2

All Systems Go!

Systems of Quadratic Equations

MATERIALS

Graphing technology

Lesson Overview

Students solve a problem in context that can be modeled by a system of equations involving a linear equation and a quadratic equation. They solve this first question graphically and discuss the number of solutions to the system and the number of solutions that make sense for the context. Students are then guided to solve a system of a linear equation and a quadratic equation algebraically, and then verify their results graphically. Students solve additional systems algebraically and graphically. They also discuss the number of possible solutions for each type of system and sketch graphs demonstrating those solutions.

Algebra 2
Systems of Equations and Inequalities

(3) The student applies mathematical processes to formulate systems of equations and inequalities, use a variety of methods to solve, and analyze reasonableness of solutions. The student is expected to:

(A) formulate systems of equations, including systems consisting of three linear equations in three variables and systems consisting of two equations, the first linear and the second quadratic.

(C) solve, algebraically, systems of two equations in two variables consisting of a linear equation and a quadratic equation.

(D) determine the reasonableness of solutions to systems of a linear equation and a quadratic equation in two variables.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G

Essential Ideas

- Systems of equations involving a linear equation and a quadratic equation can be solved both algebraically and graphically.
- A system of equations containing a linear equation and a quadratic equation may have no solution, one solution, or two solutions.

- The number of solutions for a system of equations depends on the number of points where the graphs of the two equations intersect.
- A system of equations involving a linear equation and a quadratic equation may be used to model real-world problems.

Lesson Structure and Pacing: 1 Day

Engage

Getting Started: Block That Kick!

Students are presented with a scenario that can be modeled using a system of equations consisting of a quadratic equation and a linear equation. They describe the shape and characteristics of each equation and reason about whether the two graphs intersect.

Develop

Activity 2.1: Modeling a System with a Linear and a Quadratic Equation

Students use the scenario from the Getting Started to sketch the graph of the system of equations. They identify the number of solutions and determine whether all solutions are reasonable in context of the problem situation. Students use the graph to approximate the solution to the problem.

Activity 2.2: Solving a System with a Linear and a Quadratic Equation

Students solve systems of equations consisting of a linear equation and a quadratic equation by setting the equations equal to one another, and then solving the resulting quadratic equation by factoring or using the Quadratic Formula. They verify the results graphically by identifying the coordinates of the points of intersection of the two graphs. Students sketch graphs to demonstrate that a system of equations consisting of a linear equation and a quadratic equation may have 0, 1, or 2 solutions. They then solve additional systems algebraically and verify their results graphically.

Demonstrate

Talk the Talk: System Solutions

Students determine the number of possible solutions for a system of two linear equations and a system of a linear equation and a quadratic equation. They then explain why a system consisting of a linear equation and a quadratic equation cannot have an infinite number of solutions.

ENGAGE

Getting Started: Block That Kick!

Facilitation Notes

In this activity, students are presented with a scenario that can be modeled using a system of equations consisting of a quadratic equation and a linear equation. They describe the shape and characteristics of each equation and reason about whether the two graphs intersect. The intention of this activity is for students to make sense of the context and predict the outcome. Students will use technology in the next activity to answer the question.

Ask a student to read the problem aloud. Discuss as a class.

Differentiation strategy

To assist all students not familiar with the game of football, have a student explain when punting occurs in a football game and why the blocker can knock down the football only as it travels upward.

Questions to ask

- What do the variables represent in this problem?
- What does the value 0.75 represent? Why does this value make sense in this context?
- What does the value 3 represent? Why does this value make sense in this context?
- Why would the height of the blocker's hands be decreasing?

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

As students work, look for

- Recognition that this is a simplified version of the context, dealing only with the height of the football and the height of the blocker's hands. The horizontal force from the kick is not being considered. The only concern is whether it is possible that the blocker's hands and the football can be at the same height at the same time. The assumption is that the ball and blocker can both be at the same location on the field at the same time, so if both the ball and blocker's hands are at the same height, then a block will occur.
- Attempts to use technology to graph the functions. Predictions should be made without the use of technology.

Questions to ask

- Does the parabola open upward or downward? How do you know?
- Does the parabola have a maximum or a minimum? How do you know?
- What point(s) on the parabola can you identify by viewing its equation?

- What other points on the parabola would be helpful to know in order to sketch its graph?
- How can you tell whether the graph of the line is increasing or decreasing?
- What point(s) on the line can you identify by viewing its equation?
- What other points on the line would be helpful to know in order to sketch its graph?
- Did anyone sketch a graph that looks different than this one?
- Why is there more than one way to sketch the graphs of the two functions with your limited information?
- What points should be the same in everyone's graphs?
- Do the parabola and line intersect in your sketch?
- Is it possible that the parabola and line do not intersect? What would a sketch of that case look like?
- What does the intersection of the parabola and the line represent in this problem situation?
- Can the ball be blocked only if the blocker's hand is the same height as the height of the football?
- Can you think of a case where the heights would be the same, but the ball is not blocked?

Summary

A real-world problem may be modeled by a system consisting of a linear equation and a quadratic equation.

Activity 2.1

Modeling a System with a Linear and a Quadratic Equation

DEVELOP

Facilitation Notes

In this activity, students use the scenario from the Getting Started to sketch the graph of the system of equations. They identify the number of solutions and determine whether all solutions are reasonable in context of the problem situation. Students use the graph to approximate the solution to the problem.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

Differentiation strategies

To scaffold support for students when transferring the graph from their graphing technology to the graph in the activity, suggest that they:

- Use the scale from the graph in the activity as the viewing window on their graphing technology.

- Label key characteristics, such as the intercepts and maximum.
- Refer to the table of values to select specific points.

Misconception

Students may incorrectly view the graph as the football field, with the parabola being the path of the football and the line being the running path of the player. Remind students to refer to the axes labels to interpret the graph.

Questions to ask

- How does the graph you predicted in the Getting Started compare to the graph you created with technology?
- Use the key characteristics to describe how the height of the football changes.
- Use the key characteristics to describe how the height of the blocker's hands changes.
- Where on the graph are the solutions to the system?
- How many times does the line intersect the parabola?
- Do you think a parabola and line will always intersect in two places? Explain.
- Why is the first point of intersection rather the second point of intersection important in this context?
- Using the graph, how would you describe what is happening in the football game at the first intersection point?
- Does it make sense to block the ball during the increasing interval of the parabola? Why?
- Using the graph, how would you describe what is happening in the football game at the second intersection point?
- Does it make sense to block the ball during the decreasing interval of the parabola? Why not?

Summary

The intersections of the graphs of a quadratic equation and a linear equation may have meaning in a real-world situation.

Activity 2.2
Solving a System with a Linear and a Quadratic Equation

Facilitation Notes

In this activity, students solve systems of equations consisting of a linear equation and a quadratic equation by setting the equations equal to one another, and then solving the resulting quadratic equation by factoring or using the Quadratic Formula. They verify the results graphically by identifying the coordinates of the point(s) of intersection of the two graphs. Students sketch graphs to demonstrate that a system of equations consisting of a linear equation and a quadratic equation may have 0, 1, or 2 solutions. They then solve additional systems algebraically and verify their results graphically.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Differentiation strategies

To scaffold support,

- Recommend that students enter each equation into their calculator and then use the table feature to determine the *y*-values that correspond to the *x*-values that are solutions. Along with determining the coordinate pairs that are solutions, the visual cue of seeing that a given *x*-value has the same *y*-value for both functions helps make sense of the meaning of solving a system of equations.
- Suggest that students use the solution to the system of equations as a clue of how to scale the axes.

As students work, look for

- Different sketches that demonstrate the ways in which two graphs may intersect and have the same number of solutions. When sharing responses, use student examples showing parabolas that open upward and open downward, and lines that are increasing, decreasing, and horizontal.
- Sketches of a line and parabola that appear to have no points of intersection; however, when the graphs are extended, they will intersect.

Misconception

Students may get confused when speaking about a quadratic equation, whether you are referring to the quadratic equation that was part of the system, or the quadratic equation that is the result of setting the linear and quadratic equation equal to one another. For this reason, you may want to refer to the former as the *original* quadratic equation and the latter as the *resulting* quadratic equation.

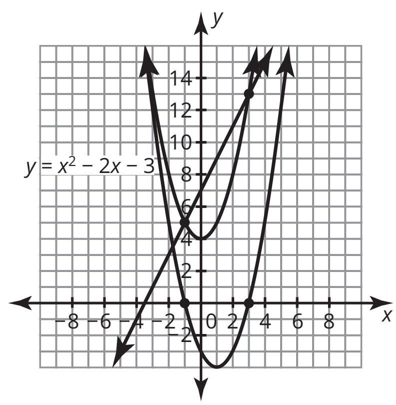
Questions to ask

- What does it mean to solve a system of equations?
- Why can you set the equations equal to each other?
- How is this process similar to solving a system of linear equations?
- If the linear equation is set equal to the quadratic equation, is the resulting equation always quadratic? Why?
- Why is it important to have all of the terms on one side of the equation and 0 on the other side of the equation?

- Did you solve the resulting quadratic equation by factoring or using the Quadratic Formula? Why did you choose that method?
- Does it matter what equation you use to solve for the values of y?
- What is the meaning of the solutions (–1, 5) and (3, 13)?
- How is the number of possible solutions to a system involving a linear equation and a quadratic equation related to what the discriminant tells you about the number of possible solutions when solving a quadratic equation?

Differentiation strategy

To extend the activity, after students graph each equation of the system and calculate the points of intersection, have them graph the resulting parabola, $y = x^2 - 2x - 3$, on the same coordinate plane. Discuss why the x-coordinates of the zeros of the resulting parabola are the same as the x-coordinates of the points of intersection of the original line and parabola.



Have students work with a partner or in a group to complete Question 3. Share responses as a class.

Questions to ask

- Did you solve the resulting quadratic equation by factoring or using the Quadratic Formula? Why did you choose that method?
- If the discriminant equals zero, how many unique real roots will the original quadratic equation have?
- If the discriminant of the resulting quadratic equation equals zero, how many solutions will the system have? What does this indicate about the relationship between the line and the parabola?
- How did you determine the y-value for your solution?
- If the discriminant equals a negative number, how many unique real roots will the original quadratic equation have?
- If the discriminant of the resulting quadratic equation equals a negative number, how many solutions will the system have? What does this indicate about the relationship between the line and the parabola?

Summary

A system of equations consisting of a linear equation and a quadratic equation can be solved algebraically or graphically. The system may have zero, one, or two solutions.

Talk the Talk: System Solutions

DEMONSTRATE

Facilitation Notes

In this activity, students determine the number of possible solutions for a system of two linear equations, and a system of a linear equation and a quadratic equation. They then explain why a system consisting of a linear equation and a quadratic equation cannot have an infinite number of solutions.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Differentiation strategies

- To assist all students,
  - Suggest that they sketch diagrams to make sense of their responses.
  - Recommend the use of double arrows on graphs that demonstrate an infinite number of solutions so that it is evident that there is a graph to represent each of the two equations.
- To extend the activity, have students make posters for reference in response to Questions 1 through 3.

Questions to ask

- What is an example of a system of equations consisting of two linear equations that has no solution? One solution? An infinite number of solutions?
- What characteristic in a system of two linear equations indicates it has no solutions? One solution? An infinite number of solutions?
- What would the graph of a system with this number of solutions look like?

Summary

The number of possible solutions for a system of equations depends on the number of points where the graphs of the two equations can intersect.

NOTES

2

All Systems Go!

Systems of Quadratic Equations

Warm Up

Solve each system of equations.

1. $\begin{cases} y = 2x - 5 \\ y = x - 1 \end{cases}$

2. $\begin{cases} y = -3x + 2 \\ y = 5x - 6 \end{cases}$

3. $\begin{cases} y = -2x + 7 \\ y = -4x + 3 \end{cases}$

4. $\begin{cases} y = 3x + 7 \\ y = x + 1 \end{cases}$

Learning goal

- Solve systems of a linear equation and a quadratic equation.

You have solved systems of linear equations graphically by determining the point of intersection and algebraically using substitution. How can you use these same methods to solve systems involving a linear and a quadratic equation?

Warm Up Answers

1. (4, 3)
2. (1, −1)
3. (−2, 11)
4. (−3, −2)

Answers

1. The graph of the height of the football over time is a parabola that opens downward. The graph of the blocker's hands over time is a decreasing line.
2. Sample answers. Yes. The height of the football is increasing, and the height of the blocker's hands are decreasing. No. If the height of the blockers hands is greater than the maximum height of the football, they might not intersect.

GETTING STARTED

Block That Kick!

A punter kicks a football. The height of the football, in meters, is modeled by the function $h(t) = -4.9t^2 + 20t + 0.75$, where t represents time, in seconds. A blocker can only attempt to knock down the football as it travels upward from the punter's foot. The height in meters of the approaching blocker's hands is modeled by the function $h(t) = -0.6t + 3$, where t represents the same time. Can the blocker knock down the football?

1. Describe the shape of the functions that model the football's height over time and the height of the blocker's hands over time.

2. Sketch a graph of the situation. Do you think it is possible for the blocker to knock down the football? Explain your reasoning.

ELL Tip

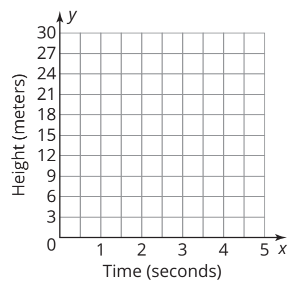
Two non-mathematical terms that appear in this section are *punter* and *blocker*. Determine students' familiarity with football, and define the root words *punt* and *block*. In the context of football, *punt* means *to drop a ball and kick it with one's foot before it hits the ground*. To *block* means *to prevent someone or something from doing something or going somewhere*. Then, discuss the terms *punter* and *blocker* as opponents in the given context.

ACTIVITY **2.1**

Modeling a System with a Linear and a Quadratic Equation

A system of equations can involve nonlinear equations, such as quadratic equations. The scenario described in the previous activity models the relationship between a quadratic and a linear equation.

1. Use technology to sketch the graph of the system described in the previous activity.



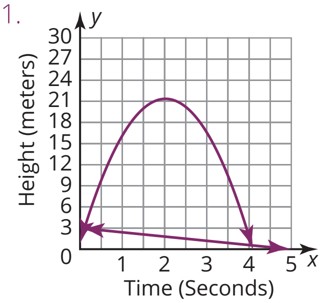
2. How many solutions does the system have? Explain your reasoning.

3. Does every solution make sense in the context of the problem situation? Explain your reasoning.

4. Use the graph to approximate at what point the blocker can block the football. Interpret your solution in the context of the problem.

Answers

1.



2. The system has two solutions. I can see from the sketch that the graphs of the equations intersect each other at exactly two points.
3. No. Only the solution represented by the point of intersection as the parabola increases makes sense because the blocker must block the football as it travels upward.
4. (0.1, 2.9); This means that the blocker can knock the football down about 0.1 second after it has been kicked at a height of 2.9 meters.

Answers

1a. $2x + 7 = x^2 + 4$

1b. $x = 3$ and $x = -1$

1c. When $x = 3$, $y = 13$; when $x = -1$, $y = 5$.

1d. The system has two solutions: (3, 13) and (−1, 5).

ACTIVITY **2.2**

Solving a System With a Linear and a Quadratic Equation

Methods for solving a system of nonlinear equations can be similar to methods for solving a system of linear equations.

1. Consider the system of a linear equation and a quadratic equation shown.

$$\begin{cases} y = 2x + 7 \\ y = x^2 + 4 \end{cases}$$

Ask yourself:

Since *y* is equal to two different expressions, can you set the expressions equal to each other?

a. Write a new equation you can use to solve this system.

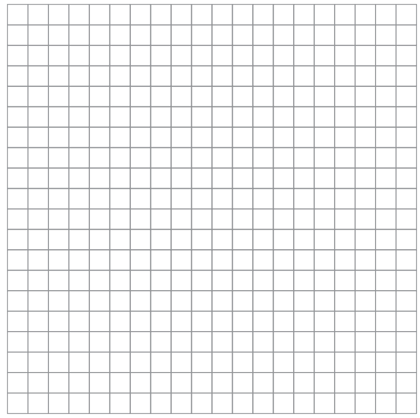
b. Solve the resulting equation for *x*.

c. Calculate the corresponding values for *y*.

d. Identify the solution(s) to the system of equations.

4 • TOPIC 3: Applications of Quadratics

e. **Graph each equation of the system and calculate the points of intersection.**

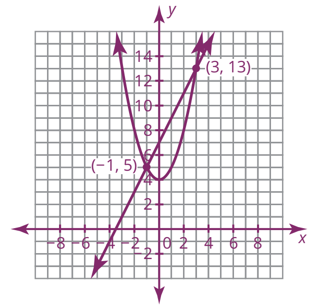


f. **What do you notice about the solutions you calculated algebraically and graphically?**

2. **Think about the graphs of a linear equation and a quadratic equation. Describe the different ways in which the two graphs can intersect and provide a sketch of each case.**

Answers

1e.

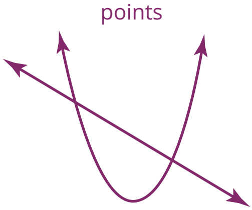


1f. The solutions are the same.

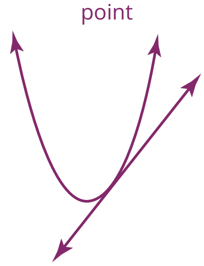
2. The graphs of a linear equation and quadratic equation can intersect at two points, at one point, or not at all. See sketches below.

2. Sample answers.

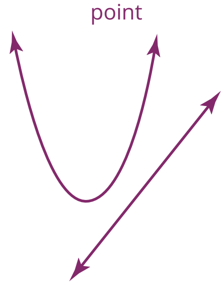
Two intersection points



One intersection point

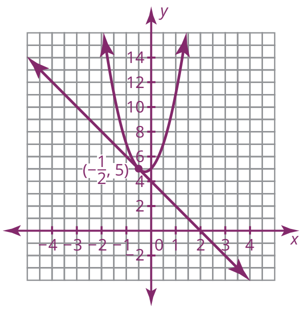


No intersection point

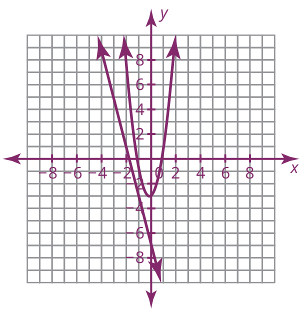


Answers

3a. The system has one solution: $(-\frac{1}{2}, 5)$.

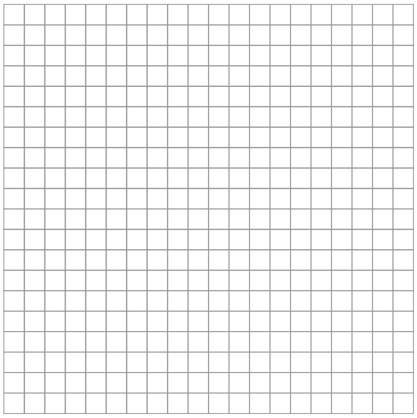


3b. $x = \frac{-5 \pm \sqrt{-23}}{6}$; The system has no real solutions.

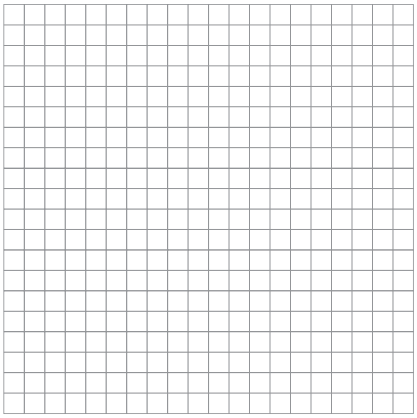


3. **Solve each system of equations algebraically over the set of real numbers. Then verify the solution graphically.**

a. $\begin{cases} y = -2x + 4 \\ y = 4x^2 + 2x + 5 \end{cases}$



b. $\begin{cases} y = -4x - 7 \\ y = 3x^2 + x - 3 \end{cases}$



NOTES

TALK the TALK

System Solutions

1. **A system of equations consisting of two linear equations has how many possible solutions?**

2. **A system of equations consisting of a linear equation and a quadratic equation has how many possible solutions?**

3. **Explain why a system of equations consisting of a linear equation and a quadratic equation cannot have an infinite number of solutions.**

Answers

1. A system of equations consisting of two linear equations can have no solution, one solution, or an infinite number of solutions.
2. A system of equations consisting of a linear equation and a quadratic equation can have no solution, one solution, or two solutions.
3. A system of equations consisting of a linear equation and a quadratic equation cannot have an infinite number of solutions because the graphs of the two equations can never be identical.