a scenario about the weights of baseballs, students sketch the graph of an absolute value function and write the corresponding absolute value equation. Once constraints are provided, students write two equations for the least and greatest acceptable values. They solve the equations by determining the points of intersection of the graphs of the absolute value function and a constant function. Later in this lesson, students will revisit this context and solve using an absolute value inequality.

Activity 5.2: Solving Absolute Value Equations

Students analyze a worked example demonstrating how to solve a linear absolute value equation by rewriting it as two separate equations without the absolute value symbol. They then examine different strategies for solving absolute value equations, identifying incorrect thinking as necessary. Students solve absolute value equations, many with more than one term within the absolute value symbol and some where the absolute value expression is not isolated on one side of the equation.

Activity 5.3: Absolute Value Inequalities

Students revisit the baseball scenario; however this time they write a linear absolute value inequality to represent it. The graph is provided, and they use it to determine whether specified values are solutions to the inequality. Students use substitution to verify their answers. They transfer the solution from the graph to a number line and write a compound inequality to represent the solution. They write an inequality to represent the values that were not solutions to their original inequality and graph the solution on a number line.

Day 2

Activity 5.4: Solving Problems with Absolute Value Functions

Students engage in another scenario that can be modeled by an absolute value inequality, but this time the scenario involves the diameter of a little league baseball. They sketch the graph of the function on a coordinate plane, approximate the solution from a graph, and then verify their solution by writing and solving an absolute value inequality. Students then deal with a modified version of the scenario, but this time they write the absolute value inequality and then verify the solution graphically.

Activity 5.5: Absolute Value and Compound Inequalities

Students rewrite absolute value inequalities as compound inequalities, solve them algebraically, and graph their solutions on a number line.

Demonstrate

Talk the Talk: Seeing Double

The baseball scenario is modified so that students must apply a transformation to their original absolute value equation. They write an equation for the new function and sketch its graph. Students also describe in general why you can rewrite an absolute value equation as two separate equations.

ENGAGE

Getting Started: Opposites Attract? Absolutely!

Facilitation Notes

In this activity, given simple absolute value equations in the form $|x| = c$, where c is some constant, students graph each solution set on a number line. Students then write absolute value equations given a solution set graphed on a number line.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

As students work, look for

Errors when the constant c is a negative value. The solution set should be empty.

Questions to ask

- How many points should appear on the graph of the solution set of the absolute value equation?
- Will an absolute value equation always have two solutions? Why or why not?
- What is an example of an absolute value equation that has a single point solution set?
- What is an example of an absolute value equation that has no solutions?
- In each case, how is the constant used to identify the solution set?
- What does the constant represent in each equation?

Differentiation strategies

- To scaffold support for all students, create a classroom number line and have students use it to identify the graph of the solution set for each equation.
- To extend the activity, ask students to create an additional number line that takes the graph of Question 2, part (b) and moves each point one unit to the right so that the points graphed are -3 and 5. Ask students how they would modify the original absolute value equation $|x| = 4$ to write an absolute value equation that demonstrates this translation. Discuss why $|x - 1| = 4$ is the answer. This also allows students to see that the answers to absolute value equations do not always have to be in the form c and $-c$.

Summary

An absolute value equation may have 0, 1, or 2 solutions.

DEVELOP

Activity 5.1
Creating an Absolute Value Function from a Situation

Facilitation Notes

In this activity, given a scenario about the weights of baseballs, students sketch the graph of an absolute value function and write the corresponding absolute value equation. Once constraints are provided, students write two equations for the least and greatest acceptable values. They solve the equations by determining the points of intersection of the graphs of the absolute value function and a constant function. Later in this lesson, students will revisit this context and solve using an absolute value inequality.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Differentiation strategies

To scaffold support,

- If the decimal values are holding students back from understanding the new concepts, have them round the values to 145 grams and 3.3 grams.
- Construct the graph as a class. Begin by asking students to locate a point on the graph that represents a baseball that is the exact target weight. Next, locate two points that are approximately one gram less/more than the target weight, and so on. The V-shape of the absolute value function will emerge. Then, ask them to locate specific points on the graph and interpret the significance of each point as it relates to the problem situation.

As students work, look for

- Different methods students use to graph the function representing the scenario.
- Statements demonstrating understanding that this function is a translation of $f(x) = |x|$.
- Different responses to Question 2, part (b). Students may use arithmetic to get the least and greatest values, with equations $x = 141.75$ and $x = 148.34$. Other students may write the equations $x - 145.045 = 3.295$ and $x - 145.045 = -3.295$. If no students write the latter equations, provide them and help students make sense of them. This will be helpful as students begin the next activity.

Misconception

Students may think the equation for the graph is $|x| = 145.045$, thinking only of the target weight rather than representing how all the manufactured baseball's weights relate to the target weight.

Questions to ask

- What does *target weight* mean? What is the target weight of a baseball?
- Why does the point representing the target weight lie on the x-axis?
- Select a point on the left and right of the target weight and explain what each point represents.
- Why do your equations include a $+3.295$ and -3.295?
- What do the equations to the two solutions represent?

Summary

Absolute value functions can be used to model real-world scenarios.

Activity 5.2

Solving Absolute Value Equations

Facilitation Notes

In this activity, students analyze a worked example demonstrating how to solve a linear absolute value equation by rewriting it as two separate equations without the absolute value symbol. They then examine different strategies for solving absolute value equations, identifying incorrect thinking as necessary. Students solve absolute value equations, many with more than one term within the absolute value symbol and some where the absolute value expression is not isolated on one side of the equation.

Ask a student to read the introduction aloud. Analyze the worked example and complete Questions 1 and 2 as a class.

Differentiation strategies

To support students who struggle,

- Provide the final step shown from the worked example in Bob's work. Show $-(x - 1) = 6$ above $-x + 1 = 6$.
- Make Martina's thinking and Bob's thinking more explicit to students, and have them write their thinking next to their work.

Martina's thinking:	Bob's thinking:
The expression inside the absolute value symbol can equal 6 or −6. Both expressions will result in 6 when the absolute value is taken.	Both the expression inside the absolute value symbol as it is written and the opposite of that expression are equal to 6.

- Suggest students choose one method and use it consistently. Note that Bob's method may require distribution of a negative sign to an algebraic expression, while Martina's method includes the negative sign with the constant value.

Questions to ask

- Why is it acceptable to rewrite the absolute value equation as two separate equations?
- Why can $|a|$ be written as $+a$ and $-a$?
- How is an absolute value equation rewritten when there is an expression with more than one term within the absolute value symbol?
- How are Martina's work and Bob's work different?
- How did Bob get the equation $-x + 1 = 6$?
- Show how Martina and Bob will get the same solution.

Have students work with a partner or in a group to complete Question 3. Share responses as a class.

As students work, look for

- Sign errors in rewriting the absolute value equation as separate equations, simplifying the result, and solving the individual equations.
- Substitution of answers back into the original absolute value equation to check the answers.
- The realization that Question 3, part (c) does not have a solution prior to attempting to solve it.

Questions to ask

- Did you use Martina's strategy or Bob's strategy to solve this problem?
- What is another way to solve this problem?
- How do you know if your answer is correct?

Have students work with a partner or in a group to complete Questions 4 and 5. Share responses as a class.

Differentiation strategy

To assist all students, select four students in advance to present Artie's, Donald's, Cho's, and Steve's work. Suggest they talk through the steps as they write them. Have students close their books while the presentations are being made and then vote on which methods they think are correct.

Questions to ask for Question 4

- How is this equation different than the ones you solved in Question 3?
- Explain how Artie and Donald started to solve the equation using different strategies.
- How are Artie's and Donald's methods alike?
- What is another way Donald could have rewritten $|x| = 9$ as two separate equations?
- Do you prefer Artie's method or Donald's method? Why?
- Do you have to isolate the absolute value expression first?
- What would you suggest to Cho to correct her work?
- Explain how Steve's work includes both Martina's and Bob's methods.

Questions to ask for Question 5

- What is another way to solve this equation?
- What would the steps look like if you rewrote the expression inside the absolute value as two different expressions first?
- What does this equation look like when the absolute value part of the equation is isolated first?
- If you want to isolate the absolute value part of the equation in $3|x - 2| = 12$, what is the first step?
- If you want to isolate the absolute value part of the equation in $35 = 5|x + 6| - 10$, what is the first step? Can you divide by 5 first or should you add 10 first? Does it make a difference?

Misconception

Students may overgeneralize and think that whenever a negative value is on the opposite side of the equation as the absolute value expression, the equation automatically has no solutions. This is true only when the absolute value expression is isolated on one side of the equation. Use the equations $|x| - 14 = -10$ and $|x| + 14 = 10$ to clarify this misconception.

Summary

To solve an absolute value equation, the equation must be rewritten as two separate equations without the absolute value symbol.

Activity 5.3
Absolute Value Inequalities

Facilitation Notes

In this activity, students revisit the baseball scenario; however this time they write a linear absolute value inequality to represent it. The graph is provided, and they use it to determine whether specified values are solutions to the inequality. Students then use substitution to verify their answers. They transfer the solution from the graph to a number line and write a compound inequality to represent the solution. They then write an inequality to represent the values that were not solutions to their original inequality and graph the solution on a number line.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

Differentiation strategy

To assist all students, create a poster of the graph provided for Question 2. Have students use green dot stickers to represent acceptable weights and red stickers to represent unacceptable weights. Ask students to add additional stickers to the graph as well. Discuss why all stickers lie on the graph of the absolute value function, the meaning of the coordinate pair of each dot, and the relationship between the color of the sticker and its position in reference to the line $y = 3.295$. Refer to this graph as students transfer the answers to the number line in Question 3.

Misconception

Students may assume all points under the line $y = 3.295$ on the coordinate plane are part of the solution set. Discuss that when using the coordinate plane, only those values that lie on the graph of the absolute value function and are below the line $y = 3.295$ make sense. Then, discuss that because only the x-values (weight) are necessary, the answers may be transferred to a number line.

Questions to ask

- How did you know which inequality symbol to use in your absolute value inequality?
- What does 3.295 grams represent in this scenario?
- Where are the acceptable values positioned on the graph? Why does this make sense?
- What are the points of intersection? How did you determine them?
- Why does the solution to $|w - 145.045| = 3.295$ give you the least acceptable weight and the greatest acceptable weight?
- Should the graph on the number line have open or closed circles? Why?

- What end values are used on the number line graph?
- Is the number line or the inequality more helpful to you in understanding the solution? Why?

Have students work with a partner or in a group to complete Question 5. Share responses as a class.

Questions to ask

- Explain how you determined this inequality?
- Should the graph on the number line have open or closed circles? Why?
- How does this number line graph compare to the number line graph in Question 3?
- What is the compound inequality that represents the solution?

Summary

The solution set to an absolute value inequality can be graphed on a number line and expressed as a compound inequality.

Activity 5.4

Solving Problems with Absolute Value Functions

Facilitation Notes

In this activity, students engage in another scenario that can be modeled by an absolute value inequality, but this time the scenario involves the diameter of a little league baseball. They sketch the graph of the function on a coordinate plane, approximate the solution from a graph, and then verify their solution by writing and solving an absolute value inequality. Students then deal with a modified version of the scenario, but this time they write the absolute value inequality and then verify the solution graphically.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Differentiation strategies

- To scaffold support for all students, allow them to use technology to graph the functions and determine the points of intersection to verify their solutions.
- To extend the activity, ask students to redo the problem, but this time use the circumference, rather than the diameter, of the baseball.

As students work, look for

- The use of the target diameter of the baseball, 7.47, as the x-intercept in the graph of the linear absolute value function.
- A horizontal line, $y = 1.27$, to describe the acceptable difference in diameter.

Questions to ask

- How is the target diameter represented in your graph? In your equation?
- What does $y = 1.27$ represent in this scenario?
- How is the horizontal line helpful in solving this problem?
- What is the significance of the point (7.47, 0) on the graph?
- Does your inequality include the endpoints? Why or why not?
- What is the significance of the intersection of the function and the line $y = 1.27$?
- What is the domain of the linear absolute value function? How does this compare to the domain of the solution?
- What is the range of the linear absolute value function? How does this compare to the range of the solution?
- Would the manufacturer keep a baseball that has a diameter of 6.2 cm? Explain why or why not. What if the diameter of the baseball was 8.2 centimeters?
- What is the diameter of a baseball that meets the specified diameter, but the manufacturer decides not to keep it?

Summary

An absolute value function can be used to model some real-world scenarios.

Activity 5.5

Absolute Value and Compound Inequalities

Facilitation Notes

In this activity, students rewrite absolute value inequalities as compound inequalities, solve them algebraically, and graph their solutions on a number line.

Ask a student to read the introduction. Discuss the table as a class.

Differentiation strategies

To scaffold support,

- Use simpler absolute value inequalities with numbers first to make sense of the rules. For example, use $|x| < 7$, $|x| \leq 7$, $|x| > 7$, and $|x| \geq 7$.

- Have students sketch graphs on number lines to accompany each solution.
- Remind students that $-c$ means the opposite of c, not necessarily that c is a negative number.

Questions to ask

- Why does it make sense that the absolute value inequalities with less than symbols are rewritten as compact inequalities?
- What would the graph of this compact inequality look like?
- Can a compact inequality be written as two separate inequalities? If so, explain how you would write them.
- Why does it make sense that the absolute value inequalities with greater than symbols are rewritten as two separate inequalities?
- Why does it make sense that the absolute value inequalities with greater than symbols have one inequality written with $> c$ and the other with $< -c$?

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

As students work, look for

- Errors solving the compact inequalities.
- Confusion graphing the solution to part (c) when written as $6\frac{1}{5} > x > -3$. Suggest students rewrite the inequality as $-3 < x < 6\frac{1}{5}$.
- Students may lose sight of the meaning of absolute value in the process of solving. In parts (d) and (e), students may not notice that the inequalities are essentially saying *all absolute values are greater than −1* (the solution set is all real numbers) and *no absolute values are less than −1* (no solution). They might ignore these somewhat obvious conclusions and instead follow a procedure to rewrite the absolute value inequality and get the same results.

Misconception

Students may assume that when graphing compact inequalities, they should always shade between the two values. This is not always the case, as they should see when attempting to graph $-4 < x < -6$ in part (e). At first glance, it may appear that x lies between -4 and -6, however, when the inequality is interpreted, x can never be greater than -4 and less than -6. There is no solution to this inequality. Discuss the fact that when a compact inequality is written with $<$ symbols, the placement of the values should appear as they would on a number line. This is not the case with $-4 < x < -6$.

Questions to ask

- What are two values that are a solution to the inequality?
- How can you check that your solution is correct?
- In part (b), did you rewrite the inequality so that the absolute value expression is on the left of the inequality symbol? If so, what is the new inequality?
- In part (c), is it necessary to isolate the absolute value expression before applying the rules to rewrite the absolute value inequality as a compound inequality? Will you get the same solution either way?
- For which inequality are you required to reverse or flip the inequality symbol?
- When graphing a compound inequality, how do you know whether to graph the union or intersection of the two inequalities?
- When graphing a compound inequality, how do you know when to use *and* or *or*?
- Are the endpoints in the graph open or closed circles? Why?
- For parts (d) and (e), how could you determine the solution without completing any algebra steps?
- Are all absolute values greater than -1?
- Are any absolute values less than -1?

Summary

When solving an absolute value inequality, first rewrite it as an equivalent compound inequality without the absolute value symbol and then solve the compound inequality.

Talk the Talk: Seeing Double

DEMONSTRATE

Facilitation Notes

In this activity, the baseball scenario is modified so that students must apply a transformation to their original absolute value equation. They write an equation for the new function and sketch its graph. Students also describe in general why you can rewrite an absolute value equation as two separate equations.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Differentiation strategy

To support students who struggle, write the original absolute value function from the first activity on the board. Discuss what the values in the function represent with respect to the problem situation and what is different about this new situation.

As students work, look for

- Whether they multiply the fraction $\frac{1}{2}$ by the y-value or the x-expression.
- Changes to the constant function rather than the absolute value function.

Misconception

Students may not understand that taking one-half of the weight difference results in a dilation factor of 2 rather than $\frac{1}{2}$. Use actual coordinate pairs from the graph and substitute values in the new equation to help students make sense of this.

Questions to ask

- What was the equation of the original function?
- What does the y-value represent?
- Why does the function $\frac{1}{2}y = |x - 145.045|$ represent the new situation?
- How can the function $\frac{1}{2}y = |x - 145.045|$ be rewritten so that the variable y is isolated?
- Is $\frac{1}{2}y = |x - 145.045|$ equivalent to $y = 2|x - 145.045|$?
- What was the A-value in the original function? What is the new A-value?
- What affect does this change in A-value have on the graph?
- What is the new least acceptable weight?
- What is the new greatest acceptable weight?
- How can the least acceptable weight and the greatest acceptable weight be used to write a compound inequality?

Summary

An absolute value function can be used to model some real-world problems. When the parameters of the problem change, transformations may be applied to the original function to solve the new problem.

5

Play Ball!

Absolute Value Equations and Inequalities

Warm Up

Evaluate each expression.

1. $|9 + (-4)|$
2. $|-1 - 5|$
3. $|4 \cdot (-6)|$
4. $|0 \div (-2)|$

Learning Goals

- Understand and solve absolute value equations.
- Solve and graph linear absolute value inequalities on number lines.
- Graph absolute value functions and use the graph to determine solutions.

Key Terms

- linear absolute value equation
- linear absolute value inequality
- equivalent compound inequality

You know what the graphs of absolute value functions look like. How can you use what you know about graphs and linear equations to solve absolute value equations and inequalities?

Warm Up Answers

1. 5
2. 6
3. 24
4. 0

Answers

1. See number lines below.

2a. $|x| = 1$

2b. $|x| = 4$

ELL Tip

Students need to understand the meaning of the term *inspection* to correctly interpret the statement at the beginning of this activity. Discuss the meaning of the term *inspection* by analyzing its root word *inspect*, which means *to look at or examine something closely*. Provide examples of the word *inspection*, or *inspect*, in real-life scenarios, such as getting a car *inspected*, *inspecting* the contents of the package, or a uniform that passes *inspection*.

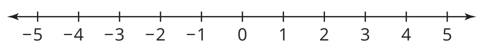
GETTING STARTED

Opposites Attract? Absolutely!

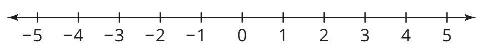
You can solve many absolute value equations using inspection.

1. Graph the solution set of each equation on the number line given.

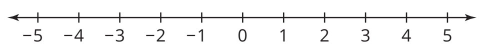
a. $|x| = 5$



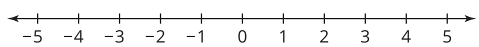
b. $|x| = 2$



c. $|x| = -3$

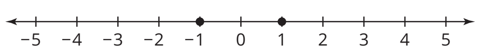


d. $|x| = 0$

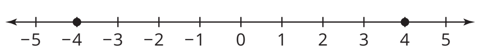


2. Write the absolute value equation for each solution set graphed.

a.

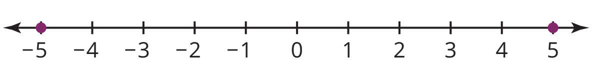


b.

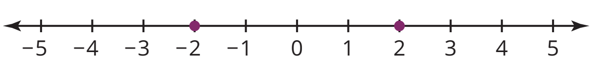


2 • TOPIC 1: Extending Linear Relationships

1a.

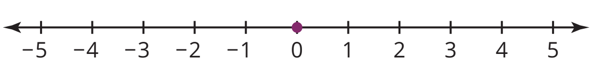


1b.



1c. No solutions

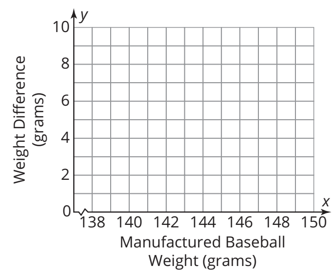
1d.



ACTIVITY **5.1** Creating an Absolute Value Function from a Situation

The official rules of baseball state that all baseballs used during professional games must be within a specified range of weights. The baseball manufacturer sets the target weight of the balls at 145.045 grams on its machines.

1. Sketch a graph that models the relationship between a manufactured baseball's weight, x, and its distance from the target weight, y. Explain how you constructed your sketch. Then write an absolute value equation to represent the situation and the graph.



2. The specified weight allows for a difference of 3.295 grams in the actual weight of a ball and the target weight. Since the weight must be within a distance of 3.295 grams from the target weight, $y = 3.295$.

a. Graph the equation $y = 3.295$ on the coordinate plane in Question 1.

b. What two equations can you write, without absolute values, to show the least acceptable weight and the greatest acceptable weight of a baseball? Explain your reasoning.

c. Use the graph to write the solutions to the equations you wrote in part (b). Show your work.

Ask yourself:

How is the function transformed from the basic function $f(x) = |x|$?

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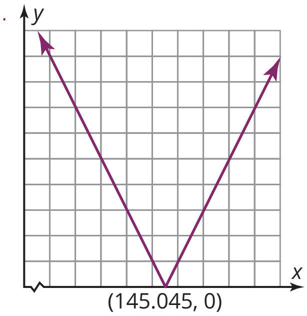
Answers

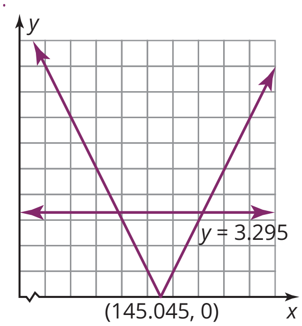
1.



At $x = 145.045$, the baseball weight's distance from the target weight, y, is 0. For each unit of decrease or increase in the baseball's weight, x, there is an equal decrease or increase in the distance from the target weight. These distances are all positive.

The graph can be represented by the function $d(x) = |x - 145.045|$.

2a.



2b. $x - 145.045 = 3.295$ and $x - 145.045 = -3.295$

2c. $x = 148.34$ or $x = 141.75$

ELL Tip

Discuss the meaning of the term *manufacturer* by analyzing its root word *manufacture*, which means *to make something*. Relate *manufacture* to a *manufacturer*, as *one who makes something*; provide synonyms such as *maker*, *producer*, and *builder*. Ask students to explain how *manufacturer* is used in the context of this activity.

Answers

1. When I add $+(a)$ and $-(a)$, I get a sum of 0.
2. Martina divided both sides of the second equation by -1, and Bob distributed the (-1).

   $x = 7$ or $x = -5$

ACTIVITY **5.2**

Solving Absolute Value Equations

The two equations you wrote can be represented by the **linear absolute value equation** $|w - 145.045| = 3.295$. To solve any absolute value equation, recall the definition of absolute value.

Worked Example

Consider this linear absolute value equation.

$$|a| = 6$$

There are two points that are 6 units away from zero on the number line: one to the right of zero, and one to the left of zero.

$$+(a) = 6 \quad \text{or} \quad -(a) = 6$$

$$a = 6 \quad \text{or} \quad a = -6$$

Now consider the case where $a = x - 1$.

$$|x - 1| = 6$$

If you know that $|a| = 6$ can be written as two separate equations, you can rewrite any absolute value equation.

$$+(a) = 6 \quad \text{or} \quad -(a) = 6$$

$$+(x - 1) = 6 \quad \text{or} \quad -(x - 1) = 6$$

1. How do you know the expressions $+(a)$ and $-(a)$ represent opposite distances?

2. Martina and Bob continued to solve the linear absolute value equation $|x - 1| = 6$ in different ways. Compare their strategies and then determine the solutions to the equation.

Martina

$(x - 1) = 6$ or $(x - 1) = -6$

Bob

$x - 1 = 6$ or $-x + 1 = 6$

3. Solve each linear absolute value equation. Show your work.

a. $|x + 7| = 3$

b. $|x - 9| = 12$

c. $|3x + 7| = -8$

d. $|2x + 3| = 0$

Ask yourself:

Before you solve each equation, think about the number of solutions each equation may have. You may be able to save yourself some work—and time!

4. Artie, Donald, Cho, and Steve each solved the equation $|x| - 4 = 5$.

Artie

$|x| - 4 = 5$

$x - 4 = 5$ $-(x) - 4 = 5$

$x = 9$ $-x = 9$

$x = -9$

Donald

$|x| - 4 = 5$

$|x| = 9$

$x = 9$ $-(x) = 9$

$x = -9$

Cho

$|x| - 4 = 5$

$(x) - 4 = 5$ $-[(x) - 4] = 5$

$x - 4 = 5$ $-x + 4 = 5$

$x = 9$ $-x = 1$

$x = -1$

Steve

$|x| - 4 = 5$

$(x) - 4 = 5$ $-(x) - 4 = -5$

$x = 9$ $-x - 4 = -5$

$-x = -1$

$x = 1$

a. Explain how Cho and Steve incorrectly rewrote the absolute value equation as two separate equations.

b. Explain the difference in the strategies that Artie and Donald used.

Answers

3a. $x = -4$ or $x = -10$

3b. $x = 21$ or $x = -3$

3c. No solutions

3d. $x = -\frac{3}{2}$

4a. Cho incorrectly applied the negative symbol to the (−4) in the second equation. Steve incorrectly applied a second negative symbol to the (−5).

4b. Artie applied the positive and negative signs to the expression inside the linear absolute value symbols and then solved each equation. Donald isolated the linear absolute value expression before applying the positive and negative symbols to the isolated expression.

Answers

5a. $x = 16$ or $x = -16$

5b. $25 = x$ or $-9 = x$

5c. $x = 6$ or $x = -2$

5d. $3 = x$ or $-15 = x$

Think about:

Consider isolating the absolute value part of the equation before you rewrite it as two equations.

5. Solve each linear absolute value equation.

a. $|x| + 16 = 32$

b. $23 = |x - 8| + 6$

c. $3|x - 2| = 12$

d. $35 = 5|x + 6| - 10$

ACTIVITY
5.3 Absolute Value Inequalities

You determined the linear absolute value equation $|w - 145.045| = 3.295$ to identify the most and least a baseball could weigh and still be within the specifications. The manufacturer wants to determine all of the acceptable weights that the baseball could be and still fit within the specifications. You can write a **linear absolute value inequality** to represent this problem situation.

1. Write a linear absolute value inequality to represent all baseball weights that are within the specifications.

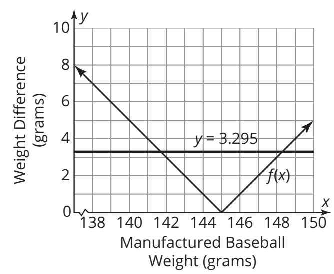
2. Use the graph to determine whether the weight of each given baseball is acceptable. Substitute each value in the inequality to verify your answer.

a. 147 grams

b. 140.8 grams

c. 148.34 grams

d. 141.75 grams



Answers

1. $|w - 145.045| \leq 3.295$

2a. Yes

2b. No

2c. Yes

2d. Yes

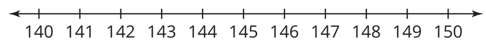
ELL Tip

Two non-mathematical terms that appear in this activity are *specifications* and *dispose*. Discuss the definition of each term: *specifications* are *detailed standards that are expected to be met when something is built*, and *dispose* means *to get rid of, or throw away*. Discuss the type of *specifications* that are referenced in this context, and why the term *disposing* makes sense in this context.

Answers

3. See number line below.
4. $141.75 \leq w \leq 148.34$
5a. $|w - 145.05| > 3.295$
5b. See number line below.

3. Use the graph on the coordinate plane to graph the inequality on the number line showing all the acceptable weights. Explain the process you used.



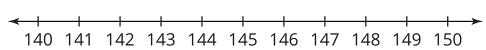
4. Complete the inequality to describe all the acceptable weights, where *w* is the baseball's weight.

$\_\_\_\_\_\_\_\_ \leq w \leq \_\_\_\_\_\_\_\_$

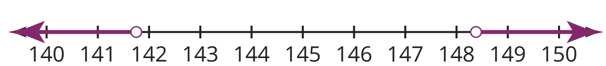
5. Raymond has the job of disposing of all baseballs that are not within the acceptable weight limits.

a. Write an absolute value inequality to represent the weights of baseballs that Raymond can dispose of.

b. Graph the inequality on the number line. Explain the process you used.



3.


5b.


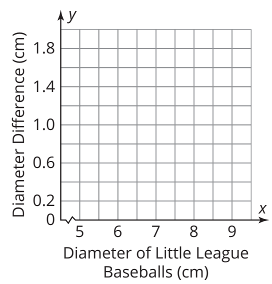
ACTIVITY **5.4**

Solving Problems with Absolute Value Functions

In Little League Baseball, the diameter of the ball is slightly smaller than that of a professional baseball.

1. For Little League baseballs, the manufacturer sets the target diameter to be 7.47 centimeters. The specified diameter allows for a difference of 1.27 centimeters.

a. Sketch the graph of the linear absolute value function, $f(d)$, on the coordinate plane.



b. Use your graph to estimate the diameters of all the Little League baseballs that fit within the specifications. Explain how you determined your answer.

c. Algebraically determine the diameters of all the baseballs that fit within the specification. Write your answer as an inequality.

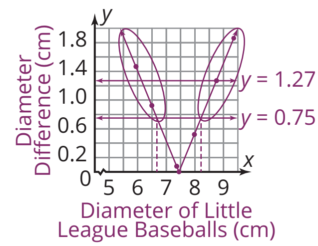
2. The manufacturer knows that the closer the diameter of the baseball is to the target, the more likely it is to be sold. The manufacturer decides to keep only the baseballs that are less than 0.75 centimeter from the target diameter.

a. Algebraically determine which baseballs will not fall within the new specified limits and will not be kept. Write your answer as an inequality.

b. How can you use your graph to determine whether you are correct?

Answers

1a.



1b. The graph of $y = 1.27$ crosses the original graph at two points. The first is between 6 and 6.5, so the smallest diameter of a baseball that meets specifications would be about 6.25 centimeters. It also crosses between 8.5 and 9 so the largest diameter of a baseball that meets the specifications would be about 8.74 centimeters.

1c. $6.2 \leq d \leq 8.74$

2a. $d \leq 6.72$ or $d \geq 8.22$

2b. The x-coordinates of the points of intersection describe the upper and lower limit of the solution set. All points less than approximately 6.5 cm are not acceptable and all points greater than approximately 8 cm are also not acceptable.

Answers

1. See number lines below.

ACTIVITY **5.5**

Absolute Value and Compound Inequalities

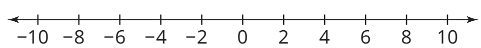
Absolute value inequalities can take four different forms, as shown in the table. To solve a linear absolute value inequality, you can first write it as an **equivalent compound inequality**.

Notice that the equivalent compound inequalities do not contain absolute values.

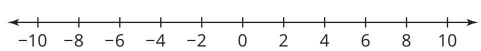
Absolute Value Inequality	Equivalent Compound Inequality
$\|ax + b\| < c$	$-c < ax + b < c$
$\|ax + b\| \le c$	$-c \le ax + b \le c$
$\|ax + b\| > c$	$ax + b < -c$ or $ax + b > c$
$\|ax + b\| \ge c$	$ax + b \le -c$ or $ax + b \ge c$

1. Solve the linear absolute value inequality by rewriting it as an equivalent compound inequality. Then graph your solution on the number line.

a. $|x + 3| < 4$



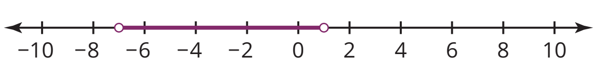
b. $6 \le |2x - 4|$



Remember:

As a final step, don't forget to check your solution.

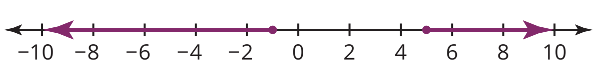
1a.



$-4 < x + 3 < 4$

$-7 < x < 1$

1b.

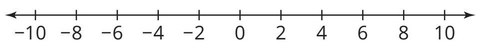


$6 \le 2x - 4$ or $-6 \ge 2x - 4$

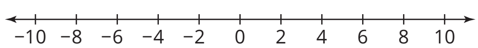
$10 \le 2x$ or $-2 \ge 2x$

$5 \le x$ or $-1 \ge x$

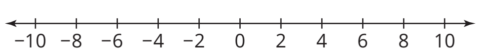
c. $|-5x + 8| + 2 < 25$



d. $|x + 5| > -1$

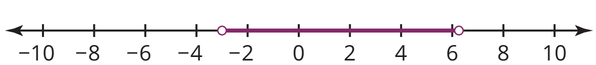


e. $|x + 5| < -1$

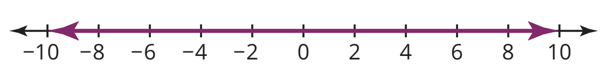


Answers

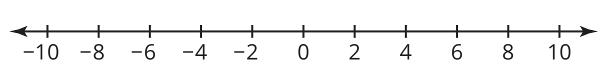
1. See number lines below.

1c. 

$$|-5x + 8| < 23$$
$$-23 < -5x + 8 < 23$$
$$-23 - 8 < -5x < 23 - 8$$
$$-31 < -5x < 15$$
$$-3 < x < \frac{31}{5}$$

1d. 

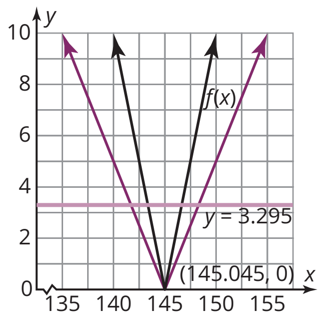
This solution is all numbers. The absolute value of any number is always greater than a negative number.

1e. 

There is no solution. The absolute value can never be less than a negative number.

Answers

1.



The transformation cuts the range of accepted weights in half. The new function is $\frac{1}{2}y = |x - 145.045|$, which is equivalent to $y = 2|x - 145.045|$. The function has been vertically dilated by a factor of 2, which is represented by a change in the A-value from 1 to 2.
The least and greatest accepted weights are $x - 145.045 = 1.6475$ and $x - 145.045 = (-1.6475)$; so $143.3975 \leq x \leq 146.6925$.

2. Sample answer.
You can rewrite an absolute value equation as two separate equations because the absolute value function indicates a distance between a value and a given location on a number line. Because the value may appear to the right or the left of the given location, two values are possible. To solve, you have to account for both.

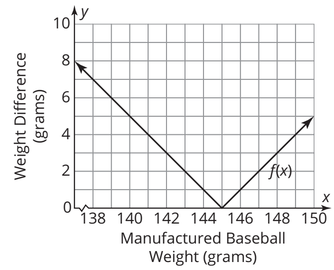
NOTES

TALK the TALK

Seeing Double

Consider the situation from the first activity: a baseball manufacturer sets the target weight of the baseballs at 145.045 grams. The specified weight allows for a certain distance, y, between the actual weight and the target weight.

1. **Suppose this distance between the target weight and the actual weight is cut in half. Describe how this represents a transformation of the original function. Sketch a graph of the new function and write the new equation.**



2. **Describe why you can rewrite an absolute value equation as two separate equations.**