5

The Root of the Problem

Solving Quadratic Equations

Warm Up

Use the Distributive Property to determine each product.

1. $(x + 1)(x + 2)$

2. $(x + 4)(x - 5)$

3. $(2x - 3)(x - 4)$

4. $(x + 2)^2$

Learning Goals

- Factor quadratic trinomials to determine the roots of quadratic equations and to rewrite quadratic functions in forms that reveal different key characteristics.
- Complete the square to determine the roots of quadratic equations of the form $ax^2 + bx + c$.
- Use the Quadratic Formula to determine roots and zeros.

Key Term

- Quadratic Formula

You have analyzed the different structures of quadratic equations. How can the structure of a quadratic equation help you determine a solution strategy? Is there a single strategy that works to solve any quadratic equation?

GETTING STARTED

Grassroots

You know how to use the Properties of Equality to solve equations in the forms shown.

$$y = x^2 + d$$
$$y = (x - c)^2$$
$$y = a(x - c)^2$$
$$y = a(x - c)^2 + d$$

1. Use Properties of Equality to solve each equation. State the property used in each step of your solution.

a. $27 = x^2 - 9$

b. $(x + 3)^2 = 121$

c. $48 = 3(x - 1)^2$

d. $\frac{1}{2}(x + 5)^2 - 18 = 0$

2. Describe the strategy Oscar used to solve part (a) in Question 1.

Oscar

$$27 = x^2 - 9$$
$$27 - 27 = x^2 - 9 - 27$$
$$0 = x^2 - 36$$
$$0 = (x - 6)(x + 6)$$

$(x - 6) = 0$ and $(x + 6) = 0$

$x = 6$ and $x = -6$

ACTIVITY **5.1**

Solving Quadratic Equations by Factoring

Let's consider strategies to solve quadratics in the form $y = ax^2 + bx + c$ using the factoring strategies you have learned.

Worked Example

You can use factoring to calculate the roots for the quadratic equation $x^2 - 4x = -3$.

$$x^2 - 4x = -3$$
$$x^2 - 4x + 3 = -3 + 3$$
$$x^2 - 4x + 3 = 0$$
$$(x - 3)(x - 1) = 0$$

$$(x - 3) = 0 \quad \text{and} \quad (x - 1) = 0$$
$$x - 3 + 3 = 0 + 3 \quad \text{and} \quad x - 1 + 1 = 0 + 1$$
$$x = 3 \quad \text{and} \quad x = 1$$

Remember:

The Zero Product Property states that if the product of two or more factors is equal to zero, then at least one factor must be equal to zero.

1. Why is 3 added to both sides in the first step of the Worked Example?

Think about:

What is the connection between the Worked Example and determining the roots from factored form, $y = a(x - r_1)(x - r_2)$?

2. Determine each student's error and then solve each equation correctly.

Angela

$x^2 + 6x = 7$

$x(x + 6) = 7$

$x = 7$ and $x + 6 = 7$

$x = 1$

Dwight

$x^2 + 5x + 6 = 6$

$(x + 2)(x + 3) = 6$

$x + 2 = 6$ and $x + 3 = 6$

$x = 4$ and $x = 3$

3. Use factoring to solve each quadratic equation, if possible.

a. $x^2 - 8x + 12 = 0$

b. $x^2 + 8x = -7$

c. $\frac{2}{3}x^2 - \frac{5}{6}x = 0$

d. $f(x) = x^2 + 10x + 12$

Think about:

What efficiency strategies did you use to solve linear equations with fractional coefficients?

4. Describe the different strategies and reasoning that Jim and Pam used to solve $4x^2 - 25 = 0$.

Jim

$$
\begin{aligned}
4x^2 - 25 &= 0 \\
4x^2 &= 25 \\
x^2 &= \frac{25}{4} \\
x &= \pm\sqrt{\frac{25}{4}} \\
x &= \pm\frac{5}{2}
\end{aligned}
$$

Pam

$4x^2 - 25 = 0$

$(2x - 5)(2x + 5) = 0$

$2x - 5 = 0$ and $2x + 5 = 0$

$2x = 5 \qquad 2x = -5$

$x = \frac{5}{2}$ and $\quad x = -\frac{5}{2}$

ACTIVITY
5.2

Completing the Square to Determine Roots

You can use the completing the square method to determine the roots of a quadratic equation that cannot be factored.

Worked Example

Complete the square to determine the roots of the equation $x^2 + 10x + 12 = 0$.

Isolate $x^2 + 10x$.

$$x^2 + 10x + 12 - 12 = 0 - 12$$
$$x^2 + 10x = -12$$

Determine the constant term that would complete the square. Add this term to both sides of the equation.

$$x^2 + 10x + \_\_\_\_ = -12 + \_\_\_\_$$
$$x^2 + 10x + 25 = -12 + 25$$
$$x^2 + 10x + 25 = 13$$

Rewrite the left side as a perfect square.

$$(x + 5)^2 = 13$$

Take the square root of each side of the equation.

$$\sqrt{(x + 5)^2} = \pm\sqrt{13}$$
$$x + 5 = \pm\sqrt{13}$$

Set the factor of the perfect square trinomial equal to each square root of the constant. Then solve for x.

$$x + 5 = \sqrt{13} \quad \text{and} \quad x + 5 = -\sqrt{13}$$
$$x = -5 + \sqrt{13} \quad \text{and} \quad x = -5 - \sqrt{13}$$
$$x \approx -1.39 \quad \text{and} \quad x \approx -8.61$$

The roots are approximately -1.39 and -8.61.

Ask yourself:

How was equality of the equation maintained through the completing the square process?

1. Complete the square to determine the roots of each equation.

a. $x^2 - 6x + 4 = 0$

b. $x^2 - 12x + 6 = 0$

2. A ball is thrown straight up from 4 feet above the ground with a velocity of 32 feet per second. The height of the ball over time can be modeled with the function $h(t) = -16t^2 + 32t + 4$. What is the maximum height of the ball?

3. Jessie is fencing in a rectangular plot outside of her back door so that she can let her dogs out to play. She has 60 feet of fencing and only needs to place it on three sides of the rectangular plot because the fourth side will be bound by her house. What dimensions should Jesse use for the plot so that the maximum area is enclosed? What is the maximum area? Draw a diagram to support your work.

ACTIVITY

5.3

Using the Quadratic Formula

The **Quadratic Formula**, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, can be used to calculate the solutions to any quadratic equation of the form $ax^2 + bx + c = 0$, where a, b, and c represent real numbers and $a \neq 0$.

Worked Example

You can use the Quadratic Formula to determine the zeros of the function $f(x) = -4x^2 - 40x - 99$.

Rewrite the function as an equation to be solved for x when $y = 0$.	$-4x^2 - 40x - 99 = 0$
Determine the values of a, b, and c.	$a = -4$, $b = -40$, $c = -99$
Substitute the values into the Quadratic Formula.	$x = \frac{-(-40) \pm \sqrt{(-40)^2 - 4(-4)(-99)}}{2(-4)}$
Perform operations to rewrite the expression.	$x = \frac{40 \pm \sqrt{1600 - 1584}}{-8}$

$$x = \frac{40 \pm \sqrt{16}}{-8}$$

$$x = \frac{40 \pm 4}{-8}$$

$$x = \frac{40 + 4}{-8} \quad \text{and} \quad x = \frac{40 - 4}{-8}$$

$$x = \frac{44}{-8} \quad \text{and} \quad x = \frac{36}{-8}$$

$$x = -5.5 \quad \text{and} \quad x = -4.5$$

The zeros of the function $f(x) = -4x^2 - 40x - 99$ are $x = -5.5$ and $x = -4.5$.

The Seaside Serpents baseball team has a new promotional activity to encourage fans to attend games: launching free T-shirts! They can launch a T-shirt in the air with an initial velocity of 91 feet per second from $5\frac{1}{2}$ feet off the ground (the height of the team mascot).

A T-shirt's height can be modeled with the quadratic function $h(t) = -16t^2 + 91t + 5.5$, where t is the time in seconds and $h(t)$ is the height of the launched T-shirt in feet. They want to know how long it will take for a T-shirt to land back on the ground after being launched (if no fans grab it before then!).

Ask yourself:

What would a sketch showing the height of the T-shirt over time look like?

1. **Why does it make sense to use the Quadratic Formula to solve this problem?**

2. **Use the Quadratic Formula to determine how long it will take for a T-shirt to land back on the ground after being launched.**

Ask yourself:

Do you think an exact solution or approximate solution is more appropriate for this context?

3. **Meredith is solving the quadratic equation $x^2 - 7x - 8 = 3$. Her work is shown.**

   a. **Identify Meredith's error.**

   b. **Determine the solution to Meredith's quadratic equation.**

Meredith

$x^2 - 7x - 8 = 3$

$a = 1,\ b = -7,\ c = -8$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-8)}}{2(1)}$$

$$x = \frac{7 \pm \sqrt{49 + 32}}{2}$$

$$x = \frac{7 \pm \sqrt{81}}{2}$$

$$x = \frac{7 \pm 9}{2}$$

$$x = \frac{7+9}{2} \text{ or } x = \frac{7-9}{2}$$

$$x = \frac{16}{2} = 8 \text{ or } x = \frac{-2}{2} = -1$$

The roots are 8 and -1.

4. **Use the Quadratic Formula to determine the zeros for each function. Round the solutions to the nearest hundredth.**

   a. $f(x) = 2x^2 + 10x - 1.02$

   b. $h(x) = 3x^2 + 11x - 2$

NOTES

TALK the TALK

Show Me the Ways

1. Determine the real roots of the quadratic equation $y = 2x^2 + 4x - 6$ using each method.

Factoring	Completing the Square
Using the Quadratic Formula	**Graphing**

$y = 2x^2 + 4x - 6$