Lesson 3

Samesies:

Comparing Multiple Representations of Functions

Lesson Overview

Students compare the graphic, verbal, numeric, and algebraic representations of a function. They group equivalent representations of functions and then identify their function families. Students analyze a tile pattern and use a table to represent the sequence and recognize patterns. They then create expressions that represent different aspects of the design. Within this same context, students show that different expressions are algebraically equivalent.

Lesson Video(s): The aligned lesson overview video(s) provide additional instruction for students on the key concepts in this lesson and can be found alongside the digital interactive student lesson.

TEKS: 2A.8A

Lesson Structure and Pacing: 1 Day

Engage

Getting Started: Odd One Out

Develop

Activity 3.1: Equivalent Representations
Activity 3.2: Equivalent Functions

Demonstrate

Talk the Talk: Equal to the Task

Getting Started: Odd One Out

Asynchronous Facilitation Notes

In this activity, students analyze relationships that are expressed using different representations. They identify a relationship that does not belong with the others and justify their choice in a free response question.

Synchronous Facilitation Notes

In this activity, students analyze relationships that are expressed using different representations. They identify a relationship that does not belong with the others and justify their choice.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Differentiation strategy

To scaffold support, suggest students use multiple representations for each relationship for the purposes of comparison.

Misconception

Students may assume there is only one correct answer to this activity. Look for and encourage a variety of responses.

Questions to ask

- Which relationships are functions?
- Which relationships have a constant first difference?
- Which relationships have a constant second difference?
- Which relationships are linear functions?
- Which relationships increase over the entire domain?
- Which relationships are continuous?
- Which relationships are discrete?
- Which relationships have graphs that are symmetric? Identify the symmetry.

Summary

Relationships between quantities can be represented in graphs, tables, equations, and contexts.

Activity 3.1:
Equivalent Representations

Asynchronous Facilitation Notes

In this activity, students will examine different functions, represented as graphs, tables, equations, or contexts. The students will be given one relation to analyze then analyze and select all equivalent relations given four options in multiple select questions. Students explain the strategies they used to determine equivalence relations within each group and how they decided if a relation is or is not a function in two separate free response questions. They also describe each function family associated with the various groupings in a fill in the blank question and how to determine the function family from a graph, table, context or an equation in a free response question.

Synchronous Facilitation Notes

In this activity, students analyze 24 different functions, represented as graphs, tables, equations, or contexts. They analyze the representations and create groups of equivalent relations. Students explain the strategies they used to choose their groups and how they decided if a relation is or is not a function. They also describe each function family associated with the various groupings.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

As students work, look for

Different strategies used to analyze the representations. The depth of the multiple select questions reveals the level of student understanding. If students create groups that contain only graphs, only equations, only tables, and only contexts, encourage them to use a strategy that incorporates different forms of equivalent representations in each group.

Differentiation strategy

To scaffold support, encourage students to compare representations when they are viewed in a similar form. For example, it may be easier to identify equivalent representations if all the representations are viewed graphically.

Questions to ask

- What does *equivalent relations* mean?
- How can a table of values, an equation, a context, and a graph be considered equivalent?
- What is an example of a relation that is not a function?
- What is the difference between a relation that is a function and a relation that is a not a function?
- How many groups did you use to sort the twenty-four representations?
- Do any of the representations fit into more than one group? Which ones?
- Did you graph all of the representations?
- Did you use the *y*-intercepts to make any decisions?
- Did you use the function families?
- Which function families are represented in this activity?
- What is the Vertical Line Test?
- Do all of the graphs pass the Vertical Line Test?
- Do the tables show only one *y*-value for each *x*-value?
- Do the equations produce only one output value for each input value?
- What is the difference between a linear function and a quadratic function?
- What is the difference between a quadratic function and an exponential function?

Summary

Equivalent relationships may be represented in different forms.

Activity 3.2:
Equivalent Functions

Asynchronous Facilitation Notes

In this activity, students analyze the first designs in a tiling pattern, describe patterns in a free response question, and organize the patterns in a fill in the blank table. Through this organization, other patterns emerge. They then answer several free response questions related to the situation and create algebraic expressions to represent the number of various colors in any given design in a fill in the blank table. Students show that two different expressions are equivalent and conclude that the function is quadratic in several free response questions. Finally, they prove that the number of white tiles plus one equals the number of gray tiles, the number of white tiles is always an even number, and the total number of tiles is always an odd number in additional free response questions.

Synchronous Facilitation Notes

In this activity, students analyze the first designs in a tiling pattern, describe patterns, and organize the patterns in a table. Through this organization, other patterns emerge. They then answer several questions related to the situation and create algebraic expressions to represent the number of various colors in any given design. Students show that two different expressions are equivalent and conclude that the function is quadratic. Finally, they prove that the number of white tiles is always an even number and the total number of tiles is always an odd number.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

As students work, look for

Whether students notice that the design number skips from 4 to 7. They may calculate Design 5 and 6 instead of Design 7 and 10.

Questions to ask

- How many tile colors are involved in this pattern?
- Why is the total number of tiles always a square number?
- How does the number of gray tiles compare to the number of white tiles?
- Is the sum of the white tiles and the gray tiles the same as the total number of tiles?
- Why is the number of white tiles always an even number?
- Why is the number of gray tiles always an odd number?
- Why is the total number of tiles always an odd number?
- How many white rectangles are in each pattern design?
- How many gray squares are in each pattern design?
- Did you need to create a visual model for Design 4?
- How did you determine the number of white tiles in Design 4? The number of gray tiles?
- How did you determine the number of white tiles in Design 7? The number of gray tiles?
- How did you determine the number of white tiles in Design 10? The number of gray tiles?
- How many tiles are used in Design 11?
- Can the hotel afford Design 11 on their budget? Why or why not?

Have students work with a partner or in a group to complete Questions 6 and 7. Share responses as a class.

As students work, look for

Common errors when using the Distributive Property.

Differentiation strategies

- To scaffold support, suggest students use technology to graph the expressions to verify their equivalence.
- To assist all students, rather than just reviewing the Worked Example, construct the argument for the equivalent relationship between $w(n) + g(n)$ and $t(n)$ as a class.

Questions to ask

- How did you determine the expression for the number of white tiles in the pattern? The number of grey tiles?
- How does your expression compare to your classmates' expression?
- How did you determine the expression for the total number of tiles in the pattern?
- What algebraic properties are used to rewrite Tonya's expression?
- How can you rewrite Tonya's expression with fewer terms?
- What algebraic properties are used to rewrite Alex's expression?
- How can you rewrite Alex's expression with fewer terms?
- How can you use a graph to verify the equivalence of the expressions?
- Do both expressions result in the same graph? What does this imply?

Ask a student to read the information that follows Question 7 aloud. Analyze the Worked Example and complete Question 8 as a class.

Have students work with a partner or in a group to complete Questions 9 and 10. Share responses as a class.

Questions to ask

- What expression represents $w(n) + 1$?
- What expression represents $g(n)$?
- Are the white tiles always arranged in rectangles?
- Does multiplying by four always result in an even number?
- Is the product of two odd numbers always an odd number?
- If an even number is represented by $2n$, what expression represents an odd number?
- What is $(2n + 1)(2n + 1)$?
- Why does $(2n + 1)(2n + 1)$ always result in an odd number?

Summary

A function can be structured in more than one way. Two functions are equivalent if their algebraic or graphical representations are the same.

Talk the Talk: Equal to the Task

Asynchronous Facilitation Notes

In this activity, students complete statements about the equivalence of functions in fill in the blank questions. They also explain their reasoning for each answer. Next, in free response questions, they analyze a table of values to determine whether it represents a function and, if so, with which function family it is associated. Finally, students analyze pairs of functions to determine whether they are equivalent in multiple choice questions.

Synchronous Facilitation Notes

In this activity, students complete statements about the equivalence of functions. Next they analyze a table of values to determine whether it represents a function and, if so, with which function family it is associated. Finally, students analyze pairs of functions to determine whether they are equivalent.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

As students work, look for

Different strategies to determine equivalent functions. Do all strategies involve graphing? Do some strategies compare table values? Do some strategies involve algebraic manipulations?

Differentiation strategy

To scaffold support, encourage students to use technology to verify equivalent expressions.

Misconception

Students may confuse the table values of a function and a non-function. If the same *x*-value maps onto multiple *y*-values, the relation described by the table of values is not a function. If multiple *x*-values map onto the same *y*-value, the relation may be a function.

Questions to ask

- How can graphs be used to verify the equivalence of two functions?
- If the equations that describe two functions produce the same graph, are the functions equivalent?
- Can a function map multiple *x*-values onto the same *y*-values?
- Can a function map the same *x*-value onto multiple *y*-values?
- Which function family is associated with a table of values with constant first differences? With constant second differences?

Summary

Two functions are equivalent if their algebraic and graphical representations are the same.