Module 1: Exploring Patterns in Linear and Quadratic Relationships

TOPIC 2: EXPLORING AND ANALYZING PATTERNS

This topic gives students opportunities to analyze and describe various patterns. Students are asked to represent algebraic expressions in different forms and use algebra and graphs to determine whether they are equivalent. Lessons provide opportunities for students to review linear, exponential, and quadratic functions using multiple representations. Students also learn to write quadratic equations given any three points. They are introduced to the complex number system and solve quadratic equations with imaginary roots.

Where have we been?

In previous courses, students gained extensive experience with multiple representations of linear, exponential, and quadratic functions. This topic serves as an opportunity for students to recall what they already know and prepare for how they will use this knowledge throughout the rest of the course.

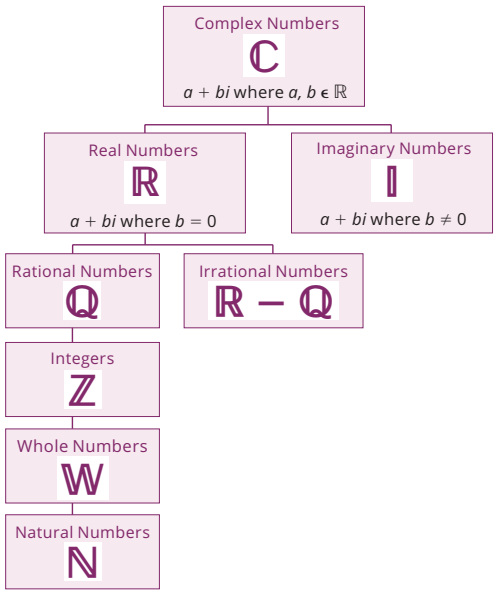
Where are we going?

Throughout the rest of this course, students will be exploring more complex functions: polynomials, rational functions, radical functions, and logarithmic functions. This topic is critical for preparing students for this work, which includes identifying key characteristics, connecting representations, writing expressions, using expressions and equations to solve for unknown values, analyzing graphical representations, and making connections to the real world.

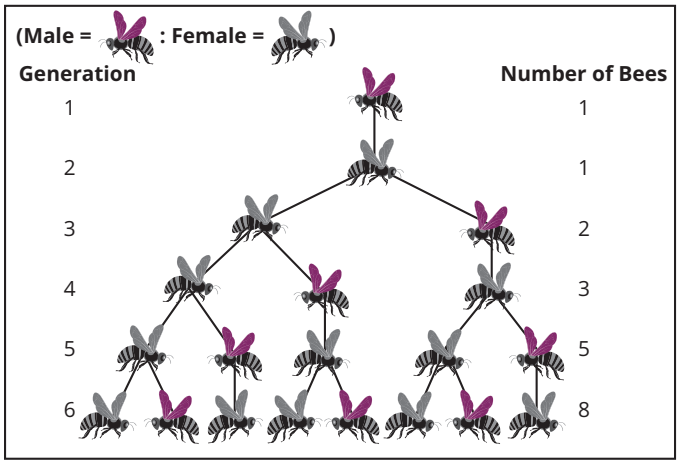
The Complex Number System

The set of complex numbers is the set of all numbers written in the form $a + bi$, where a and b are real numbers. The term a is the real part of a complex number, and the term bi is the imaginary part of a complex number. The set of complex numbers is represented by the notation $\mathbb{C}$.

The set of imaginary numbers is represented by the notation $\mathbb{I}$. A pure imaginary number is a number of the form $a + bi$, where a is equal to 0 and b is not equal to 0.



Droning On and On



You can find patterns everywhere! And sometimes a pattern's beauty isn't evident until you describe it using mathematics. Consider a pattern found in nature—the family tree of a male drone bee. Female bees have two parents, a male and a female, whereas male bees have just one parent, a female. In this family tree the parents appear below the original male drone bee. The total number of bees in each generation follows the pattern 1, 1, 2, 3, 5, 8, ... What makes this particular pattern fascinating is that it seems to appear everywhere! This pattern is called the Fibonacci sequence and you can find it in flowers, seashells, pineapples, art, architecture, and even in your DNA! Do you see the pattern? If so, name the next three terms.

Talking Points

Quadratic functions can be an important topic to know about for college admissions tests.

Here is an example of a sample question:

What are the coordinates of the vertex of the parabola whose equation is $y = 2x^2 + 4x - 5$?

If you represent the quadratic equation as $ax^2 + bx + c$, then the x-coordinate of the vertex is $\frac{-b}{2a} = \frac{-4}{2(2)} = -1$. The y-coordinate of the vertex is then $2(-1)^2 + 4(-1) - 5 = -7$.

So, the vertex is at $(-1, -7)$.

Key Terms

relation

A relation is a mapping between a set of input values and a set of output values.

function

A function is a relation such that for each element of the domain, there exists exactly one element in the range.

factored form of a quadratic equation

An equation of the form $f(x) = a(x - r_1)(x - r_2)$, where a does not equal 0.

vertex form of a quadratic equation

An equation of the form $f(x) = a(x - h)^2 + k$, where a does not equal 0.