Extending Linear Relationships

Topic 1 Overview

How is *Extending Linear Relationships* organized?

Extending Linear Relationships advances students' ability to solve systems of equations. The topic begins with a review of a system of two linear equations in two variables, including the graphing, substitution, and elimination methods for solving systems of two linear equations. Students recall what the solution to a system of equations means, as well as the cases of no solution or infinite solutions. The methods of solving are then applied to systems of three linear equations in three variables. Students then learn Gaussian elimination as an algorithm for solving linear systems of equations.

Students also use systems of linear inequalities to model optimal solutions to real-world situations. They write linear inequalities to represent constraints in a given scenario and then combine inequalities to create a system that encompasses all of the constraints. Students use graphical representations to determine solutions. Finally, they explore linear programming, where they use the vertices of the solution region to determine maximum or minimum values.

Next, students are introduced to matrices and explore their properties. Students then discover the identity matrix and use technology to calculate inverse matrices. Finally, students use matrices to solve systems of linear equations in three variables. They learn to use matrix equations to identify whether systems of equations have one solution, no solution, or infinitely many solutions.

Students calculate the absolute value of given values before considering the linear absolute value function. To help students understand the structure of a linear absolute value function and its graph, students think about an absolute value function as a linear function that has a reflection across the x-axis (or the D-value as the functions get more complex). They first graph the function $f(x) = x$; next they graph $f(x) = |x|$ and discuss how the graph changes. The process is repeated for $f(x) = |-x|$. Now comfortable with the shape of the graph of an absolute value function, students explore transformations of the A-, B-, C-, and D-values. They make generalizations about the effect of these transformations on the graph of a given function and its key characteristics. Students move from these abstract experiences with transformation to solving and graphing linear absolute value equations and inequalities based on real-world situations. Students begin with a graphical representation of a linear absolute value function and use a horizontal line to solve an equation or an inequality. With a solid visual understanding of the structure of a linear absolute value function, students then solve these equations and inequalities using what they know about Properties of Equalities and compound inequalities.

What is the entry point for students?

Students have experience solving linear systems in two variables. In previous courses, students learned to solve systems of linear equations graphically and algebraically. They understand that solutions are located where the graphs of the equations intersect and that the solutions make each equation true. In a previous course, students expanded on their ability to solve systems, adding linear combinations to their repertoire of strategies. This topic uses this knowledge to solve more complex systems, including those containing three variables and three equations. Students have transformed linear functions, focusing primarily on vertical dilations, vertical translations, and reflections. They have learned that absolute value is a measure of magnitude and have calculated the absolute value of a given number. They combine this knowledge to graph and transform linear absolute value equations. Students use their knowledge of solving linear equations in one variable to solve absolute value equations. They have previously solved compound linear inequalities and build on those experiences to solve absolute value inequalities.

How does a student demonstrate understanding?

Students will demonstrate understanding in *Extending Linear Relationships* if they can:

- Define a system of equations and solution of a system.
- Solve a system of equations using graphing, substitution, linear combinations, and matrices.
- Explain why some systems have no solutions or infinitely many solutions.
- Explain that a point of intersection on the graph of a system of equations represents a solution to both equations.
- Use Gaussian elimination to solve a system of three equations and three variables algebraically.
- Model a real-world scenario with a system of equations or inequalities.
- Interpret solutions to a system of equations or inequalities in terms of a context.
- Graph a system of linear inequalities on a coordinate plane and locate points that satisfy the system.
- Represent a system of linear equations as a matrix equation.
- Solve a system of three linear equations using inverse matrices on a graphing calculator.
- Graph an absolute value function.
- Describe the key characteristics of the graph of a linear absolute value function.
- Transform an absolute value function and a linear piecewise function, noting the results of changing the A-, B-, C-, and D-values.
- Solve absolute value equations and inequalities using a graph or the Properties of Equality.

Why is *Extending Linear Relationships* important?

Throughout the rest of this course, students will use the connection between graphical and algebraic representations to determine solutions to problems. Knowing how to represent and solve systems with matrices extends students' ability to solve much larger systems with more than two equations. Students who choose to pursue engineering, computer science, economics, and mathematics fields will use systems to analyze and solve complex problems and will learn to program machines to complete the calculations. The ability to set up models and analyze solutions in terms of the context are critical skills in these fields of study and careers.

How do the activities in *Extending Linear Relationships* promote student expertise in the mathematical process standards?

All Carnegie Learning topics are written with the goal of creating mathematical thinkers who are active participants in classroom discourse, so productive habits of mind are evident in all lessons. Students are expected to make sense of problems and work towards solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

Students make use of structure as they relate the process of matrix multiplication to solving a system of equations in three variables. They also make use of structure as they use transformations to graph linear absolute value functions. Students use mathematics to model typical problems in context as they represent real-world scenarios with a system of equations or inequalities.

Materials Needed

Graphing Technology
Technology that can operate with matrices
Masking Tape
Markers

New Tools and Notation

You can solve matrix equations using a process that is similar to the one you use to solve linear equations.

Worked Example

Solving Linear Equations	**Description**	**Solving Matrix Equations**
$\frac{2}{3}x = 8$		$\begin{cases} 2x + y = -6 \\ x + 3y = 7 \end{cases}$ $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ 7 \end{bmatrix}$ $A \cdot X = B$
$\left(\frac{3}{2}\right)\left(\frac{2}{3}\right)x = \left(\frac{3}{2}\right)18$	Multiply each side by the multiplicative inverse.	$A^{-1} = \begin{bmatrix} \frac{3}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix}$ $\begin{bmatrix} \frac{3}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix} \begin{bmatrix} -6 \\ 7 \end{bmatrix}$ $A^{-1} \cdot A \cdot X = A^{-1} \cdot B$
$1x = 27$	A number times its multiplicative inverse equals the identity.	$\begin{bmatrix} \left(\frac{3}{5}\right)2 + \left(-\frac{1}{5}\right)1 & \left(\frac{3}{5}\right)1 + \left(-\frac{1}{5}\right)3 \\ \left(-\frac{1}{5}\right)2 + \left(\frac{2}{5}\right)1 & \left(-\frac{1}{5}\right)1 + \left(\frac{2}{5}\right)3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} =$ $\begin{bmatrix} \left(\frac{3}{5}\right)(-6) + \left(-\frac{1}{5}\right)7 \\ \left(-\frac{1}{5}\right)(-6) + \left(\frac{2}{5}\right)7 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \left(-\frac{18}{5}\right) + \left(-\frac{7}{5}\right) \\ \left(\frac{6}{5}\right) + \left(\frac{14}{5}\right) \end{bmatrix}$ $I \cdot X = A^{-1} \cdot B$
$x = 27$	A quantity multiplied by the identity equals itself.	$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ 4 \end{bmatrix}$ $X = A^{-1} \cdot B$

Learning Together

ELPS: 1.A, 1.C, 1.D, 1.E, 1.F, 1.G, 2.C, 2.D, 2.E, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.E, 3.F, 4.A, 4.B, 4.C, 4.G, 4.K, 5.B, 5.E, 5.F, 5.G

Lesson	Lesson Name	TEKS	Days	Highlights
1	Gauss in Das Haus Solving Systems of Equations	2A.3A 2A.3B 2A.3C 2A.3D	2	Students solve systems comprised of linear and quadratic equations. They begin by solving a system of two linear equations graphically and algebraically. Students then use substitution to solve a system that is comprised of a quadratic equation and a linear equation. In each case, they use graphs to determine the number of possible solutions to that type of system. Students practice solving systems of two equations in real-world and mathematical problems. Students then solve systems of three linear equations in three variables using substitution and Gaussian elimination, both in and out of context.
2	Make the Best of It Optimization	2A.3A 2A.3E 2A.3F 2A.3G	2	Students move from solving systems of equations to solving systems of inequalities. They model problems in context requiring several inequalities to be graphed on the same coordinate plane. Students recognize that the solution to a system of inequalities is the intersection of the solutions to each inequality. Then, through a context, they are introduced to linear programming as a process to determine the optimal solution to a system of linear inequalities. Students use linear programming to solve problems and explain the difference between the solution to a system of linear inequalities and the solution to an equation calculated by linear programming.

Lesson	Lesson Name	TEKS	Days	Highlights
3	Systems Redux Solving Matrix Equations	2A.3B	2	Students are introduced to identity and inverse matrices. They express a system of equations as a matrix equation. Students relate solving a matrix equation to solving a linear equation, and the use of technology to solve a matrix equation. As a culminating activity, they model a scenario with a system of equations, convert it to a matrix equation, solve the matrix equation using technology, and interpret the solution in terms of the scenario.
4	Putting the V in Absolute Value Defining Absolute Value Functions and Transformations	2A.2A 2A.6C 2A.7I	3	Students are already familiar with the general shape of the graphs of absolute value functions, and they have studied transformations of linear functions. In this lesson, students experiment with the absolute value function family. They expand their understanding of transformations to include horizontal translations and dilations. Students interpret functions in the form $f(x) = A(B(x - C)) + D$. They distinguish between the effects of changing values inside the argument of the function (the *B*- and *C*-values) and changing values outside the function (the *A*- and *D*-values). At the end of the lesson, students summarize the impact of transformations on the domain and range of the absolute value function.
5	Play Ball! Absolute Value Equations and Inequalities	2A.6D 2A.6E 2A.6F	2	Students begin this lesson by graphing the solution sets of simple absolute value equations on number lines and writing simple absolute value equations given their number line graphs. They then investigate absolute value functions using a real-world context. First, students write an absolute value equation to represent the context and solve it graphically. They then learn through a worked example and student work how to solve absolute value equations and practice this skill. Students revisit the real-world context; however, this time they write an absolute value inequality and solve it graphically. Students are provided compound inequalities that are equivalent to absolute value inequalities and they use these relationships to solve and graph absolute value inequalities.

Suggested Topic Plan

*1 Day Pacing = 45 min. Session

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: 2A.3A, 2A.3B, 2A.3C, 2A.3D **LESSON 1** **Gauss in Da Haus** **GETTING STARTED** **ACTIVITY 1** **ACTIVITY 2**	**LESSON 1** continued **ACTIVITY 3** **ACTIVITY 4** **TALK THE TALK**	**MATHia** Use LiveLab and Reports to monitor students' progress	TEKS: 2A.3A, 2A.3E, 2A.3F, 2.A,3G **LESSON 2** **Make the Best of It** **GETTING STARTED** **ACTIVITY 1**	**LESSON 2** continued **ACTIVITY 2** **TALK THE TALK**

Day 6	Day 7	Day 8	Day 9	Day 10
MATHia Use LiveLab and Reports to monitor students' progress	TEKS: 2A.3B **LESSON 3** **Systems Redux** **GETTING STARTED** **ACTIVITY 1**	**LESSON 3** continued **ACTIVITY 2** **TALK THE TALK**	**MATHia** Use LiveLab and Reports to monitor students' progress	TEKS: 2A.2A, 2A.6C, 2A.7I **LESSON 4** **Putting the V in Absolute Value** **GETTING STARTED** **ACTIVITY 1**

Day 11	Day 12	Day 13	Day 14	Day 15
LESSON 4 continued **ACTIVITY 2** **ACTIVITY 3**	**LESSON 4** continued **ACTIVITY 4** **ACTIVITY 5** **TALK THE TALK**	**MATHia** Use LiveLab and Reports to monitor students' progress	TEKS: 2A.6D, 2A.6E, 2A.6F **LESSON 5** **Play Ball!** **GETTING STARTED** **ACTIVITY 1** **ACTIVITY 2** **ACTIVITY 3**	**LESSON 5** continued **ACTIVITY 4** **ACTIVITY 5** **TALK THE TALK**

Day 16	Day 17
MATHia Use LiveLab and Reports to monitor students' progress	**END OF TOPIC ASSESSMENT**

Assessments

There is one assessment aligned to this topic: End of Topic Assessment.