1

Ahead of the Curve

Solving Quadratic Inequalities

Warm Up

Determine the solution of each quadratic equation.

1. $x^2 - 100 = -64$

2. $x^2 + 3x + 5 = 15$

3. $4x^2 + 12x = 7$

4. $x^2 + 4x - 3 = 5$

Learning Goals

- Solve a quadratic inequality by calculating the roots of the quadratic equation which corresponds to the inequality and testing values within intervals determined by the roots.
- Connect the graphical representation of a quadratic function and the solution to a corresponding quadratic inequality represented on a number line.
- Use interval notation to record the solutions to quadratic inequalities.

You have interpreted the solution sets to linear inequalities on a coordinate plane. You have also solved quadratic equations using a variety of methods. How can you interpret the solutions sets to quadratic inequalities on a coordinate plane using what you know about solving quadratic equations?

GETTING STARTED

It Has Its Ups and Downs

Remember:

A vertical motion model is a quadratic equation of the form $y = -16t^2 + v_0 t + h_0$.

A firework is shot straight up into the air with an initial velocity of 500 feet per second from 5 feet off the ground. The graph of the function that represents this situation is shown.

1. Use the graph to approximate when the firework will be at each given height off the ground.

a. 0 feet

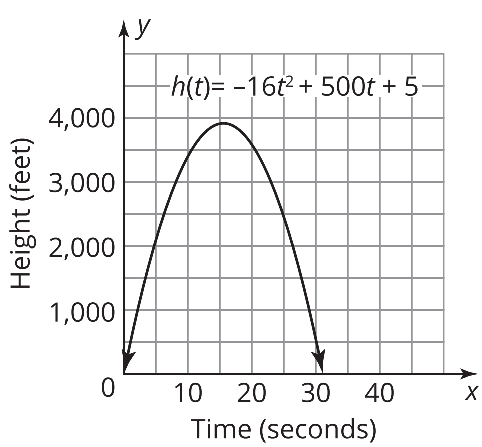
b. 1000 feet

c. 2500 feet

d. 3900 feet

2. Describe any patterns you notice for the number of times the firework reaches a given height.

3. Draw a horizontal line on the graph to represent when the firework is 2000 feet off the ground.



a. When is the firework higher than 2000 feet? Circle this portion of the graph.

b. When is the firework below 2000 feet? Draw a box around this portion of the graph.

c. Write a quadratic inequality that represents the times when the firework is below 2000 feet.

ACTIVITY
1.1 Solving Quadratic Inequalities

Just like with the other inequalities you have studied, the solution to a quadratic inequality is the set of values that satisfy the inequality.

Worked Example

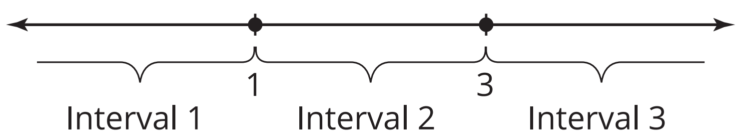
Let's determine the solution of the quadratic inequality $x^2 - 4x + 3 < 0$.

Write the corresponding quadratic equation. $\quad x^2 - 4x + 3 = 0$

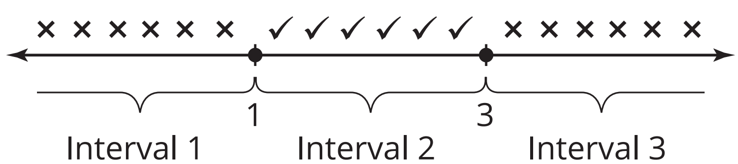
Calculate the roots of the quadratic equation using an appropriate method.

$$(x - 3)(x - 1) = 0$$
$$(x - 3) = 0 \quad \text{or} \quad (x - 1) = 0$$
$$x = 3 \quad \text{or} \quad x = 1$$

Plot the roots to divide the number line into three regions.



Choose a value from each interval to test in the original inequality.



Interval 1	Interval 2	Interval 3
Try $x = 0$	Try $x = 2$	Try $x = 4$
$0^2 - 4(0) + 3 < 0$	$2^2 - 4(2) + 3 < 0$	$4^2 - 4(4) + 3 < 0$
$3 < 0$ ✗	$4 - 8 + 3 < 0$	$16 - 16 + 3 < 0$
	$-1 < 0$ ✓	$3 < 0$ ✗

Identify the solution set as the interval(s) in which your test value satisfies the inequality.

Interval 2 satisfies the original inequality, so the solution includes all numbers between 1 and 3.

Solution: $x \in (1, 3)$

The symbol $\in$ is read "is an element of," "is in," or "belongs to." The notation $x \in (1, 3)$ has the same meaning as $1 < x < 3$.

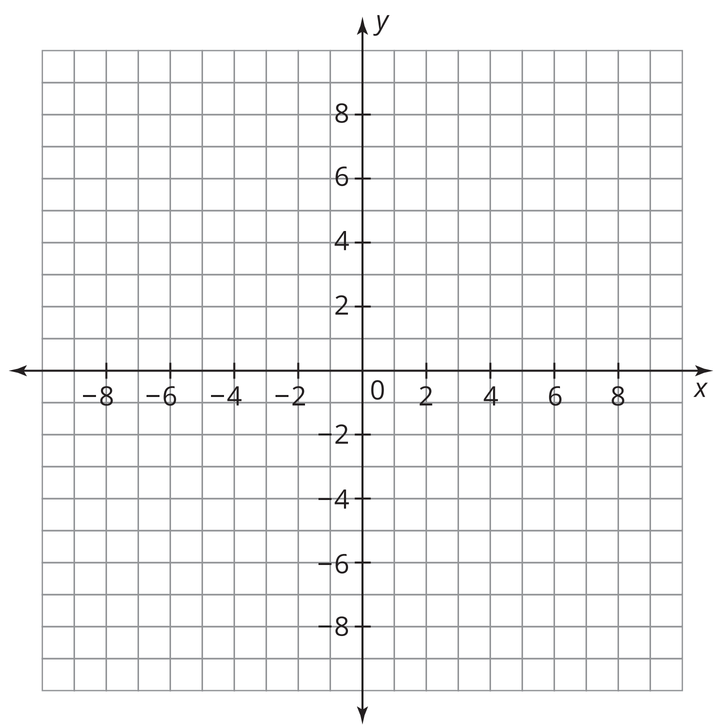
1. Analyze the Worked Example.

The notation $x \in [1, 3]$ means the same as $1 \leq x \leq 3$.

a. How would the solution set change if the inequality was less than or equal to? Explain your reasoning.

b. How would the solution set change if the inequality was greater than or equal to? Explain your reasoning.

2. Graph $y = x^2 - 4x + 3$ on the coordinate plane shown and label the roots of the equation and the vertex. Then describe how the graph supports that the solution set for the associated quadratic inequality $x^2 - 4x + 3 < 0$ is $1 < x < 3$.



Jeff correctly determined the roots of the quadratic inequality $2x^2 - 14x + 27 \geq 7$ to be $x = 5$ and $x = 2$. However, he incorrectly determined the solution set. His work is shown.

Jeff

$$2x^2 - 14x + 27 = 7$$
$$2x^2 - 14x + 20 = 0$$
$$2(x^2 - 7x + 10) = 0$$
$$2(x - 5)(x - 2) = 0$$
$$x = 5 \quad \text{or} \quad x = 2$$

$x = 1$	$x = 3$	$x = 6$
$2(1)^2 - 14(1) + 27 \geq 7$	$2(3)^2 - 14(3) + 27 \geq 7$	$2(6)^2 - 14(6) + 27 \geq 7$
$2 - 14 + 27 \geq 7$	$18 - 42 + 27 \geq 7$	$72 - 84 + 27 \geq 7$
$15 \geq 7$ ✓	$3 \geq 7$ ✗	$15 \geq 7$ ✓

Solution: $x \in (-\infty, 1]$ or $x \in [6, \infty)$

3. Describe Jeff's error. Then, determine the correct solution set for the inequality.

Ask yourself:

When testing values from each interval, could you use the factored form of the inequality rather than the original inequality?

ACTIVITY

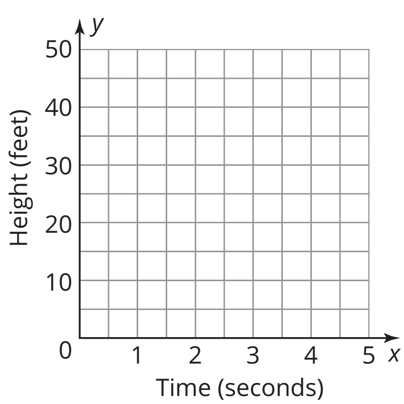
1.2

Modeling Quadratic Inequalities

A water balloon is launched from a machine upward from a height of 10 feet with an initial velocity of 46 feet per second.

1. Identify the variables and write a quadratic function to represent this situation.

2. Use technology to sketch the graph of the function.



3. Draw a horizontal line on the graph to represent when the balloon is 30 feet off the ground.

a. Circle the portion of the graph that represents when the balloon is above 30 feet.

b. Write and solve an inequality to determine when the balloon is above 30 feet. Use the graph to explain your solution.

4. Determine when the balloon is at or below 43 feet. Interpret your solution in terms of the model you graphed.

NOTES

TALK the TALK

Boom! Boom!

In the Getting Started activity, a firework was shot straight up into the air with an initial velocity of 500 feet per second from 5 feet off the ground. The function representing the situation was identified as $h(t) = -16t^2 + 500t + 5$. You determined the firework would be above 2000 feet between about 5 seconds and 27 seconds.

Suppose a second firework was shot straight up into the air with an initial velocity of 500 feet per second from the ground.

1. **Predict whether the second firework will be above 2000 feet for more time, less time, or the same amount of time as the first firework.**

2. **Write a quadratic inequality to represent when the second firework will be above 2000 feet.**

3. **Determine when the second firework will be above 2000 feet.**

NOTES

4. **Was your prediction made in Question 1 correct?**

5. **Use technology to compare the graph of the first firework to the graph of the second firework. What do you notice?**