Skills Practice

Name ______________________________ **Date** ____________________

I. Solving Quadratic Inequalities

A. Determine the roots of each quadratic inequality. Use the interval method to determine the solution set of the inequality. Round your answer to the nearest thousandth if necessary.

1. $x^2 - 7x + 16 \geq 10$

2. $x^2 + 7x - 2 < -12$

3. $x^2 + x - 15 < 4$

4. $-x^2 + 11x - 21 \leq 2$

5. $-x^2 + 4x - 5 \leq -2$

6. $-x^2 - 3x + 14 > -3$

B. A water balloon is thrown upward from a height of 5 feet with an initial velocity of 35 feet per second. The quadratic function $h(t) = -16t^2 + 35t + 5$ represents the height of the balloon, h, in feet t seconds after it is thrown. Use this information to answer each question. Round your answer to the nearest thousandth.

1. How long does it take for the balloon to reach the ground?

2. Determine when the balloon is less than 10 feet above the ground.

3. Determine when the balloon is more than 10 feet above the ground.

4. Determine when the balloon is less than 20 feet above the ground.

5. Determine when the balloon is more than 20 feet above the ground.

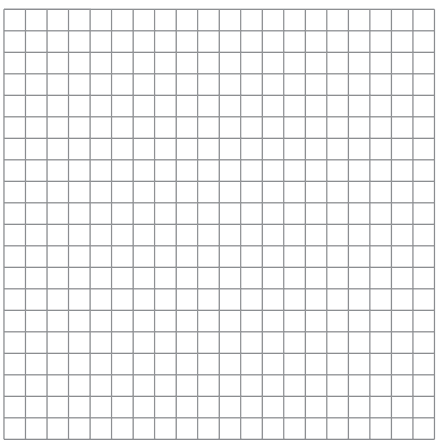
6. Determine when the balloon is less than 30 feet above the ground.

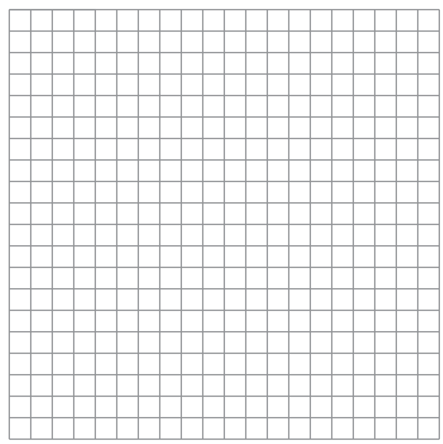
II. Systems of Linear and Quadratic Equations

A. Solve each system of equations algebraically. Then verify each solution graphically.

1. $\begin{cases} y = x^2 - 6x + 7 \\ y = 2x \end{cases}$

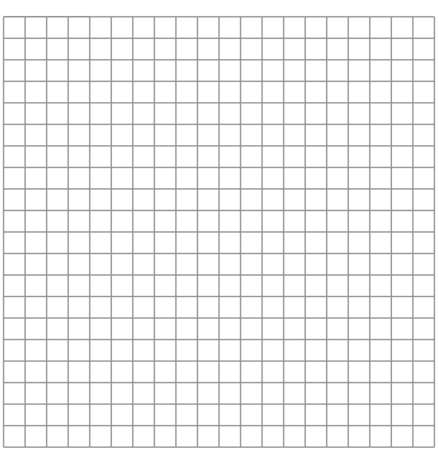
2. $\begin{cases} y = x^2 - 3x + 1 \\ y = x - 3 \end{cases}$

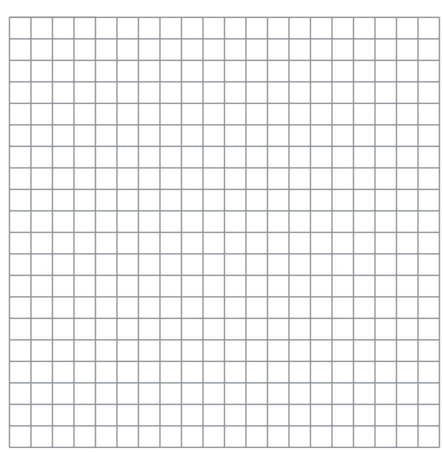




3. $\begin{cases} y = 2x^2 + 16x + 24 \\ y = -x - 2 \end{cases}$

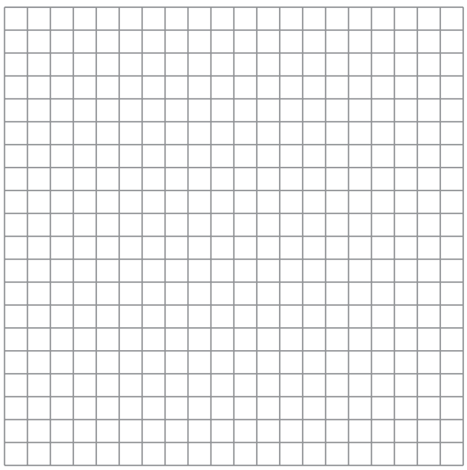
4. $\begin{cases} y = x^2 + 6x - 6 \\ y = 3x + 1 \end{cases}$

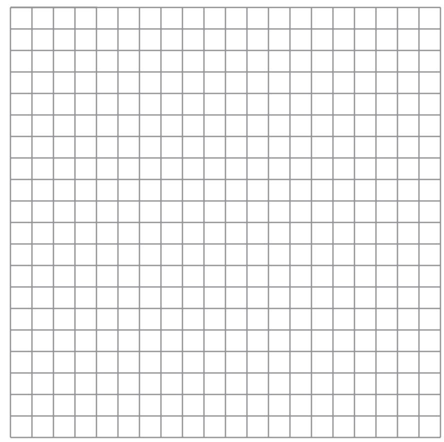




5. $\begin{cases} y = 4x^2 + 6x + 3 \\ y = -6x - 6 \end{cases}$

6. $\begin{cases} y = 3x^2 + 24x + 50 \\ y = 4x + 1 \end{cases}$





B. Write and solve a system of equations for each situation. Then explain if the solution(s) to the system make sense in the problem situation.

1. A student throws a ball straight upward from a height of 5 meters with an initial velocity of 40 feet per second. The student is in the gymnasium, where the height of the ceiling is 30 feet. Will the ball hit the ceiling? Recall that vertical motion can be modeled by the equation $h(t) = -16t^2 + v_0 t + h_0$, where h is the height, in feet, of the object after t seconds, v_0 is the initial velocity, and h_0 is the initial height.

2. Free Skate is a company that specializes in making skateboards. Each month, the company must keep track of their costs and revenue. Free Skate's costs consist of fixed costs that include rent, utilities, and workers' salaries, as well as the variable cost to make the skateboards. Its costs can be represented by the function $C(x) = 20x + 700$. The company's revenue for every skateboard sold can be represented by the function $R(x) = 150x - 0.6x^2$. Determine the break-even point(s) for the month.

3. Another company, LoveSk8, decides they want to compete with Free Skate from question 2. LoveSk8 determines its monthly costs will be represented by the function $C(x) = 17x + 800$. LoveSk8 predicts its revenue will be represented by the function $R(x) = 60x - 0.7x^2$. Determine how many skateboards LoveSk8 must make to break even each month.

4. A professional base jumper jumps off the Perrine Bridge in Twin Falls, Idaho. When he jumps, he free falls for a few seconds and then opens his parachute so that he can land safely. Before he opens the parachute, the equation $h(t) = -4.9t^2 + t + 450$ models his height in meters t seconds after he jumps. The equation $h(t) = -5t + 325$ represents his height after he opens the parachute. How long after jumping did he open the parachute?

5. A punter kicks a football. The height of the football, in meters, is modeled by the function $h(t) = -4.9t^2 + 8.5t + 0.25$, where t represents time, in seconds. A blocker can only attempt to knock down the football as it travels upward from the punter's foot. The height in meters of the approaching blocker's hands is modeled by the function $h(t) = -0.4t + 3$, where t represents the same time. Will the blocker knock down the football? If so, when will the blocker knock it down?

6. Steven and his sister are playing a guessing game. Steven tells his sister that he is thinking of two negative integers. The first number minus the second number is -4. The square of the first number minus 4 times the second number is equal to 61.

III. Inverse of Linear and Quadratic Functions

A. Determine the corresponding point on the graph of the inverse of each function.

1. Given that (2, 5) is a point on the graph of $f(x)$, what is the corresponding point on the graph of its inverse?

2. Given that (–3, 1) is a point on the graph of $f(x)$, what is the corresponding point on the graph of its inverse?

3. Given that (–4, –1) is a point on the graph of $f(x)$, what is the corresponding point on the graph of its inverse?

4. Given that (0, 8) is a point on the graph of $f(x)$, what is the corresponding point on the graph of its inverse?

5. Given that (1, −7) is a point on the graph of $f(x)$, what is the corresponding point on the graph of its inverse?

6. Given that (−6, 0) is a point on the graph of $f(x)$, what is the corresponding point on the graph of its inverse?

B. Complete each table. Write an equation to represent the relationship. Write an equation for the inverse of the problem situation.

1. One foot is equivalent to 12 inches.

Feet	Inches
1	
2	
3	
4	
5	

2. One meter is equivalent to 100 centimeters.

Meters	Centimeters
1	
2	
3	
4	
5	

3. One pint is equivalent to 2 cups.

Pints	Cups
2	
4	
6	
8	
10	

4. Four quarters is equivalent to 1 dollar.

Quarters	Dollars
4	
16	
32	
64	
128	

5. Three feet is equivalent to 1 yard.

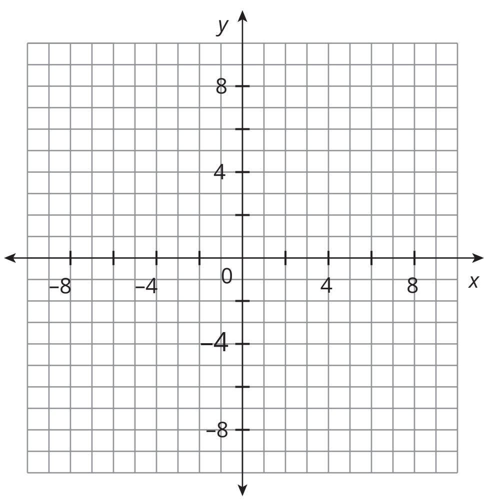
Feet	Yards
3	
9	
12	
18	
24	

6. One U.S. dollar is equivalent to 20 Mexican pesos.

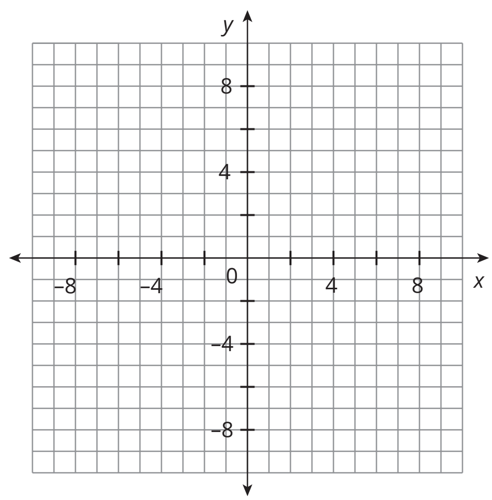
Dollars	Pesos
1	
2	
3	
4	
5	

C. Determine the inverse of each function. Graph the original function and its inverse.

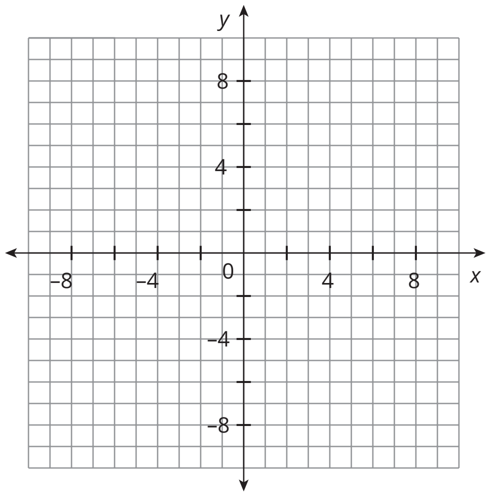
1. $f(x) = 4x$



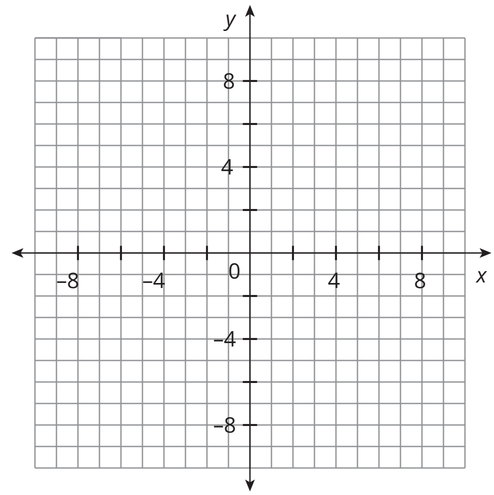
2. $f(x) = \frac{1}{3}x$



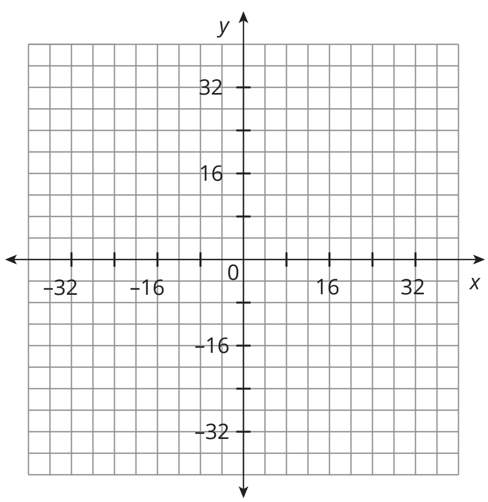
3. $f(x) = 2x + 1$



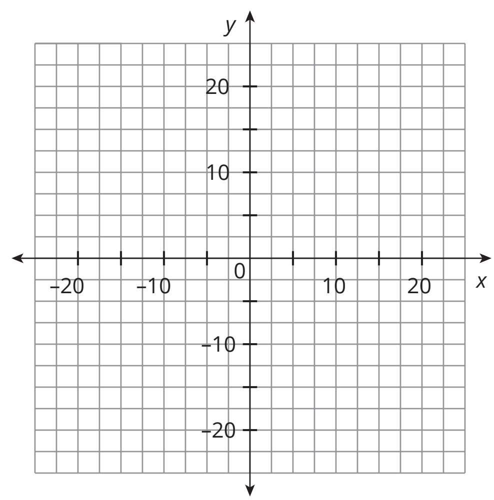
4. $f(x) = -6x - 2$



5. $f(x) = \frac{2}{3}x - 8$



6. $y = 12$



D. Determine the equation of the inverse for each quadratic function.

1. $f(x) = 7x^2$

2. $f(x) = -x^2$

3. $f(x) = 6x^2 + 11$

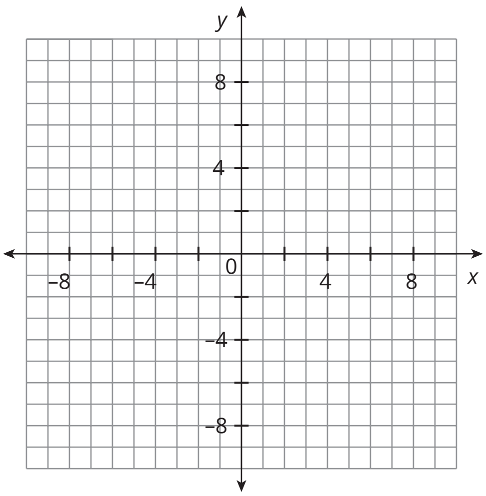
4. $f(x) = 2x^2 - 12$

5. $f(x) = -4x^2 - 6$

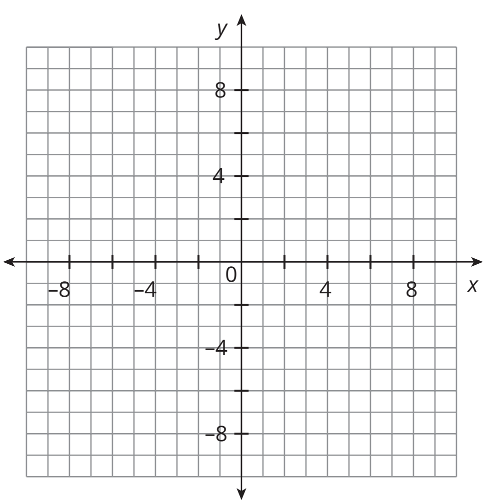
6. $f(x) = -3x^2 + 20$

E. Determine the equation of the inverse for each given function. Graph the function and its inverse. Restrict the domain of the original function and the inverse so that the inverse is also a function.

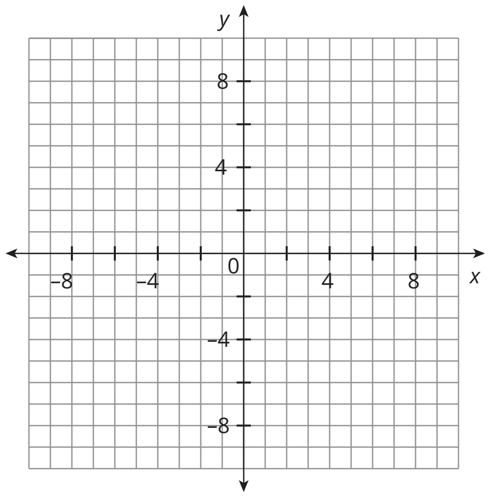
1. $f(x) = 2x^2$



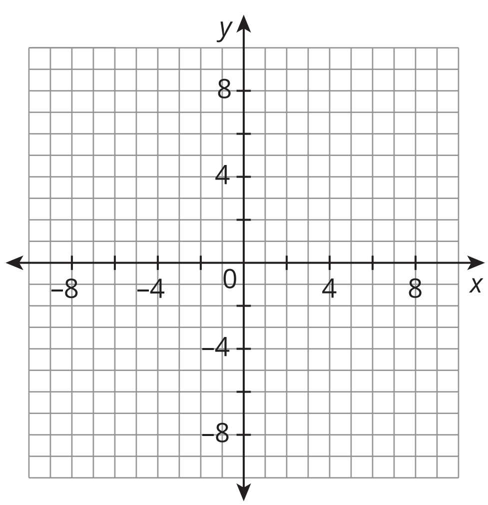
2. $f(x) = x^2 + 3$



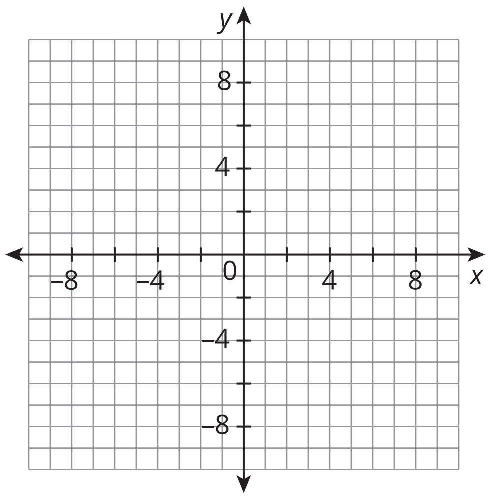
3. $f(x) = -4x^2 - 2$



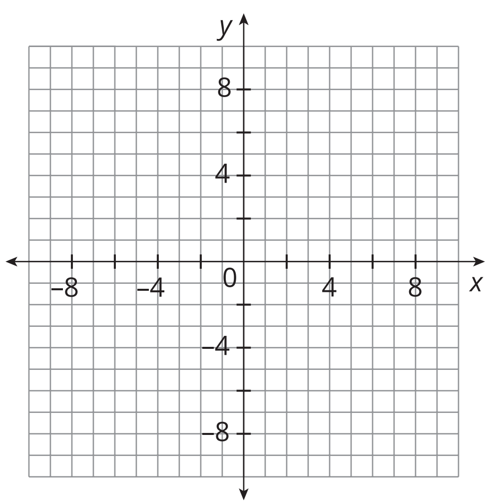
4. $f(x) = 3x^2 - 4$



5. $f(x) = -\frac{1}{2}x^2$

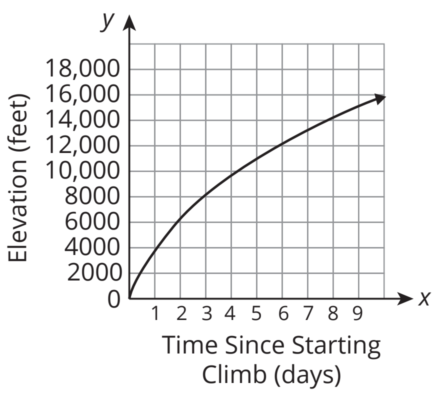


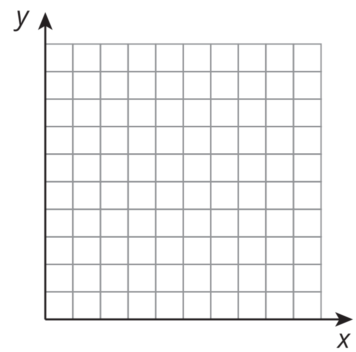
6. $f(x) = -x^2 + 5$



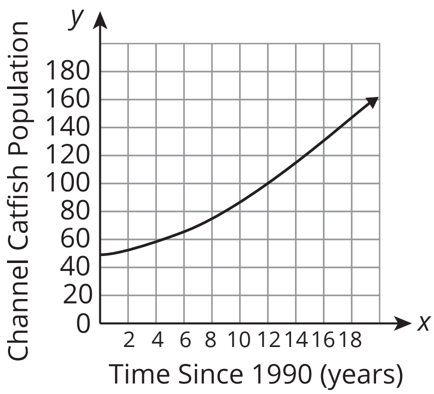
F. The function modeling each problem situation is given. Sketch the inverse of each function on the grid provided and label the axes. Then, describe the domain and range of both functions as they relate to the problem situation.

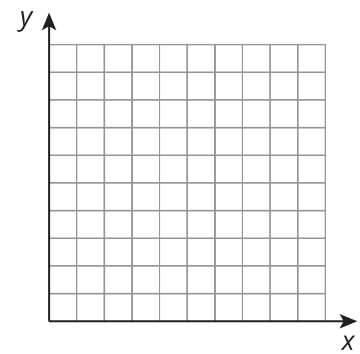
1. The function models a mountain climber's elevation over his 10-day mountain climb.



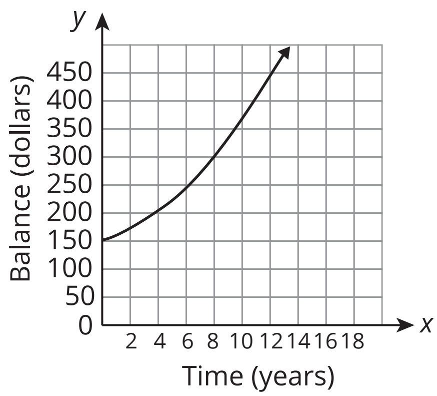


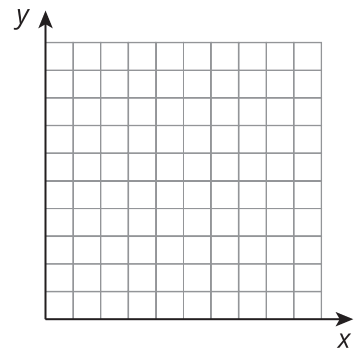
2. In the year 1990, park rangers stocked an unpopulated lake with 50 channel catfish. The function models the channel catfish population over a period of time.



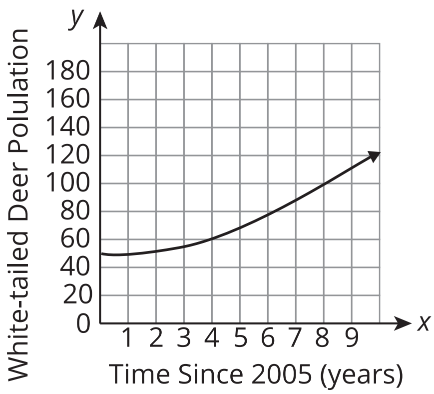


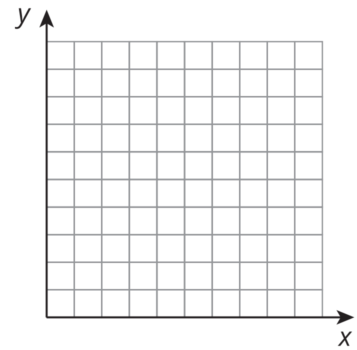
3. Daniel deposited $150.00 in a savings account. The function models the balance in the savings account in dollars.



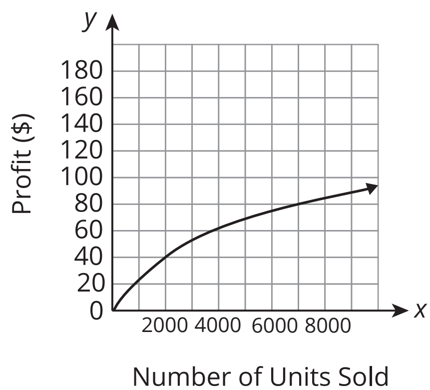


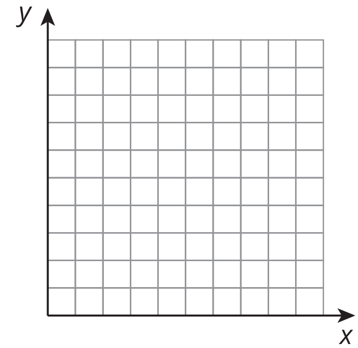
4. In the year 2005, a park had a population of 47 white-tailed deer. The function models the population of white-tailed deer in a park over a period of time.



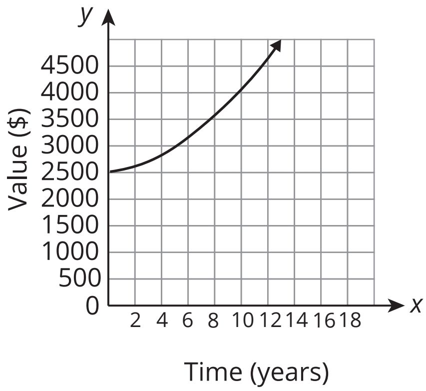


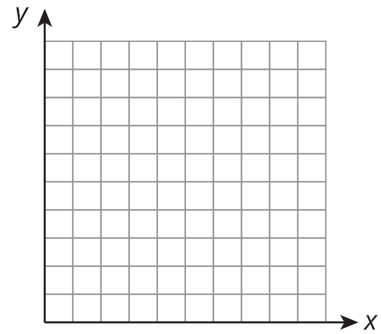
5. A company's profit over a period of a year is modeled on the graph below.





6. Lance bought a rare baseball memorabilia item 15 years ago for $2500. The function models the value of the item over time.





IV. Using Quadratic Functions to Model Data

A. Use the given regression equation to answer each question.

1. A farmer finds that his crop yield per acre can be modeled by the quadratic regression equation $y = -0.02x^2 + 1.08x + 3.89$, where x represents the amount of fertilizer applied in pounds per hundred square feet and y represents the crop yield in bushels. What is the approximate yield when 20 pounds of fertilizer are applied per hundred square feet?

2. The growth of soybean plants in inches over a certain time is tested with different amounts of fertilizer. The growth can be modeled by the quadratic regression equation $y = -0.001x^2 + 0.12x + 5.6$, where y represents the growth in inches and x represents the amount of fertilizer per plant in milligrams. How much growth could be expected from a plant given 40 milligrams of fertilizer?

3. The height of a ball thrown at an angle is measured photographically. The height of the ball can be modeled by the quadratic regression equation $h = -16.2t^2 + 46t + 4$, where h is the height of the ball after t seconds. At what time should the ball reach a height of 30 feet?

4. The quadratic regression equation $y = -20x^2 + 600x + 2250$ models the relationship between the selling price of a necklace, x, in dollars and the profit earned each month, y, in dollars. To reach a profit of $300 per month from the necklaces, what should the selling price be?

5. A company manufactures bicycles. The relationship between the number of bicycles made and the cost to produce those bicycles can be modeled by the quadratic regression equation $y = x^2 + 36.7x + 4756$, where y represents the total cost to make the bicycles and x represents the number of bicycles made. What would be the cost to manufacture 50 bicycles?

6. A model rocket is launched into the air. The height of the model rocket can be modeled by the quadratic regression equation $y = -4.1x^2 + 272.8x + 2.7$, where y represents the height of the model rocket in feet and x represents the time in seconds. At what time should the model rocket reach a height of 2000 feet?

B. Use technology to determine the quadratic regression equation for each data set. Round decimals to the nearest thousandth.

1.

x	y
1	22
6	6
20	20
42	64
80	99

2.

x	y
12	38
24	12
30	3
40	16
54	54

3.

x	y
25	60
50	80
75	140
100	210
125	250
150	322
175	400

4.

x	y
10	14
20	36
30	70
40	120
60	240
70	350

5.

x	y
0.1	8
0.2	12
0.3	14
0.4	16
0.5	18
0.6	10
0.7	2

6.

x	y
0.1	2.2
0.2	3.1
0.3	4
0.4	4.8
0.5	5.4
0.6	6
0.7	4.2

C. Use technology to determine the *y*-intercept, *x*-intercept(s), and vertex of the graph of each given quadratic regression equation. Then determine what these values mean in terms of the problem situation and tell whether the values make sense.

1. The speed, s, of a car and the car's average fuel efficiency, G, in miles per gallon at that speed can be modeled by the quadratic regression equation $G = -0.014s^2 + 1.502s - 9.444$.

2. An athlete throws a disc upward at an angle. The height in feet, h, of the disc can be modeled by the quadratic regression equation $h = -0.002x^2 + 0.440x + 5.621$, where x represents the distance in feet that the disc has traveled horizontally.

3. Martin and his friend are playing catch with a baseball. Martin tosses the ball to his friend, but he overthrows it, and it hits the ground. The height in feet, h, of the baseball can be modeled by the quadratic regression equation $h = -0.005x^2 + 0.752x + 5.220$, where x represents the distance in feet that the baseball has traveled horizontally.

4. A company's profit in dollars, p, can be modeled by the quadratic regression equation $p = -116.938x^2 + 4010.166x - 10{,}590.863$, where x represents the number of years since the company was started.

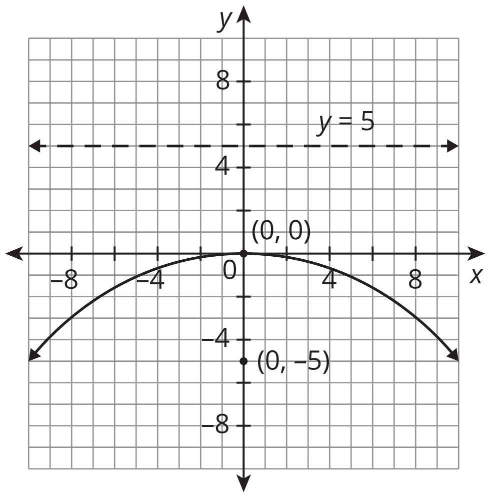
5. The temperatures, t, in degrees Fahrenheit recorded during a 10-hour winter snowstorm can be modeled by the quadratic regression equation $t = 0.74x^2 - 8.12x + 35.07$, where x represents the number of hours the storm has lasted.

6. A company's daily profit, p, from selling calculators can be modeled by the quadratic regression equation $p = -9.67x^2 + 1625.20x - 48{,}793.33$, where x represents the price of the calculator.

V. Equation of a Parabola

A. For the graph of each parabola centered at the origin, write its equation in conic form. Show your work for deriving the equation. Then, identify the axis of symmetry and concavity of the parabola.

1.

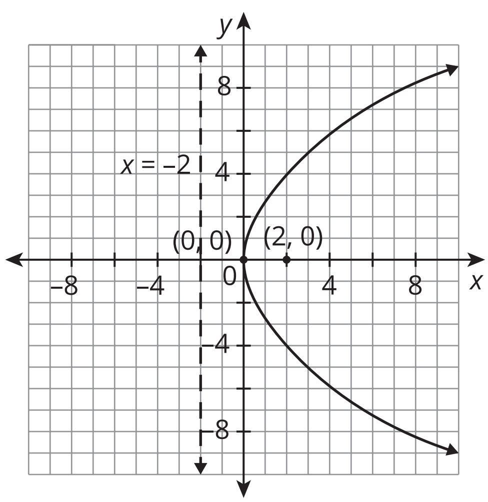


Equation:

Axis of Symmetry:

Concavity:

2.

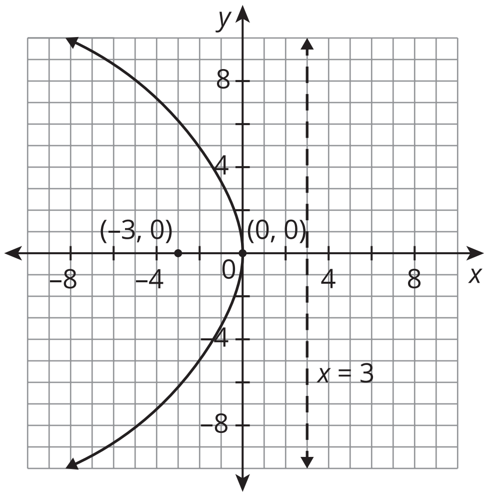


Equation:

Axis of Symmetry:

Concavity:

3.

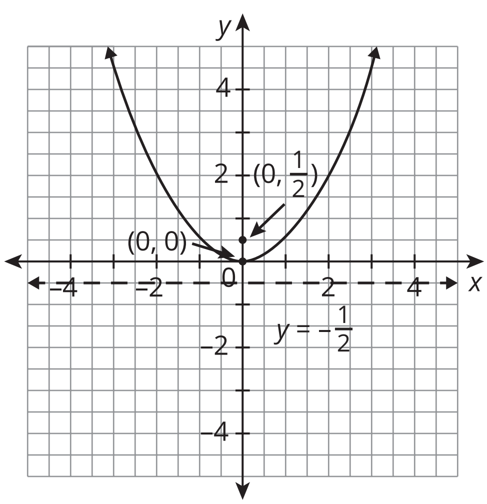


Equation:

Axis of Symmetry:

Concavity:

4.

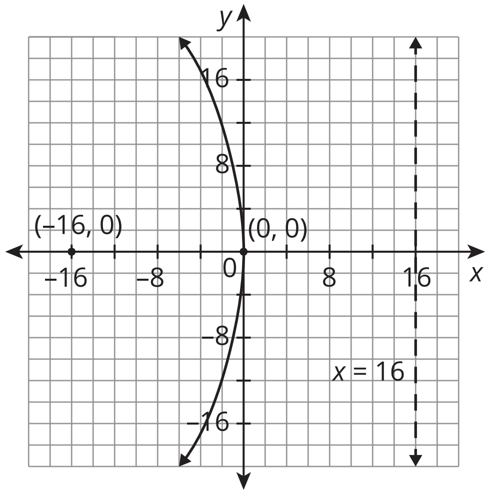


Equation:

Axis of Symmetry:

Concavity:

5.

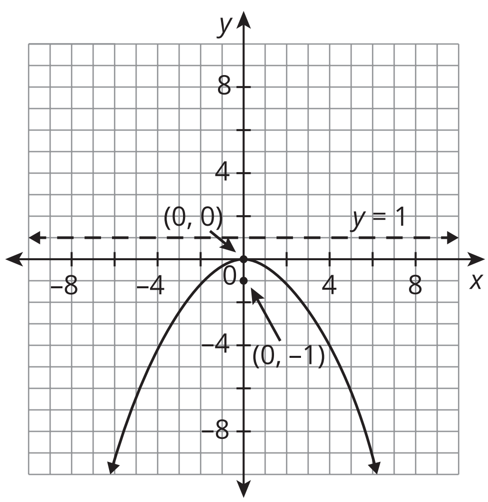


Equation:

Axis of Symmetry:

Concavity:

6.



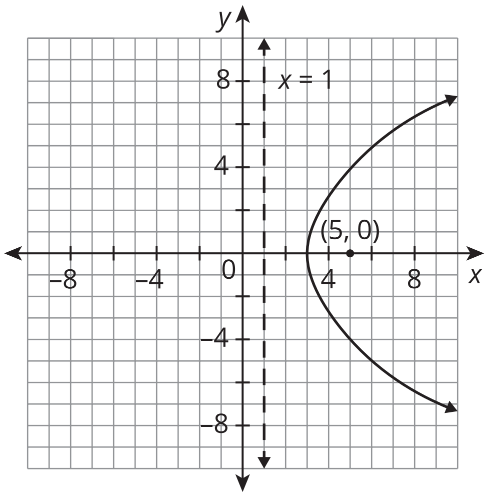
Equation:

Axis of Symmetry:

Concavity:

B. Write the conic form equation of each parabola. Identify the vertex and axis of symmetry. Show your work for deriving the equation.

1.



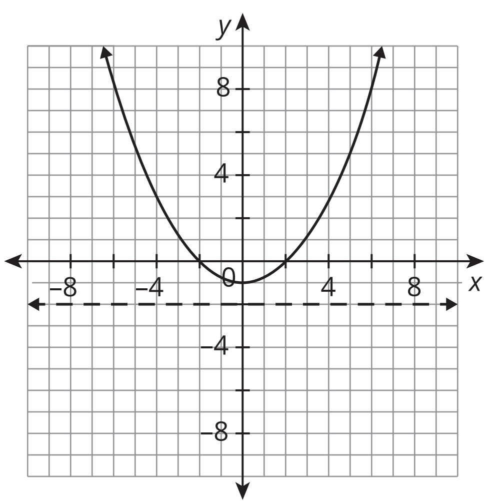
Vertex:

Axis of Symmetry:

p–value:

Equation:

2.



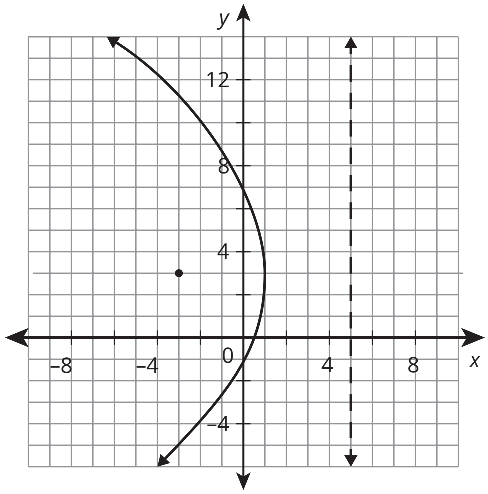
Vertex:

Axis of Symmetry:

p–value:

Equation:

3.



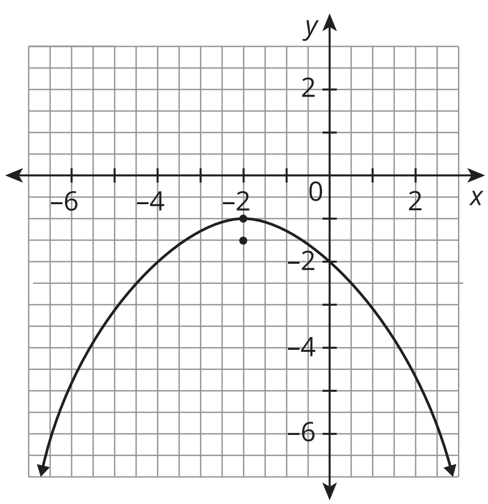
Vertex:

Axis of Symmetry:

p–value:

Equation:

4.



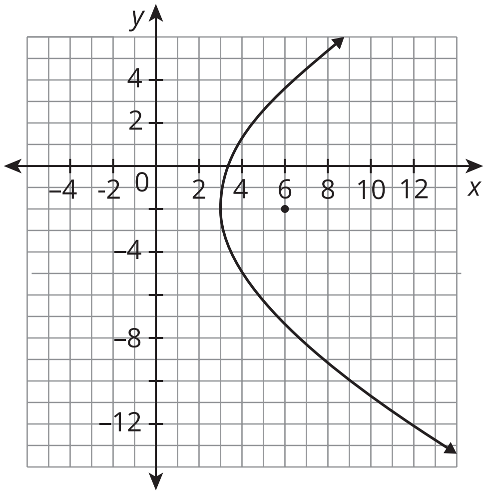
Vertex:

Axis of Symmetry:

p–value:

Equation:

5.



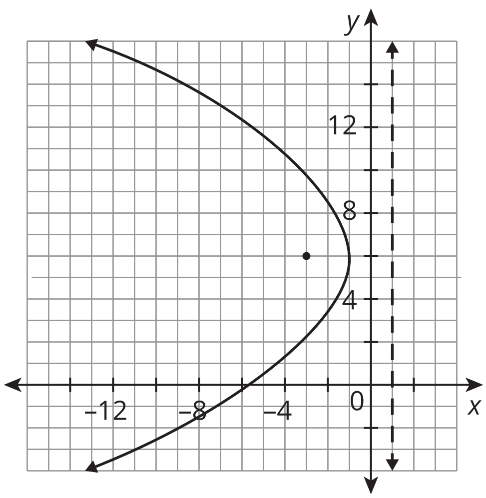
Vertex:

Axis of Symmetry:

p–value:

Equation:

6.



Vertex:

Axis of Symmetry:

p–value:

Equation:

C. Determine the equation of a parabola with the given vertex and focus.

1. A parabola has a vertex at (0, 0) and a focus at (0, –2).

2. A parabola has a vertex at (3, 0) and a focus at (5, 0).

3. A parabola has a vertex at (–2, –3) and a focus at (–2, –6).

4. A parabola has a vertex at (4, –2) and a focus at (–1, –2).

5. A parabola has a vertex at (10, 7) and a focus at (10, 18).

6. A parabola has a vertex at (−5, 8) and a focus at (2, 8).

D. Determine the equation of a parabola with the given focus and directrix. Explain your method for determining the equation.

1. A parabola has a focus at (−1, −5) and a directrix at $y = 3$.

2. A parabola has a focus at (5, 0) and a directrix at $x = -1$.

3. A parabola has a focus at (−3, −8) and a directrix at $y = 6$.

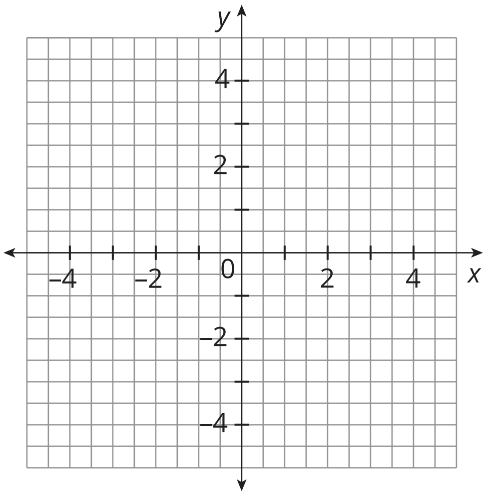
4. A parabola has a focus at (−7, −5) and a directrix at $x = -3$.

5. A parabola has a focus at (3, 3) and a directrix at $y = 5$.

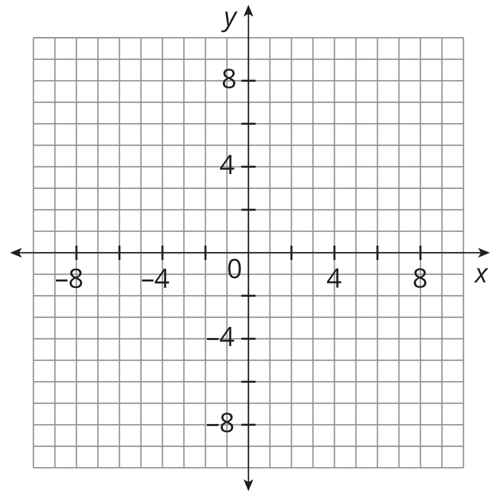
6. A parabola has a focus at (−10, 1) and a directrix at $y = -1$.

E. Sketch each parabola.

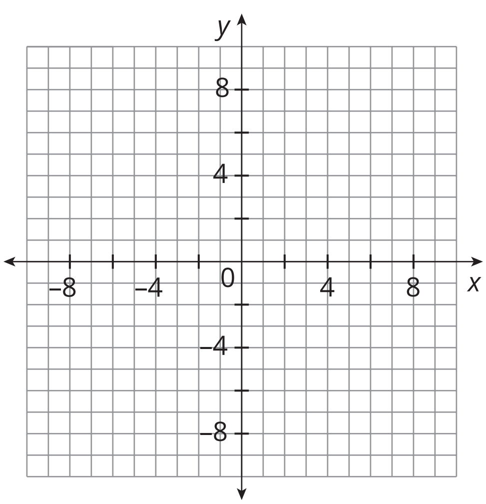
1. $x^2 = -4y$



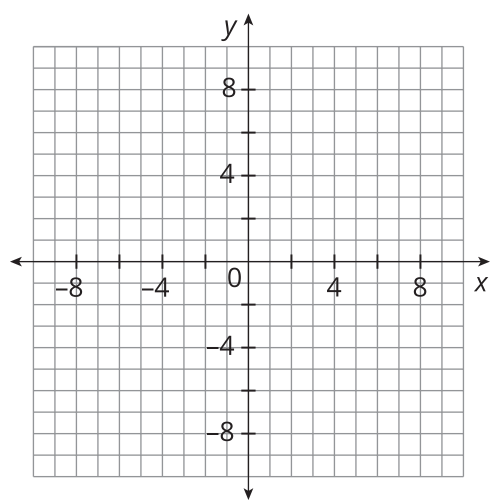
2. $y^2 = 16x$



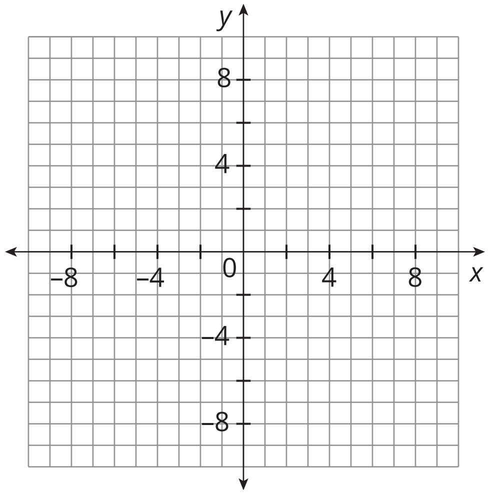
3. $y^2 = -20x$



4. $x^2 = 6y$



5. $(x - 1)^2 = 16(y - 3)$



6. $(y + 2)^2 = -4(x - 1)$