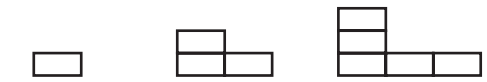
2

The Cat's Out of the Bag!

Generating Algebraic Expressions

Warm Up

Consider the block designs.



Design 1 Design 2 Design 3

1. Describe and then draw Design 4.

2. Describe the observable pattern.

Learning Goals

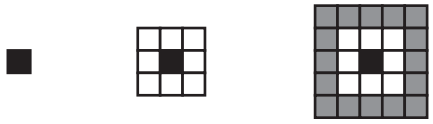
- Generate algebraic expressions using geometric patterns.
- Represent algebraic expressions in different forms.
- Determine whether expressions are equivalent.
- Identify patterns as linear, exponential, or quadratic using a visual model, a table of values, or a graph.

You have described geometric patterns using words. How can you write an algebraic expression to represent a pattern? And how do you know whether two expressions are equivalent?

Blast from the Very Recent Past

Miriam owns a flooring company. Her latest job involves tiling a square room in a pattern of alternating black, white, and gray tiles as shown.

Design 1 Design 2 Design 3

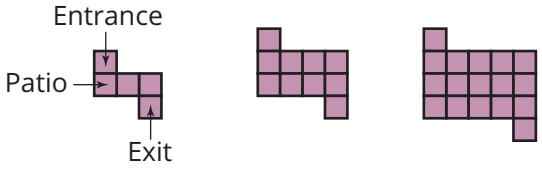


The class president, vice president, and treasurer of a high school count the ballots for the homecoming king election. The day after the ballot counting, each ballot counter tells two of their friends in the senior class the election result. The following day, each of the ballot counter's friends shares the election result with two of their friends in the senior class. This pattern continues for the entire week leading up to the pep rally.

Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
3	6	12	24	48	96	192

Maureen and Matthew are designing their backyard patio. There will be an entrance and exit off the front and back of the patio. The sequence shown represents different designs depending on the size of the patio.

Design 1 Design 2 Design 3



Consider the scenarios from the previous lesson.

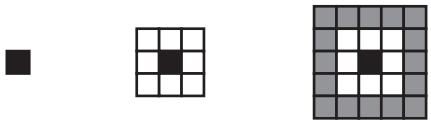
1. **Describe the growth pattern for each scenario.**

2. **Describe how the quantities change relative to one another.**

ACTIVITY
2.1 Using Patterns to Build Expressions

Miriam's flooring business is booming! She decides to hire several employees to help lay out her tile designs. It will be necessary for Miriam to describe her tile designs in a clear manner so that all of the employees can create them correctly. Miriam's square floor design uses alternating black, white, and gray tiles.

Design 1 Design 2 Design 3



1. **Describe the pattern of new tiles that are added to each design. Is the pattern linear, quadratic, or exponential? Explain your reasoning.**

Think about:

Describe the pattern in terms of the number of new tiles that must be added to each new square floor design.

2. **Write an expression to represent the number of new tiles that must be added to an $n \times n$ square floor design to build the next design. Let n represent the number of tiles along each edge of the square.**

3. **Describe which values for n make sense in this problem situation.**

4. Ramone determined an expression to represent this pattern. His expression and explanation are shown. Explain why Ramone's expression is incorrect.

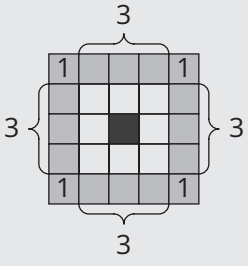
Ramone

Design	1	2	3
New Tiles	0	8	16

The expression $8(n - 1)$ represents Miriam's square floor pattern. I noticed that the number of new tiles is increasing by 8 in each new design.

Miriam asks her employees to determine the number of new tiles added to Design 2 to create Design 3. Each employee describes a unique method to determine the number of additional tiles needed to create Design 3.

5. Represent each of her employee's explanations with an algebraic expression that describes how many new tiles must be added to an $n \times n$ square to build the next design.



Wilma

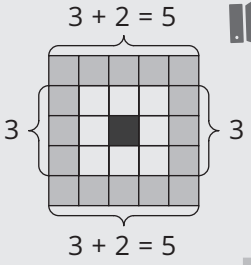
I must add 3 tiles to each of the four sides of the white square, which is $4 \cdot 3$ tiles. Then I must add 1 tile at each corner. So the number of additional tiles added to the Design 2 square floor design is $4 \cdot 3 + 4$.

Expression: _____

Howard

I must add 5 tiles to two of the sides and 3 tiles to the other two sides. The number of additional tiles added to the Design 2 square floor design is $2(3 + 2) + 2 \cdot 3$.

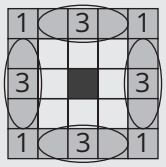
Expression: _____



Tyler

I need to add 3 tiles four times and then add the four corner tiles. The number of additional tiles added to the Design 2 square floor design is $3 + 3 + 3 + 3 + 4$.

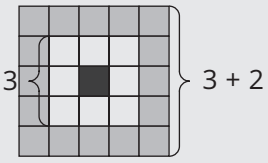
Expression: _____



Tamara

The way I look at it, I really have two squares. The original square for Design 2 has $3 \cdot 3$ tiles. The newly formed Design 3 square has $5 \cdot 5$ tiles. So, the number of additional tiles added to the Design 2 square floor design is $5 \cdot 5 - 3 \cdot 3$.

Expression: _____



6. Which expression do you think Miriam should use? Explain your reasoning.

Think about:

Does the expression you determined match one of the expressions Miriam's employees determined?

7. **Michael and Louise analyze the expressions they wrote for each employee. They both determined that the expression to represent Tamara's method is $(n + 2)^2 - n^2$. Michael claims that this expression is quadratic because of the n^2 term. Louise disagrees and says the expression is linear because the pattern grows by a constant amount. Who is correct? Explain your reasoning.**

8. **Use each expression you determined in Question 5 to calculate the number of tiles that must be added to squares with side lengths of 135 tiles to create the next design.**

Wilma's expression: **Howard's expression:**

Tyler's expression: **Tamara's expression:**

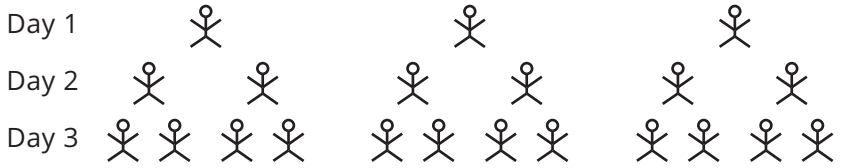
9. **Wilma tells Miriam that since all of the expressions resulted in the same solution, any of the expressions can be used to determine the number of additional tiles needed to make more $n \times n$ designs. Miriam thinks that the employees need to use more values in the expressions than just one to make this conclusion. Who is correct? Explain your reasoning.**

Recall that two or more algebraic expressions are equivalent if they produce the same output for all input values. You can verify that two expressions are equivalent by using properties to rewrite the two expressions as the same expression.

10. **Use algebraic properties to show that Wilma, Howard, Tyler, and Tamara's expressions are equivalent. Justify your reasoning.**

ACTIVITY
2.2 Patterns of Growth

The visual model shown represents the number of new seniors who learn about the homecoming election result each day that passes.

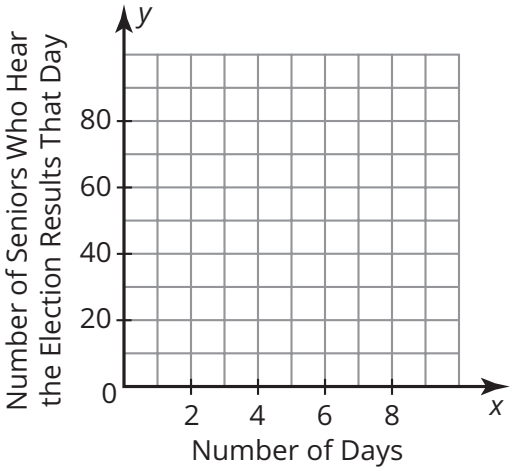


1. Analyze the pattern.

Number of Days That Pass	Number of Seniors Who Hear the Results That Day
1	
2	
3	
4	
5	
6	
n	

a. Complete the table to summarize the number of seniors who learn about the election result each day. Then write an expression to represent the number of seniors who learn about the election result on the *n*th day. Finally, describe how each part of your expression relates to the visual model.

b. Create a graph of the data from your table on the coordinate plane shown. Then draw a smooth curve to model the relationship between the number of days that pass and the number of seniors who hear the senior election results that day.



2. Do all the points on the smooth curve make sense in terms of this problem situation? Why or why not?

3. **Describe this pattern as linear, exponential, or quadratic. Then write the corresponding equation. How does each representation support your answer?**

When you model a relationship on a coordinate plane with a smooth curve, it is up to you to consider the situation and interpret the meaning of the data values shown.

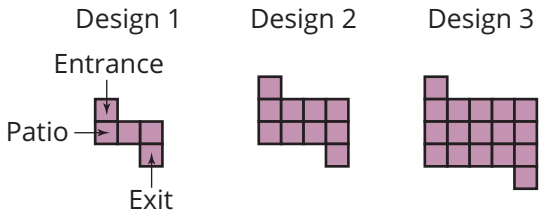
4. **Describe the key characteristics of your graph. Explain each characteristic algebraically and in terms of this problem situation.**

5. **After how many days will 500 new seniors learn about the election results?**

6. **Determine the number of seniors who hear the election results on the 12th day. Does your answer make sense in the context of this problem situation? Explain your reasoning.**

ACTIVITY
2.3
Increasing and Decreasing Patterns

The model shown represents the first three designs Maureen and Matthew could use. Each square represents 1 square foot.



1. **Describe the pattern as linear, exponential, or quadratic. Explain your reasoning.**

Maureen and Matthew each write different expressions to represent the patio designs.

2. **Describe how each term in Maureen's expression represents the visual model.**

Think about:

How can Maureen use subtraction when the number of tiles in each term is increasing?

Maureen

$(n + 2)^2 - 2(n + 1)$

3. **Consider Matthew's expression.**

Matthew
$n^2 + 2n + 2$

   a. **Use technology to verify graphically the equivalence of the two expressions.**

   b. **Identify the parts of the graph that represent this problem situation.**

   c. **Verify the equivalence of the two expressions algebraically.**

4. **To accommodate outdoor furniture, a grill, and a shed, the patio must have an area of at least 125 square feet (not including the walkways). What is the smallest design Matthew can build and still have enough space for these items?**

How is the number of tiles in each design related to the one that came before it?

NOTES

TALK the TALK

You Know the Type

In this activity, you examined three different patterns and wrote a function to describe each.

1. **Describe the similarities and differences among linear, exponential, and quadratic functions.**