Applications of Quadratics
Topic 3 Overview

How is *Applications of Quadratics* organized?

Applications of Quadratics provides students with an opportunity to review what they have learned about quadratics in Algebra 1 by modeling and solving problems for situations involving quadratics. They start with a real-world problem that can be modeled by a quadratic inequality. Students have solved linear inequalities, and they apply and expand on those experiences to solve quadratic inequalities. From there, students are given a scenario that can be modeled by a system of one quadratic function and one linear function. They use technology to graph the system and determine the solutions and then use substitution to solve a system algebraically. Students are presented with a real-world situation and use familiar strategies to fit a quadratic regression. Students explore inverse functions and determine inverses from a graph, table, and equation. They consider a real-world situation that requires a restricted domain and use the inverse to solve problems. Finally, students explore parabolas as a conic section and write the general and standard equations.

What is the entry point for students?

Students are very familiar with inequalities. They have graphed and solved linear inequalities in one and two variables, as well as systems of linear inequalities. They use these skills to graph and solve quadratic inequalities, interpreting the solution set in the same way. Students first solved systems of linear equations graphically in middle school and algebraically in Algebra 1. In *Systems of Equations and Inequalities*, students advanced their graphical and algebraic techniques for solving systems of linear equations. Students know that a graph represents the solutions to the function it models and that the intersection point(s) of two graphs represent the solution(s) shared by both functions. With systems of quadratic equations, students use the same solving techniques; the only difference is whether it is a linear equation or a quadratic equation that they solve after eliminating a variable. Using their experience with linear regressions from Algebra 1, students are ready to use regressions to make predictions for data sets that can be modeled quadratically.

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in *Applications of Quadratics* if they can:

- Identify quantities represented in real-world situations modeled by quadratic equations.
- Write and solve a quadratic equation that best models a real-world situation.
- Graph a quadratic inequality on a coordinate plane with appropriate labels and scales.
- Interpret the solution set of a quadratic inequality.
- Graph and solve a system comprised of one linear equation and one quadratic equation.
- Solve linear and quadratic systems using graphing and substitution.
- Use technology to determine the function of best fit for a data set.

- Make predictions using a quadratic regression that represents a data set.
- Use technology to graph square root functions and determine solutions.
- Identify quantities represented in real-world situations modeled by quadratic equations.
- Write the inverse of a quadratic function in function notation by restricting the domain.
- Write the general and standard form of a parabola.

Why is *Applications of Quadratics* important?

As students improve their graphical and algebraic skills and increase their knowledge of different function families, they are able to model more complicated and interesting problems from real-world situations. *Applications of Quadratics* is important, as it provides students with an opportunity to bring together the techniques that they have learned in Algebra 1 and build upon this foundation throughout this course. Students' knowledge of first- and second-degree polynomials will be used and expanded upon as they encounter cubics, quartics, rational functions, and logarithmic functions. Modeling advanced scenarios will be heavily used by students who continue on to calculus and post-secondary mathematics.

How do the activities in *Applications of Quadratics* promote student expertise in the mathematical process standards?

All Carnegie Learning topics are written with the goal of creating mathematical thinkers who are active participants in class discourse, so the mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work toward solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

Students model with mathematics throughout this topic. They use reasoning as they decontextualize scenarios by defining variables, writing equations, and solving for unknowns. The reasoning is extended when they contextualize their solutions, making sense of them in terms of a given situation. Students make appropriate use of the tools they have mastered throughout this module and course to solve problems.

Materials Needed

Calculator or graphing technology
Patty Paper
Compass
Straightedge

New Tools and Notation

A function is a **one-to-one function** if both the function and its inverse are functions.

A linear function is sometimes a one-to-one function, and an exponential function is always a one-to-one function. Neither a quadratic function nor an absolute value function is ever a one-to-one function.

Learning Together

ELPS: 1.A, 1.C, 1.D, 1.E, 1.F, 1.G, 2.C, 2.D, 2.E, 2.G, 2.H, .2I, 3.A, 3.B, 3.C, 3.D, 3.E, 3.F, 4.A, 4.B, 4.C, 4.G, 4.K, 5.B, 5.E, 5.F, 5.G

Lesson	Lesson Name	TEKS	Days	Highlights
1	Ahead of the Curve: Solving Quadratic Inequalities	2A.4H	1	Students analyze a Worked Example to calculate the solution set of a quadratic inequality by first solving for the roots of its related quadratic equation, then determining which interval(s) created by the roots satisfy the inequality. They use both a number line and coordinate plane to select the correct intervals and then make connections between those methods. Students solve a problem in context requiring the use of a quadratic inequality, and they also use a transformation to make comparisons within a context. Throughout this lesson, students use the Quadratic Formula, technology, and inequality or interval notation.
2	All Systems Go!: Systems of Quadratic Equations	2A.3A 2A.3C 2A.3D	1	Students solve a problem in context that can be modeled by a system of equations involving a linear equation and a quadratic equation. They solve this first question graphically and discuss the number of solutions to the system and the number of solutions that make sense for the context. Students are then guided to solve a system of a linear equation and a quadratic equation algebraically, and then verify their results graphically. Students solve additional systems algebraically and graphically. They also discuss the number of possible solutions for each type of system and sketch graphs demonstrating those solutions.
3	The Ol' Switcharoo: Inverses of Linear and Quadratic Functions	2A.2B 2A.2C	3	Inverses of functions are introduced in this lesson. A Worked Example highlights how to determine the inverse of a linear function algebraically. Students use this example to determine other inverses of functions. They then create the graph of the inverse of a linear function by reflecting the original function across the line $y = x$ using patty paper. This process is repeated for quadratic functions. The term *one-to-one function* is defined, and students determine whether the inverse of a function is also a function. A graphic organizer is completed to summarize the definition and representations of inverses functions.

Lesson	Lesson Name	TEKS	Days	Highlights
4	Modeling Behavior: Using Quadratic Functions to Model Data	2A.2C 2A.4E 2A.7I 2A.8A 2A.8B 2A.8C	1	Students begin the lesson by determining a quadratic regression equation to model a set of data and use the regression equation to make predictions. Next, they are given a quadratic equation that models a context, but this time students see the need for an inverse equation because they must solve for the independent variable when the dependent variable is provided. Throughout the lesson, students identify the independent and dependent quantities and domain and range of functions in order to make sense of an inverse of function.
5	Going the Equidistance: Equation of a Parabola	2A.4B	3	The focus and directrix of a parabola are introduced through an exploratory activity. Students use concentric circles to plot points that are equidistant from both a line and a point not on the line, then connect these equidistant points to form a parabola. A parabola is described as a conic section, and the terms *locus of points, parabola*, *focus*, and *directrix* are given. Students construct a directrix and a focus above the directrix on patty paper and complete multiple folds of the focus onto the line to create a parabola. *Concavity* and the *vertex* of a *parabola* are defined. Through investigations students conclude that any point on a parabola is equidistant from the focus and the directrix. The focus and directrix are then used to write the equation of a parabola, and the general and standard form of a parabola are given. Students derive the standard form of a parabola algebraically to make sense of the constant p in the equation and use this constant to graph parabolas. The Distance Formula is used to determine the equation of points that are equidistant from a given focus and a given directrix where the vertex is a point other than the origin. Students apply characteristics of parabolas to solve real-world problem situations.

Suggested Topic Plan

*1 Day Pacing = 45 min. Session

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: 2A.4H **LESSON 1** **Ahead of the Curve** GETTING STARTED ACTIVITY 1 ACTIVITY 2 TALK THE TALK	TEKS: 2A.3A, 2A.3C, 2A.3D **LESSON 2** **All Systems Go!** GETTING STARTED ACTIVITY 1 ACTIVITY 2 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress	TEKS: 2A.2B, 2A.2C **LESSON 3** **The Ol' Switcharoo** GETTING STARTED ACTIVITY 1 ACTIVITY 2	**LESSON 3** continued ACTIVITY 3 ACTIVITY 4
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
LESSON 3 continued ACTIVITY 5 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress	**MID-TOPIC ASSESSMENT**	TEKS: 2A.2C, 2A.4E, 2A.7I, 2A.8A, 2A.8B, 2A.8C **LESSON 4** **Modeling Behavior** GETTING STARTED ACTIVITY 1 ACTIVITY 2 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress
Day 11	**Day 12**	**Day 13**	**Day 14**	**Day 15**
TEKS: 2A.4B **LESSON 5** **Going the Equidistance** GETTING STARTED ACTIVITY 1 ACTIVITY 2	**LESSON 5** continued ACTIVITY 3 ACTIVITY 4	**LESSON 5** continued ACTIVITY 5 ACTIVITY 6 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress	**END OF TOPIC ASSESSMENT**

Assessments

There are two assessments aligned to this topic: Mid-Topic Assessment and End of Topic Assessment.