Applications of Quadratics Summary

KEY TERMS

- restrict the domain
- one-to-one function
- inverse of a function
- conic section
- locus of points
- parabola
- focus
- directrix
- vertex of a parabola
- concavity
- general form of a parabola
- standard form of a parabola

LESSON 1 Ahead of the Curve

Just like with the other inequalities you have studied, the solution to a quadratic inequality is the set of values that satisfy the inequality.

For example, consider the inequality $x^2 - 4x + 3 < 0$.

Write the corresponding quadratic equation. $x^2 - 4x + 3 = 0$

Calculate the roots of the quadratic equation using an appropriate method.

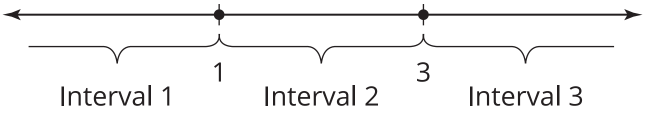
$(x - 3)(x - 1) = 0$

$(x - 3) = 0$ or $(x - 1) = 0$

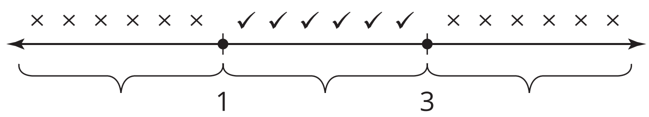
$x = 3$ or $x = 1$

Summary

Plot the roots to divide the number line into three regions.



Choose a value from each interval to test in the original inequality. Identify the solution set as the interval(s) in which your test value satisfies the inequality.



Interval 1	Interval 2	Interval 3
Try $x = 0$	Try $x = 2$	Try $x = 4$
$0^2 = 4(0) + 3 < 0$	$2^2 = 4(2) + 3 < 0$	$4^2 - 4(4) + 3 < 0$
$3 < 0$ ×	$4 - 8 + 3 < 0$	$16 - 16 + 3 < 0$
	$-1 < 0$ ✓	$3 < 0$ ×

Interval 2 satisfies the original inequality, so the solution includes all numbers between 1 and 3.
Solution: $x \in (1, 3)$, or $1 < x < 3$.

LESSON 2

All Systems Go!

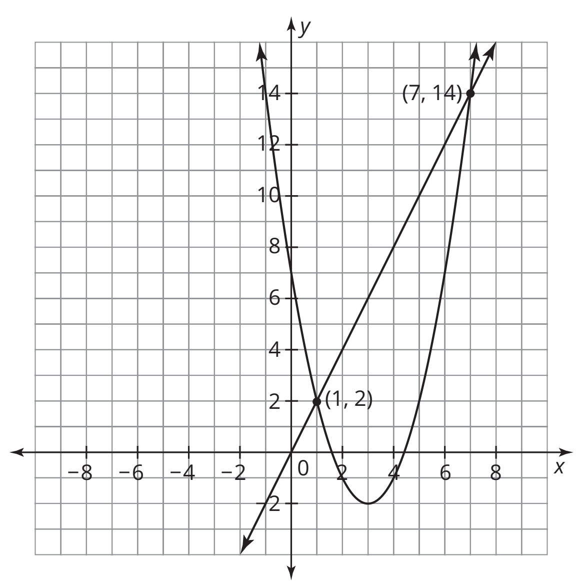
The graph of a linear equation and the graph of a quadratic equation can intersect at two points, at one point, or not at all.

The method to determine the solution or solutions of a system involving a quadratic equation and a linear equation is similar to solving a system of two linear equations. First, substitute one equation into the other. Then, solve the resulting equation for *x* and calculate the corresponding values for *y*. These values represent the point(s) of intersection. Finally, graph each equation of the system to verify the points of intersection.

$$\begin{cases} y = x^2 - 6x + 7 \\ y = 2x \end{cases}$$

$2x = x^2 - 6x + 7$
$0 = x^2 - 8x + 7$
$0 = (x - 7)(x - 1)$

$x - 7 = 0$	$x - 1 = 0$
$x = 7$	$x = 1$
$y = 2(7)$	$y = 2(1)$
$y = 14$	$y = 2$



The system has two solutions: (7, 14) and (1, 2).

LESSON 3

The Ol' Switcharoo

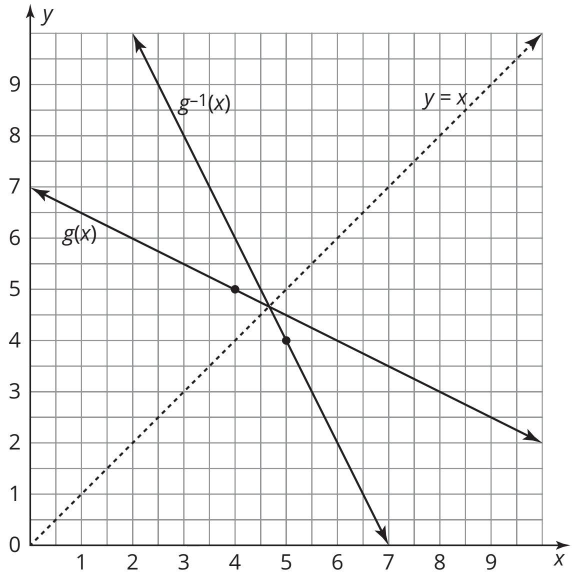
A function takes an input value, performs some operation(s) on this value, and creates an output value. The inverse of a function takes the output value, performs some operation(s) on this value, and arrives back at the original function's input value. In other words, an **inverse of a function** is a function that "undoes" another function.

Given a function $f(x)$, you can determine the inverse function algebraically by following these steps:

Step 1: Replace $f(x)$ with y.
Step 2: Switch the x and y variables.
Step 3: Solve for y.
Step 4: If y is a function, replace y with $f^{-1}(x)$.

The graph of the inverse of a function is a reflection of the function across the line $y = x$.

For example, given that (4, 5) is a point on the graph of $g(x)$, the corresponding point on the graph of $g^{-1}(x)$ is (5, 4).



A function is a **one-to-one function** if both the function and its inverse are functions.

A linear and exponential function are always one-to-one functions, but a quadratic function is not. Therefore, you need to restrict the domain of the original quadratic function so that the inverse is also a function. To **restrict the domain** of a function means to define a new domain for the function that is a subset of the original domain.

For example, you can determine the equation of the inverse of the basic quadratic function $f(x) = x^2$ by first restricting the domain to $x > 0$, replacing $f(x)$ with y, switching the x and y variables, and solving for y.

$$f(x) = x^2$$
$$y = x^2$$
$$x = y^2$$
$$y = \sqrt{x}$$

The inverse of $f(x) = x^2$ is $f^{-1}(x) = \sqrt{x}$ for $x > 0$.

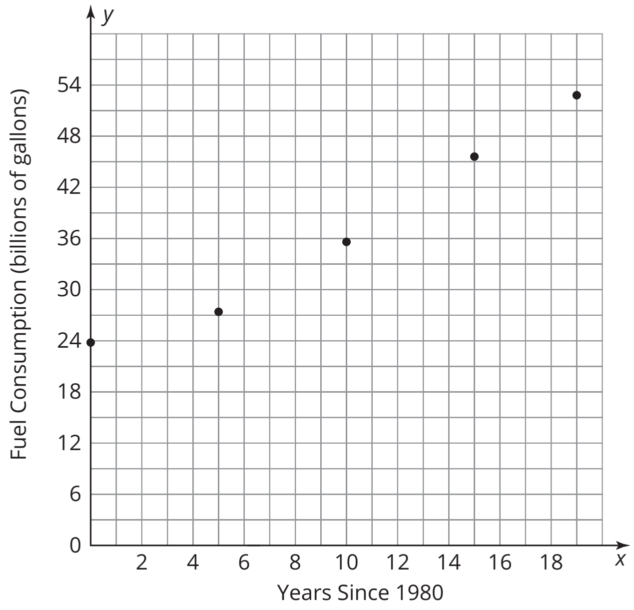
LESSON 4

Modeling Behavior

Quadratic regression equations can be used to model real-world situations and make predictions.

For example, as vans, trucks, and SUVs have increased in popularity, the fuel consumption of these types of vehicles has also increased.

Years Since 1980	Fuel Consumption (billions of gallons)
0	23.8
5	27.4
10	35.6
15	45.6
19	52.8



The quadratic regression equation that best fits the data is $y = 0.0407x^2 + 0.809x + 23.3$. The r^2 value for the quadratic regression fit is 0.996. Just as with linear and exponential regressions, the equation can be used to make predictions for the data.

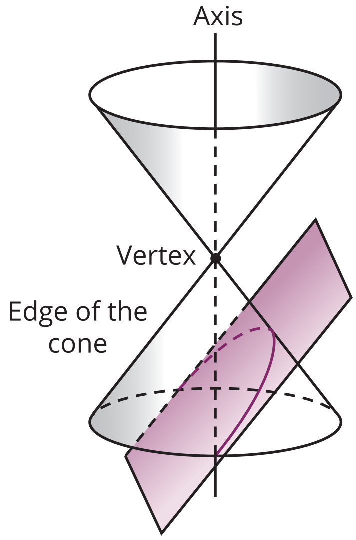
For example, you can predict the fuel consumption in the year 2020 by substituting $x = 40$ into the regression equation.

$y = 0.0407(40)^2 + 0.809(40) + 23.3$
$y \approx 121$

In 2020, fuel consumption will be about 121 billion gallons.

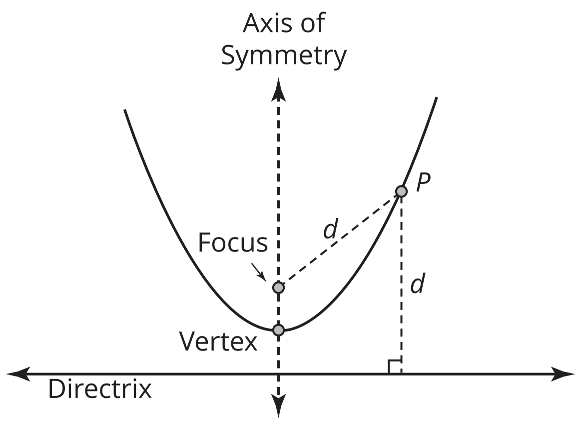
LESSON 5 Going the Equidistance

You previously studied parabolas as quadratic functions. You analyzed equations and graphed parabolas based on the position of the vertex and additional points determined using x-values on either side of the axis of symmetry.

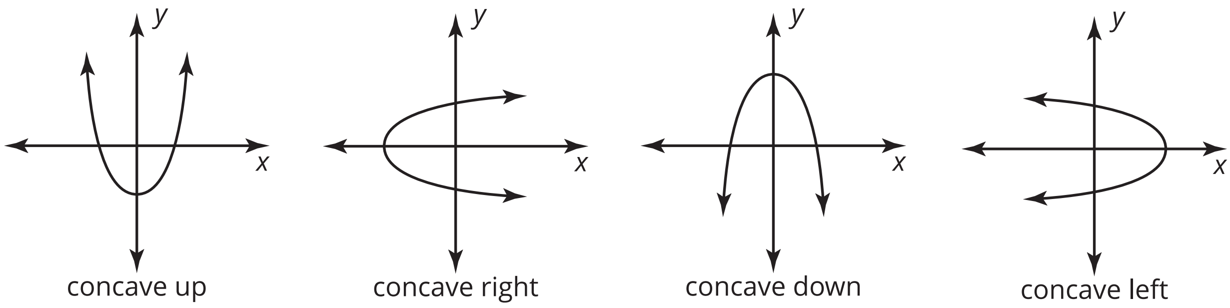


A parabola can also be described as a conic section.

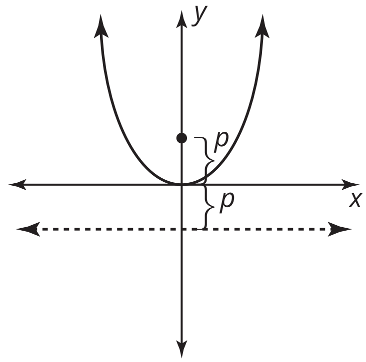
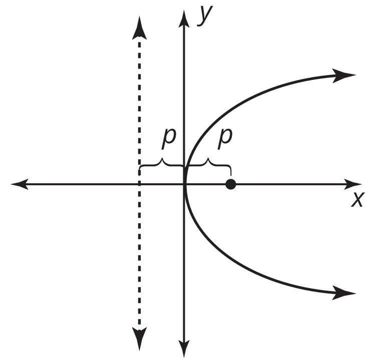
A **locus of points** is a set of points that share a property. A **parabola** is the set of all points in a plane that are equidistant from a focus and a directrix. The **focus** is a point which lies inside the parabola on the axis of symmetry. The **directrix** is a line that is perpendicular to the axis of symmetry and lies outside the parabola and does not intersect the parabola. Thus, a parabola is equidistant from the focus and the directrix. The **vertex of a parabola** is the point on the axis of symmetry which is exactly midway between the focus and the directrix. It is also the point where the parabola changes direction.



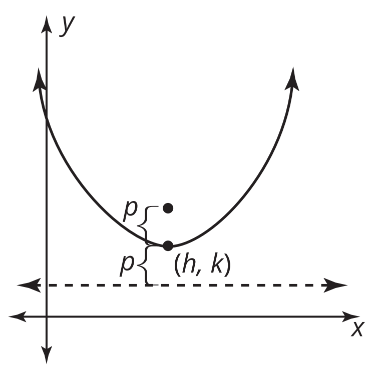
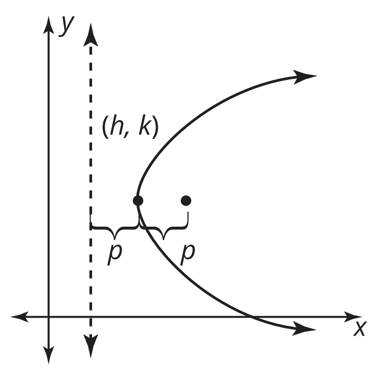
The **concavity** of a parabola describes the orientation of the curvature of the parabola. A parabola can be concave up, concave down, concave right, or concave left, as shown.



The **general form of a parabola** centered at the origin is an equation of the form $Ax^2 + Dy = 0$ or $By^2 + Cx = 0$. The **standard form of a parabola** centered at the origin is an equation of the form $x^2 = 4py$ or $y^2 = 4px$, where p represents the distance from the vertex to the focus.

Parabola		
Graph		
Equation of Parabola	$x^2 = 4py$	$y^2 = 4px$
Orientation of Parabola	vertical	horizontal
Axis of Symmetry	y-axis	x-axis
Coordinates of Vertex	(0, 0)	(0, 0)
Coordinates of Focus	$(0, p)$	$(p, 0)$
Equation of Directrix	$y = p$	$x = p$
Concavity	up or down	right or left

The standard forms of parabolas with a vertex at (h, k) are $(x - h)^2 = 4p(y - k)$ and $(y - k)^2 = 4p(x - h)$.

Parabola		
Graph		
Equation of Parabola	$(x - h)^2 = 4p(y - k)$	$(y - k)^2 = 4p(x - h)$
Orientation of Parabola	vertical	horizontal
Axis of Symmetry	$x = h$	$y = k$
Coordinates of Vertex	(h, k)	(h, k)
Coordinates of Focus	$(h, k + p)$	$(h + p, k)$
Equation of Directrix	$y = k - p$	$x = h - p$
Concavity	up or down	right or left