Exploring and Analyzing Patterns

Topic 2 Overview

How is *Exploring and Analyzing Patterns* organized?

Exploring and Analyzing Patterns provides opportunities for students to analyze and describe various patterns. Students are asked to represent algebraic expressions in different forms and use algebra and graphs to determine whether they are equivalent. Lessons provide opportunities for students to review linear, exponential, and quadratic functions using multiple representations.

The first lesson introduces students to three patterns and asks students to analyze and extend each pattern. In the next lesson, students write algebraic expressions to represent each pattern. They use equivalent forms of the expressions, graphs, and tables to explore characteristics of each pattern, solve for unknown terms, and describe each as linear, exponential, or quadratic.

Students then consider multiple equivalent representations of functions. They sort tables, graphs, scenarios, and equations into groups of equivalent relationships and then identify the function family associated with each. Students consider another tiling pattern and show how to prove algebraically and graphically that two functions are equivalent.

The three forms of a quadratic equation are reviewed, and students match nine equations with their corresponding graphs and sort the equations according to form. They consider how to determine the key characteristics of the function given the form of the equation. Students then learn to write quadratic equations given key points before using systems to write a quadratic equation given any three points.

Finally, students recall how to solve quadratic equations using Properties of Equality and square roots, factoring and the Zero Product Property, completing the square, and the Quadratic Formula. They then consider quadratic functions with no real roots. Introduced to the complex number system, students next solve quadratic functions with imaginary roots.

What is the entry point for students?

In previous courses, students gained extensive experience with multiple representations of linear, exponential, and quadratic functions. This topic serves as an opportunity for students to recall what they already know and prepare for how they will use this knowledge throughout the rest of the course. They are familiar with solving quadratic equations and have considered the case in which the graph of a quadratic equation does not cross the *x*-axis, yielding no real roots. When learning to solve quadratic equations with imaginary roots, students build upon these experiences and expand their knowledge of the real number system to the complex number system.

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in *Exploring and Analyzing Patterns* if they can:

- Interpret the real-world meaning of the terms, factors, and coefficients of an expression.
- Identify variables and quantities represented in a real-world situation.
- Write an expression to model a real-world situation.
- Create a graph, equation, table, or verbal description that indicates the key characteristics of the function.
- Use properties to rewrite expressions in different but equivalent forms to better interpret their meaning.
- Explain why equivalent expressions are equivalent.
- Explain that there are three forms of quadratic equations: standard, vertex, and factored forms.
- Identify the form in which a given quadratic function is written.
- Convert between the three forms of a quadratic function.
- Identify the key characteristics that are easily identifiable in each of the three forms.
- Factor, multiply polynomials, or complete the square to rewrite expressions.
- Determine whether given inputs and/or outputs make sense in a problem situation.
- Compare the properties of two functions represented differently.
- Write an explicit expression for a pattern that grows linearly, exponentially, or quadratically.
- Rewrite a quadratic expression using factoring or completing the square.
- Solve quadratic equations using the Properties of Equality and square roots, factoring, completing the square, and the Quadratic Formula.
- Explain that complex solutions for a quadratic equation result when the discriminant is negative in the Quadratic Formula or when the graph of the equation does not intersect the *x*-axis.
- Use substitution to solve a system of equations comprised of one linear and one quadratic function.
- Use graphing to determine the approximate solution of a system of equations comprised of one linear and one quadratic function.
- Understand that $i = \sqrt{-1}$.
- Identify a complex number in the form $a + bi$.
- Use properties to add, subtract, and multiply complex numbers.
- Solve quadratic equations with real number coefficients and imaginary solutions.
- Factor a quadratic expression using complex numbers.

Why is *Exploring and Analyzing Patterns* important?

Throughout the rest of this course, students will be exploring more complex functions: polynomials, rational functions, radical functions, and logarithmic functions. *Exploring and Analyzing Patterns* is a critical topic for

preparing students for this work, which includes identifying key characteristics, determining whether given representations are equivalent, connecting representations, writing expressions, using expressions and equations to solve for unknown values, analyzing graphical representations, and making connections to the real world.

How do the activities in *Exploring and Analyzing Patterns* promote student expertise in the mathematical process standards?

All Carnegie Learning topics are written with the goal of creating mathematical thinkers who are active participants in class discourse, so the mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work toward solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

Students model with mathematics throughout this topic, creating physical models of real-world problems, representing them with tables, graphs, and equations, and drawing conclusions based on their analysis. They must decontextualize given situations to represent them symbolically and then identify important quantities that make sense in context. While determining solutions graphically often requires approximations, students must attend to the precision of their graphs to answer questions and build new functions. Finally, students use graphical representations to look for and make use of the structure of polynomial functions.

Students use reasoning to make sense of quantities and their relationships in real-world situations. They express these relationships using graphs, tables, and expressions, creating coherent and equivalent representations. As they analyze tiling patterns, students notice repeated calculations and use the structure of the pattern to model it with an expression. They continue to make use of the structure of quadratics when they rewrite them in different forms and identify the key characteristics that are indicated in each form. Finally, when considering representations of functions and solution strategies, students justify their conclusions, communicate their reasoning, and analyze the arguments of others.

Materials Needed

Graphing technology
Tape or glue
Scissors

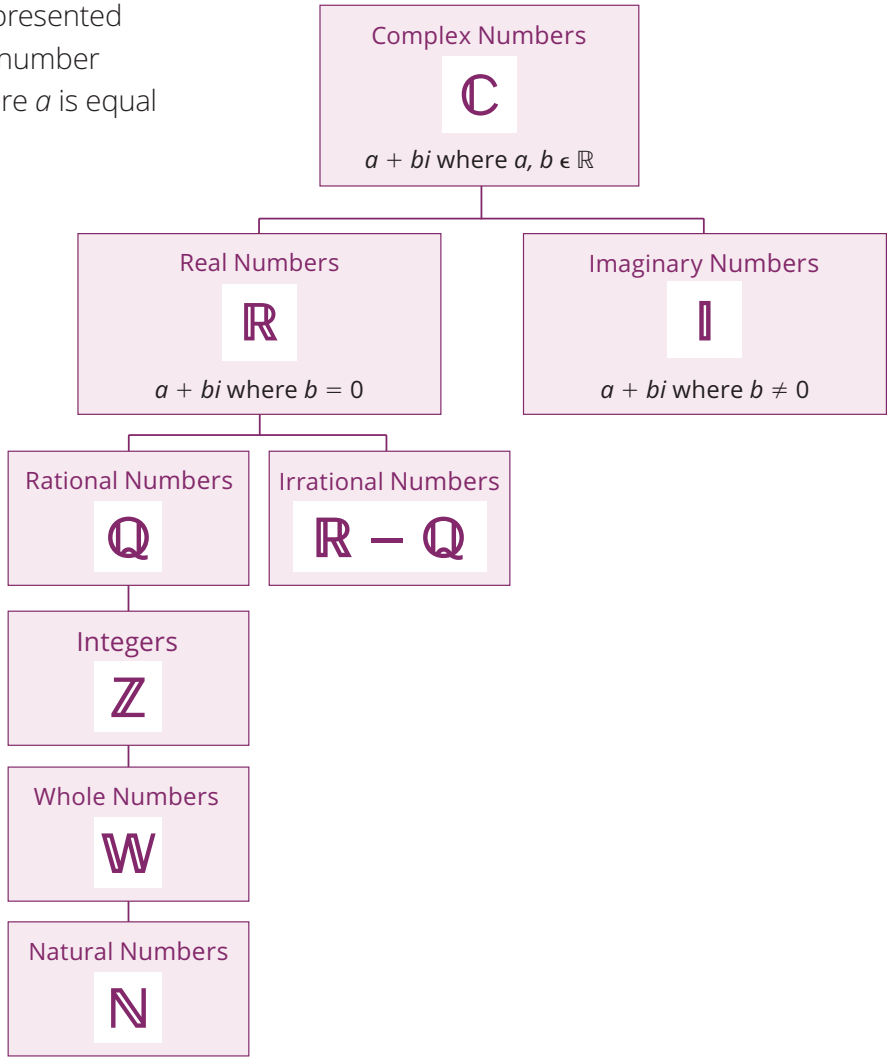
New Tools and Notation

The Complex Number System

The set of complex numbers is the set of all numbers written in the form $a + bi$, where a and b are real numbers. The term a is the real part of a complex number, and the term bi is the imaginary part of a complex number. The set of complex numbers is represented by the notation $\mathbb{C}$.

The set of imaginary numbers is the set of all numbers written in the form $a + bi$, where a and b are real numbers and b is not equal to 0.

The set of imaginary numbers is represented by the notation $\mathbb{I}$. A pure imaginary number is a number of the form $a + bi$, where a is equal to 0 and b is not equal to 0.



Learning Together

ELPS: 1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G

Lesson	Lesson Name	TEKS	Days	Highlights
1	Patterns: They're Grrrrrowing!: Observing Patterns	2A.8C	1	Students investigate three scenarios that illustrate sequences. They analyze diagrams representing each sequence, describe observable patterns, sketch other terms or designs in the sequence, and then answer questions relevant to the problem situation. Tables and graphs are used to organize data and help recognize patterns as they emerge.
2	The Cat's Out of the Bag!: Generating Algebraic Expressions	2A.5B 2A.8A 2A.8C	2	This lesson revisits the three scenarios from the previous lesson. Students write equivalent algebraic expressions for each of the scenarios. They use algebraic properties and graphical representations to show that the expressions are equivalent. Students describe the similarities and differences among linear, exponential, and quadratic functions.
3	Samesies: Comparing Multiple Representations of Functions	2A.8A	2	Students compare the graphic, verbal, numeric, and algebraic representations of a function. They group equivalent representations of functions and then identify their function families. Students analyze a tile pattern and use a table to represent the sequence and recognize patterns. They then create expressions that represent different aspects of the design. Within this same context, students show that different expressions are algebraically equivalent.

Lesson	Lesson Name	TEKS	Days	Highlights
4	True to Form: Forms of Quadratic Functions	2A.3A 2A.3B 2A.4A 2A.4D 2A.7B	2	Students match quadratic equations with their graphs using key characteristics. The standard form, the factored form, and the vertex form of a quadratic equation are reviewed as is the concavity of a parabola. Students then sort each of the equations with their graphs depending on the form in which the equation is written, while identifying key characteristics of each function such as the axis of symmetry, the *x*-intercept(s), concavity, the vertex, and the *y*-intercept. Next, students analyze graphs of parabolas in relation to a pair of numberless axes and select possible functions that could model the graph. A Worked Example shows that a unique quadratic function is determined when the vertex and a point on the parabola are known, or the roots and a point on the parabola are known. Students are given information about a function and use it to determine the most efficient form (standard, factored, vertex) to write the function. They then use the key characteristics of a graph and reference points to write a quadratic function, if possible. Finally, students analyze a Worked Example that demonstrates how to write and solve a system of equations to determine the unique quadratic function given three points on the graph. They then use this method to determine the quadratic function that models a problem situation and use it to answer a question about the situation.
5	The Root of the Problem: Solving Quadratic Equations	2A.4F	2	Students solve quadratic equations of the form $y = ax^2 + bx + c$. They first factor trinomials and use the Zero Product Property. Students then use the method of completing the square to determine the roots of a quadratic equation that cannot be factored. Finally, students use the Quadratic Formula to solve problems in real-world and mathematical problems.

Lesson	Lesson Name	TEKS	Days	Highlights
6	*i* Want to Believe: Imaginary and Complex Numbers	2A.4F 2A.7A	2	Students are introduced to imaginary numbers to calculate the square root of a negative number, and imaginary numbers are placed within the complex number system. They apply the Commutative Property, the Associative Property, and the Distributive Property to add, subtract, and multiply complex numbers. Students use the structure of quadratic equations in the form $y = ax^2 + c$, vertex form, and in standard form, as well as the discriminant and the graph to determine whether the roots of an equation are real or imaginary. They solve quadratic equations that have imaginary roots. They apply the Fundamental Theorem of Algebra to make sense of the fact that a quadratic equation can have two unique real number solutions, two equal real number solutions, or two imaginary solutions.

Suggested Topic Plan

*1 Day Pacing = 45 min. Session

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: 2A.8C **LESSON 1** **Patterns: They're Grrrrowing!** GETTING STARTED ACTIVITY 1 ACTIVITY 2 ACTIVITY 3 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress	TEKS: 2A.5B, 2A.8A, 2A.8C **LESSON 2** GETTING STARTED ACTIVITY 1	**LESSON 2** continued ACTIVITY 2 ACTIVITY 3 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
TEKS: 2A.8A **LESSON 3** **Samesies** GETTING STARTED ACTIVITY 1	**LESSON 3** continued **Samesies** ACTIVITY 2 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress	**MID-TOPIC ASSESSMENT**	TEKS: 2A.3A, 2A.3B, 2A.4A, 2A.4D, 2A.7B **LESSON 4** **True to Form** GETTING STARTED ACTIVITY 1 ACTIVITY 2
Day 11	**Day 12**	**Day 13**	**Day 14**	**Day 15**
LESSON 4 continued ACTIVITY 3 ACTIVITY 4 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress	TEKS: 2A.4F **LESSON 5** **The Root of the Problem** GETTING STARTED ACTIVITY 1 ACTIVITY 2	**LESSON 5** continued ACTIVITY 3 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress
Day 16	**Day 17**	**Day 18**	**Day 19**	
TEKS: 2A.4F, 2A.7A **LESSON 6** ***i* want to Believe** GETTING STARTED ACTIVITY 1 ACTIVITY 2 ACTIVITY 3	**LESSON 6** continued ACTIVITY 4 ACTIVITY 5 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress	**END OF TOPIC ASSESSMENT**	

Assessments

There are two assessments aligned to this topic: Mid-Topic Assessment and End of Topic Assessment.