1 Exploring Patterns in Linear and Quadratic Relationships

Module Pacing: 48 Days

Topic 1: Extending Linear Relationships

ELPS: 1.A, 1.C, 1.D, 1.E, 1.F, 1.G, 2.C, 2.D, 2.E, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.E, 3.F, 4.A, 4.B, 4.C, 4.G, 4.K, 5.B, 5.E, 5.F, 5.G

Topic Pacing: 17 days

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
1	**Gauss in Das Haus** Solving Systems of Equations	Students solve systems comprising linear and quadratic equations. They begin by solving a system of two linear equations graphically and algebraically. Students then use substitution to solve a system that is composed of a quadratic equation and a linear equation. In each case, they use graphs to determine the number of possible solutions to that type of system. Students practice solving systems of two equations in real-world and mathematical problems. Students then solve systems of three linear equations in three variables using substitution and Gaussian elimination, both in and out of context.	• A system of equations composed of two linear equations can have zero, one, or an infinite number of solutions. • A system of equations composed of one linear equation and one quadratic equation can have zero, one, or two solutions. • To solve a system of three linear equations in three variables, you can solve one equation for a variable, substitute that expression into the other two equations, and then solve the resulting system of two equations in two variables using any method. • *Gaussian elimination* is an algorithm that can be used to solve systems of three linear equations in three variables. It involves using linear combinations of the equations in the system to isolate one variable per equation.	**2A.3A** **2A.3B** 2A.3C 2A.3D	2
2	**Make the Best of It** Optimization	Students move from solving systems of equations to solving systems of inequalities. They model problems in context requiring several inequalities to be graphed on the same coordinate plane. Students recognize that the solution to a system of inequalities is the intersection of the solutions to each inequality. Then, through a context, they are introduced to linear programming as a process to determine the optimal solution to a system of linear inequalities. Students use linear programming to solve problems and explain the difference between the solution to a system of linear inequalities and the solution to an equation calculated by linear programming.	• The solution of a system of inequalities is the intersection of the solutions to each inequality. Every point in the intersection satisfies all inequalities in the system. • The optimal solution to a system of inequalities can be determined through a process called *linear programming*. • In linear programming, the vertices of the solution region of the system of linear inequalities are substituted into a given equation to determine the optimal solution. • Systems of equations can be used to model real-world problems.	**2A.3A** 2A.3E 2A.3F 2A.3G	2

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
3	**Systems Redux** Solving Matrix Equations	Students are introduced to identity and inverse matrices. They express a system of equations as a matrix equation. Students relate solving a matrix equation to solving a linear equation, and the use of technology to solve a matrix equation. As a culminating activity, they model a scenario with a system of equations, convert it to a matrix equation, solve the matrix equation using technology, and interpret the solution in terms of the scenario.	• The multiplicative *identity matrix*, I, is a square matrix such that for any matrix A, $A \bullet I = A$. • The *multiplicative inverse of a matrix* of a square matrix A is designated as A^{-1} and is a matrix such that $A \bullet A^{-1} = I$. • Matrices can be used to solve a system of equations in the form $Ax + By + Cz = D$ by writing the system as a *matrix equation* in the form $A \bullet X = B$, where A represents the *coefficient matrix*, X represents the *variable matrix*, and B represents the *constant matrix*. • Technology can be used to solve matrix equations.	**2A.3B**	2
4	**Putting the V in Absolute Value** Defining Absolute Value Functions and Transformations	Students are already familiar with the general shape of the graphs of absolute value functions, and they have studied transformations of linear functions. In this lesson, students experiment with the absolute value function family. They expand their understanding of transformations to include horizontal translations and interpret functions in the form $f(x) = A(B(x - C)) + D$. They distinguish between the effects of changing values inside the argument of the function (the *B*- and *C*-values) and changing values outside the function (the *A*- and *D*-values). At the end of the lesson, students summarize the impact of transformations on the domain and range of the absolute value function.	• An absolute value function is a function of the form $f(x) = \|x\|$. • A function $g(x)$ of the form $g(x) = f(x) + D$ is a vertical translation of the function $f(x)$. The value $\|D\|$ describes the number of units the graph of $f(x)$ is translated up or down. If $D > 0$, the graph is translated up; if $D < 0$, the graph is translated down. • A function $g(x)$ of the form $g(x) = Af(x)$ is a vertical dilation of the function $f(x)$. For $\|A\| > 1$, the graph is vertically stretched by a factor of $\|A\|$ units; for $0 < \|A\| < 1$, the graph vertically compresses by a factor of $\|A\|$ units. For $A < 0$, the graph also reflects across the *x*-axis. • A function $g(x)$ of the form $g(x) = f(x - C)$ is a horizontal translation of the function $f(x)$. The value $\|C\|$ describes the number of units the graph of $f(x)$ is translated right or left. If $C > 0$, the graph is translated to the right; if $C < 0$, the graph is translated to the left. • A function $g(x)$ of the form $g(x) = f(Bx)$ is a horizontal dilation of the function $f(x)$. For $\|B\| > 1$, the graph is horizontally compressed by a factor of $\frac{1}{\|B\|}$. For $0 < \|B\| < 1$, the graph will be horizontally stretched by a factor of $\frac{1}{\|B\|}$. For $B < 0$, the graph also reflects across the *y*-axis. • Transforming a function by changing the *A*- or *D*-values affects the output of the function, *y*. • Transforming a function by changing the *B*- or *C*-values affects the input of the function, *x*.	**2A.2A** 2A.6C 2A.7I	3

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
5	**Play Ball!** Absolute Value Equations and Inequalities	Students begin this lesson by graphing the solution sets of simple absolute value equations on number lines and writing simple absolute value equations given their number line graphs. They then investigate absolute value functions using a real-world context. First, students write an absolute value equation to represent the context and solve it graphically. They then learn through a Worked Example and student work how to solve absolute value equations and practice this skill. Students revisit the real-world context; however, this time they write an absolute value inequality and solve it graphically. Students are provided compound inequalities that are equivalent to absolute value inequalities and they use these relationships to solve and graph absolute value inequalities.	• *Linear absolute value equations* have 0, 1, or 2 solutions. The solution set for *linear absolute value inequalities* may contain all real numbers, a subset of the real numbers represented by a compound inequality, or no solutions. • Linear absolute value inequalities can be rewritten as equivalent compound inequalities. • Linear absolute value equations and inequalities can be used to represent real-world situations.	2A.6D **2A.6E** 2A.6F	2
End of Topic Assessment					1
Learning Individually with Skills Practice or MATHia *Schedule these days strategically throughout the topic to support student learning.*					5

Topic 2: Exploring and Analyzing Patterns

ELPS: 1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G

Topic Pacing: 16 days

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
1	**Patterns: They're Grrrrrowing!** Observing Patterns	Students investigate three scenarios that illustrate sequences. They analyze diagrams representing each sequence, describe observable patterns, sketch other terms or designs in the sequence, and then answer questions relevant to the problem situation. Tables and graphs are used to organize data and help recognize patterns as they emerge.	• Sequences are used to show observable patterns. • Patterns are used to solve problems. • Functions can be used to describe patterns.	**2A.8C**	1
2	**The Cat's Out of the Bag!** Generating Algebraic Expressions	This lesson revisits the three scenarios from the previous lesson. Students write equivalent algebraic expressions for each of the scenarios. They use algebraic properties and graphical representations to show that the expressions are equivalent. Students describe the similarities and differences among linear, exponential, and quadratic functions.	• Two or more algebraic expressions are equivalent if they produce the same output for all input values. • You can use the properties of a graph to prove two algebraic expressions are equivalent.	2A.5B 2A.8A **2A.8C**	2

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
3	**Samesies** Comparing Multiple Representations of Functions	Students compare the graphic, verbal, numeric, and algebraic representations of a function. They group equivalent representations of functions and then identify their function families. Students analyze a tile pattern and use a table to represent the sequence and recognize patterns. They then create expressions that represent different aspects of the design. Within this same context, students show that different expressions are algebraically equivalent.	• A *relation* is a mapping between a set of input values and a set of output values. • A *function* is a relation such that for each element of the domain there exists exactly one element in the range. • *Function notation* is a way to represent functions algebraically. The function $f(x)$ is read as "f of x" and indicates that x is the input and $f(x)$ is the output. • Tables, graphs, and equations are used to model function and non-function situations. • Equivalent expressions can be determined algebraically and graphically. • Graphing technology can be used to verify equivalent function representations.	2A.8A	**1**
		Mid-Topic Assessment			**0**
4	**True to Form** Forms of Quadratic Functions	Students match quadratic equations with their graphs using key characteristics. The standard form, the factored form, and the vertex form of a quadratic equation are reviewed as is the concavity of a parabola. Students then sort each of the equations with their graphs according to the form in which the equation is written, identifying key characteristics of each function. Next, students analyze graphs of parabolas on numberless axes and select possible functions that could model the graph. A Worked Example shows that a unique quadratic function is determined when the vertex and a point on the parabola are known, or the roots and a point on the parabola are known. Students are given information about a function and use it to determine the most efficient form to write the function. They then use the key characteristics of a graph and reference points to write a quadratic function, if possible. Finally, students analyze a Worked Example that demonstrates how to write and solve a system of equations to determine the unique quadratic function given three points on the graph. They then use this method to determine the quadratic function that models a problem situation and use it to answer a question about the situation.	• The *standard form of a quadratic function* is written as $f(x) = ax^2 + bx + c$, where a does not equal 0. • The *factored form of a quadratic function* is written as $f(x) = a(x - r_1)(x - r_2)$, where a does not equal 0. • The *vertex form of a quadratic function* is written as $f(x) = a(x - h)^2 + k$, where a does not equal 0. • The *concavity of a parabola* describes whether a parabola opens up or opens down. A parabola is concave down if it opens downward and is concave up if it opens upward. • A graphical method to determine a unique quadratic function involves using key points and the vertical distance between each point in comparison to the points on the basic function. • An algebraic method to determine a unique quadratic function involves writing and solving a system of equations, given three reference points.	**2A.3A** **2A.3B** 2A.4A 2A.4D 2A.7B	2

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
5	**The Root of the Problem** Solving Quadratic Equations	Students solve quadratic equations of the form $y = ax^2 + bx + c$. They first factor trinomials and use the Zero Product Property. Students then complete the square to determine the roots of a quadratic equation that cannot be factored. Finally, students use the Quadratic Formula to solve problems in and out of context.	• One method of solving quadratic equations in the form $0 = ax^2 + bx + c$ is to factor the trinomial expression and use the Zero Product Property. • When a quadratic equation in the form $0 = ax^2 + bx + c$ is not factorable, completing the square is an alternative method of solving the equation. • The *Quadratic Formula*, $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$, can be used to solve any quadratic equation written in general form, $0 = ax^2 + bx + c$, where a, b, and c represent real numbers and $a \neq 0$. • A system of equations containing two quadratic equations can be solved algebraically and graphically. • The Quadratic Formula, substitution, and factoring are used to algebraically solve systems of equations.	**2A.4F**	2
6	***i* Want to Believe** Imaginary and Complex Numbers	Students are introduced to imaginary numbers to calculate the square root of a negative number, and imaginary numbers are placed within the complex number system. They apply the Commutative Property, the Associative Property, and the Distributive Property to add, subtract, and multiply complex numbers. Students use the structure of quadratic equations in the form $y = ax^2 + c$, in vertex form, and in standard form, as well as the discriminant and the graph to determine whether the roots of an equation are real or imaginary. They solve quadratic equations that have imaginary roots. They apply the Fundamental Theorem of Algebra to make sense of the fact that a quadratic equation can have two unique real number solutions, two equal real number solutions, or two imaginary solutions.	• Equations with no solution in one number system may have solutions in a larger number system. • *The number i* is a number such that $i^2 = -1$. • The set of complex numbers is the set of all numbers written in the form $a + bi$, where a and b are real numbers and b is not equal to 0. • The Commutative Property, the Associative Property, and Distributive Properties apply to complex numbers. • Functions that do not intersect the x-axis have *imaginary zeros*. • When the discriminant of a quadratic equation is a negative number, the equation has two *imaginary roots*. • The *Fundamental Theorem of Algebra* states that any polynomial equation of degree n must have n complex roots or solutions.	**2A.4F** 2A.7A	2
		End of Topic Assessment			1
		Learning Individually with Skills Practice or MATHia *Schedule these days strategically throughout the topic to support student learning.*			5

Topic 3: Applications of Quadratics

ELPS: 1.A, 1.C, 1.D, 1.E, 1.F, 1.G, 2.C, 2.D, 2.E, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.E, 3.F, 4.A, 4.B, 4.C, 4.G, 4.K, 5.B, 5.E, 5.F, 5.G

Topic Pacing: 15 days

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
1	**Ahead of the Curve** Solving Quadratic Inequalities	Students analyze a Worked Example to calculate the solution set of a quadratic inequality by first solving for the roots of its related quadratic equation, then determining which interval(s) created by the roots satisfy the inequality. They use both a number line and coordinate plane to select the correct intervals and then make connections between those methods. Students solve a problem in context requiring the use of a quadratic inequality, and also use a transformation to make comparisons within a context. Throughout this lesson, students use the Quadratic Formula, technology, and inequality or interval notation.	• A horizontal line drawn across the graph of a quadratic function intersects the parabola at exactly two points, except at the vertex, where it intersects the parabola at exactly one point. • The *solution set of a quadratic* inequality is determined by first solving for the roots of the quadratic equation, and then determining which interval(s) created by the roots will satisfy the inequality. A combination of algebraic and graphical methods may be the most efficient solution method. • Quadratic inequalities can be used to model some real-world contexts. The effects of translations of quadratic functions can be used to make comparisons within a context.	2A.4H	1
2	**All Systems Go!** Systems of Quadratic Equations	Students solve a problem in context that can be modeled by a system of equations involving a linear equation and a quadratic equation. They solve this first question graphically and discuss the number of solutions to the system and the number of solutions that make sense for the context. Students are then guided to solve a system of a linear equation and a quadratic equation algebraically, and then verify their results graphically. Students solve additional systems algebraically and graphically. They also discuss the number of possible solutions for each type of system and sketch graphs demonstrating those solutions.	• Systems of equations involving a linear equation and a quadratic equation can be solved both algebraically and graphically. • A system of equations containing a linear equation and a quadratic equation may have no solution, one solution, or two solutions. • The number of solutions for a system of equations depends on the number of points where the graphs of the two equations intersect. • A system of equations involving a linear equation and a quadratic equation may be used to model real-world problems.	**2A.3A** 2A.3C 2A.3D	1

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
3	**The Ol' Switcharoo** Inverses of Linear and Quadratic Functions	Students are introduced to the *inverse of a function*. A Worked Example demonstrates how to determine the inverse of a linear function algebraically. Students use this example to determine other inverses of functions. They then create the graph of the inverse of a linear function by reflecting the original function across the line $y = x$ using patty paper. This process is repeated for quadratic functions. The term *one-to-one function* is defined, and students determine whether the inverse of a function is also a function. A graphic organizer is completed to summarize the definition and representations of inverse functions.	• Inverses of functions can be determined algebraically and graphically. • The *inverse of a function* is determined by replacing $f(x)$ with y, switching the x and y variables, and solving for y. • The graph of the inverse of a function is a reflection of that function across the line $y = x$. • To *restrict the domain* of a function means to define a new domain for the function that is a subset of the original domain. • A *one-to-one function* is a function in which its inverse is also a function. • For a one-to-one function $f(x)$, the notation for its inverse is $f^{-1}(x)$.	2A.2B **2A.2C**	3
Mid-Topic Assessment					1
4	**Modeling Behavior** Using Quadratic Functions to Model Data	Students begin the lesson by determining a quadratic regression equation to model a set of data and use the regression equation to make predictions. Next, they are given a quadratic equation that models a context, but this time, students see the need for an inverse equation because they must solve for the independent variable when the dependent variable is provided. Throughout the lesson, students identify the independent and dependent quantities and domain and range of functions in order to make sense of an inverse of function.	• Some data in context can be modeled by a quadratic regression equation. The regression equation can be used to make predictions; however, there may be limitations on the domain depending on the context. • When a problem requires using a given function to determine the independent quantity when a dependent quantity is given, determining the inverse of the original function may be a more efficient way to handle the situation.	**2A.2C** 2A.4E 2A.7I 2A.8A 2A.8B **2A.8C**	1

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
5	**Going the Equidistance** Equation of a Parabola	The focus and directrix of a parabola are introduced through an exploratory activity. Students use concentric circles to plot points that are equidistant from both a line and a point not on the line, then connect these equidistant points to form a parabola. A parabola is described as a conic section, and the terms *locus of points, parabola, focus*, and *directrix* are given. Students construct a directrix and a focus above the directrix on patty paper and complete multiple folds of the focus onto the line to create a parabola. The *concavity* and *vertex of a parabola* are defined. Through investigations, students conclude that any point on a parabola is equidistant from the focus and the directrix. The focus and directrix are then used to write the equation of a parabola, and the general and standard form of a parabola are given. Students derive the standard form of a parabola algebraically to make sense of the constant p in the equation and use this constant to graph parabolas. The Distance Formula is used to determine the equation of points that are equidistant from a given focus and a given directrix, where the vertex is a point other than the origin. Students apply characteristics of parabolas to solve real-world problem situations.	• A parabola is the *locus of points* in a plane that are equidistant from a fixed point (the *focus*) and a fixed line (the *directrix*). • The focus and directrix of a parabola can be used to derive the equation of the parabola. • Parabolas can be described by their *concavity*. • The standard form for the equation of a parabola with vertex at the origin can be written in the form $x^2 = 4py$ (symmetric with respect to the y-axis) or $y^2 = 4px$ (symmetric with respect to the x-axis), where p is the distance from the vertex to the focus. • The standard form for the equation of a parabola with vertex at the origin, $x^2 = 4py$ or $y^2 = 4px$, can be derived using the Distance Formula and the definitions of focus, directrix, and parabola. • In the standard form for the equation of a parabola centered at the origin, $x^2 = 4py$ or $y^2 = 4px$, the value of p is positive when the parabola is concave up or concave right and the value of p is negative when the parabola is concave down or concave left. • The standard forms of parabolas with vertex (h, k) are $(x - h)^2 = 4p(y - k)$ and $(y - k)^2 = 4p(x - h)$. • The characteristics of parabolas can be used to solve real-world problems.	**2A.4B**	3
End of Topic Assessment					1
Learning Individually with Skills Practice or MATHia *Schedule these days strategically throughout the topic to support student learning.*					4

2 Analyzing Structure

Module Pacing: 20 Days

Topic 1: Composing and Decomposing Functions

ELPS: 1.A, 1.C, 1.D, 1.E, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.F, 4.K, 5.E

Topic Pacing: 10 days

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
1	**Blame It on the Rain** Modeling with Functions	Given a real-world situation, students analyze different lengths and widths of a cross-sectional area to determine the dimensions of the maximum area. Students create tables of values, equations, and graphs to represent each situation. They then identify the function that represents the cross-sectional area of the drain as quadratic and the two factors that represent the length and width of the drain as linear. Students interpret the intercepts and axis of symmetry of the graph in terms of the problem situation. The Modeling Process is defined, and students describe how they used these steps in modeling the drain problem.	• Tables, graphs, and equations can be used to model real-world situations. • A function created by the product of two linear factors is a quadratic function. • The steps of the modeling process are Notice and Wonder, Organize and Mathematize, Predict and Analyze, and Test and Interpret.	2A.8A	**0**
2	**Folds, Turns, and Zeros** Transforming Function Shapes	Students consider the effect of transforming functions by non-constant factors. First, they translate a constant function by a factor of *x*, identifying key characteristics of the transformed function. Students then dilate linear functions by non-constant factors to create quadratic functions. Again, students analyze the new zeros created by each transformation and observe how the factors affect the intervals of increase and decrease of the transformed function. Students repeat this analysis by dilating a degree-2 function to create a degree-3 function. Finally, they summarize what they have observed regarding dilations, linear factors, zeros, and the behavior of graphs at zeros.	• Functions can be translated and dilated by non-constant values, which apply a different transformation to each point of the function. • The linear factors of a function indicate the locations of the zeros of the function composed of those functions. • When a linear function is dilated vertically by multiplying the function by another linear function, the resulting function is a degree-2 function. • When a quadratic function is dilated vertically by multiplying the function by a linear function, the resulting function is a degree-3 function. • The graph of a function behaves differently at zeros described by linear factors and factors of degree 2. • The linear factors of a function can be used to sketch the graph of a function.	**2A.2A**	2

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
3	**Planting the Seeds** Exploring Cubic Functions	Students investigate the dimensions of a rectangular sheet of copper that is used to create planters when squares are removed from each corner and the sides are then folded upward. They analyze several sizes of planters and calculate the volume of each size. Students then write a volume function in terms of the height, length, and width and graph the function using technology. Using key characteristics, including relative maximum or minimum values, students analyze the graph and use a horizontal line to solve for specific values. Students differentiate the domain and range of the problem situation from the domain and range of the cubic function. Students then write and analyze the volume formula for a cylindrical planter. They compare a cubic function composed of three linear factors to a cubic function composed of a linear factor and a quadratic factor.	• *Cubic functions* can be used to model real-world contexts such as volume. • The general form of a cubic function is written as $f(x) = ax^3 + bx^2 + c + d$, where $a \neq 0$. • A *relative maximum* is the highest point in a particular section of a graph, while a *relative minimum* is the lowest point in a particular section of a graph. • A cubic function may be created by the product of three linear functions or the product of a quadratic function and a linear function.	**2A.2A** 2A.7I	2
4	**The Zero's the Hero** Decomposing Cubic Functions	Students investigate the multiplicity of the zeros of a polynomial function. They use these zeros, with multiplicity, to show the decompositions of quadratic and cubic functions into their linear and quadratic factors and reconstruct the product functions using these factors. Students review multiplying binomials in order to build polynomial expressions algebraically as well as graphically. They compare degree-1, degree-2, and degree-3 polynomials.	• The Fundamental Theorem of Algebra states that a degree n polynomial has, counted with *multiplicity*, exactly n zeros. • The Zero Product Property states that if the product of two or more factors is equal to zero, then at least one factor must be equal to zero. • The graph of a function written in factored form and the graph of a function written in general form is the same graph when the functions are equivalent. • Graphing is a strategy used to determine whether functions are equivalent. • The product of three linear functions is a cubic function, and the product of a quadratic function and a linear function is a cubic function. • Quadratic and cubic functions can be decomposed and analyzed in terms of their zeros.	**2A.2A** 2A.7B	2
End of Topic Assessment					1
Learning Individually with Skills Practice or MATHia *Schedule these days strategically throughout the topic to support student learning.*					3

Topic 2: Characteristics of Polynomial Functions

ELPS: 1.A, 1.C, 1.D, 1.E, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.F, 4.K, 5.E

Topic Pacing: 10 days

Some of the content of this topic goes beyond the scope of the course standards. The content is included to enhance students' understanding of mathematics and to provide opportunities for extension.

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
1	**Odds and Evens** Power Functions	Students investigate power functions described by the equation $P(x) = ax^n$. They sketch the graphs of power functions with negative coefficients. Students then explore even and odd functions and determine graphically whether given polynomial functions are even, odd, or neither. They use transformations involving the *A*- and *B*-values to see that even functions are symmetric about the *y*-axis, thus satisfy $f(x) = f(-x)$, and odd functions are symmetric about the origin, thus satisfy $f(x) = -f(-x)$. The content of this lesson goes beyond the scope of the grade level standards. The content is included to enhance students' understanding of mathematics. This lesson is optional.	• A *power function* is a function of the form $P(x) = ax^n$, where n is a non-negative integer. • If a graph is *symmetric about a line*, the line divides the graph into two identical parts. • A function is *symmetric about a point* if each point on the graph has a point the same distance from the central point, but in the opposite direction. • When a point of symmetry is the origin, the graph is reflected across the *x*-axis and the *y*-axis. If (x, y) is replaced with $(-x, -y)$, the function remains the same. • The graph of an *even function* is symmetric about the *y*-axis, thus $f(x) = f(-x)$. • The graph of an *odd function* is symmetric about the origin, thus $f(x) = -f(-x)$.	P.2D	**0**
2	**Math Class Makeover** Transformations of Polynomial Functions	Students recall the transformational function form $g(x) = Af(B(x - C)) + D$ and use it to graph polynomial functions, write equations for transformed functions, and explain the effects of transformations on given functions. The general form of a polynomial function is given. Tables are used to organize the effects of transformations on the basic cubic functions as well as simple polynomial functions. Graphs of functions that have undergone multiple transformations are given, and students write the appropriate equation to describe each graph.	• The function $g(x) = Af(B(x - C)) + D$ is the transformation function form, where the constants *A* and *D* affect the output values of the function and the constants *B* and *C* affect the input values of the function. • The graph of an even function is symmetric about the *y*-axis. • The graph of an odd function is symmetric about the origin. • Some transformations affect the symmetry of the *polynomial function*.	**2A.2A** 2A.6A	2

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
3	**Poly-Frog** Key Characteristics of Polynomial Functions	Students investigate the key characteristics of a polynomial function. They analyze a data set in context, its quartic regression equation, and graph. Students then investigate the relationship between the power of a polynomial function and its total number of zeros; they represent the number of real zeros, imaginary zeros, and zeros with multiplicity graphically. Student then consider various equations, characteristics, and graphs to solidify their understanding of polynomial functions.	• An *n*th-degree odd polynomial has zero or an even number of *extrema*. An *n*th-degree even polynomial has an odd number of extrema. In either case, the maximum number of extrema is $n - 1$. • A polynomial function changes direction at each of its extrema. For that reason, the number of extrema and the number of changes of direction in the graph of the function are equal. • A polynomial with an even power has an even number of intervals of increase or decrease. A polynomial with an odd power has an odd number of intervals of increase or decrease. • The combination of real and imaginary roots of a polynomial function are equal to the degree of the polynomial and can be used to help determine the shape of its graph.	**2A.2A** 2A.7I	2
4	**Build-a-Function** Building Cubic Functions	Students begin by selecting a set of linear and quadratic functions whose product builds a cubic function with specified key characteristics. Students reason that a cubic function may have 0 or 2 imaginary zeros and that an infinite number of cubic functions can be written from a given set of zeros. They build a polynomial function given a set of zeros and given a graph, describing the characteristics of the function and comparing both processes.	• Cubic functions can be the product of three linear functions or the product of a quadratic function and a linear function. • A cubic function may have 0 or 2 imaginary zeros. • An infinite number of functions can be written from a given set of zeros. • A unique function can be written from the graph of a function. • Functions of degree *n* are composed of factors whose degree sum to *n*. • A polynomial function may have a combination of real and imaginary zeros.	**2A.2A** 2A.6A	1
5	**Leveled Up** Analyzing Polynomial Functions	A cubic function is used to model the profit of a business over a period of time. Students analyze the key characteristics of a sketch of the graph to answer questions relevant to the problem situation. They then use a graph of the same function to estimate when the company would achieve different profit levels. Next, the *average rate of change* of a function is defined, and a Worked Example demonstrates how to calculate the average rate of change for a specified time interval. Students practice calculating the average rate of change over different time intervals. Finally, they write a scenario to match the graph of a new polynomial, taking into account the key characteristics.	• The *average rate of change* of a function is the ratio of the change in the dependent variable to the change in the independent variable over a specified interval. • The formula for average rate of change is $\frac{f(b) - f(a)}{b - a}$ for an interval (a, b). The expression $b - a$ represents the change in the input of the function f. The expression $f(b) - f(a)$ represents the change in the function f as the input changes from a to b.	**2A.2A**	1
End of Topic Assessment					1
Learning Individually with Skills Practice or MATHia *Schedule these days strategically throughout the topic to support student learning.*					3

3 Developing Structural Similarities

Module Pacing: 15 Days

Topic 1: Relating Factors and Zeros

ELPS: 1.A, 1.D, 1.E, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.K, 5.B, 5.E

Topic Pacing: 9 days

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
1	**Satisfactory Factoring** Factoring Polynomials to Identify Zeros	Students investigate methods to factor polynomial expressions, such as factoring out the greatest common factor, chunking, grouping, and using quadratic form. They also review how to factor perfect square trinomials and the difference of two squares. Students analyze Worked Examples and student work that demonstrate the methods. They factor polynomials over the set of real numbers and over the set of complex numbers. Students then use the factors to determine the zeros to sketch the functions.	• The graphs of all polynomials that have a monomial GCF that includes a variable will pass through the origin. • Analyzing the structure of a polynomial may help you determine which factoring method may be most helpful. • Chunking is a method of factoring a polynomial in quadratic form that does not have common factors in all terms. Using this method, the terms are rewritten as a product of two terms, the common term is substituted with a variable, and then it is factored as is any polynomial in quadratic form. • Factoring a perfect square trinomial can occur in two forms: $a^2 - 2ab + b^2 = (a + b)^2$ or $a^2 + 2ab + b^2 = (a + b)^2$. • Factoring by grouping is a method of factoring a polynomial that has four terms in which not all terms have a common factor. The terms can be first grouped together in pairs that have a common factor, and then factored again. • Factoring by using quadratic form is a method of factoring a polynomial of degree 4 of the form, $ax^4 + bx^2 + c$. • Factoring the difference of squares is in the form: $a^2 - b^2 = (a + b)(a - b)$.	**2A.2A** 2A.7D **2A.7E**	3

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
2	**Conquer Division** Polynomial Division	Students analyze the graph of a cubic function that appears to have one real zero. The Factor Theorem is stated and a Worked Example demonstrates how to determine whether a linear expression is a factor of the cubic function. Polynomial long division is introduced and a Worked Example is provided. They perform polynomial long division to determine the quadratic function that is the other factor, and use this information to determine the imaginary zeros and rewrite the cubic function as a product of linear factors. Students factor binomials using the difference of squares. A Worked Example demonstrates how to determine the factor formula for the difference of cubes, and they then determine the factor formula for the sum of cubes. The Remainder Theorem is stated and students use the theorem to answer questions involving polynomial division with remainders. They analyze a Worked Example to determine unknown coefficients in polynomial functions by applying the Factor Theorem. Finally, a Worked Example of synthetic division is provided and compared to polynomial long division. Students use the algorithm to determine the quotient in several problems.	• Factors of polynomials divide into a polynomial without a remainder. • A polynomial equation of degree n has n roots over the complex number system and can be written as the product of n factors of the form $(ax + b)$. • *Polynomial long division* is an algorithm for dividing one polynomial by another of equal or lesser degree. • *Synthetic division* is a shortcut method for dividing a polynomial by a linear expression of the form $(x - r)$. • The *Factor Theorem* states that a polynomial function $p(x)$ has $x - r$ as a factor if and only if the value of the function at r is 0, or $p(r) = 0$. • The *Remainder Theorem* states that when any polynomial equation or function $f(x)$ is divided by a linear expression of the form $(x - r)$, the remainder is $R = f(r)$ or the value of the function when $x = r$. • The difference of cubes can be written in factored form as $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$. The sum of cubes can be written in factored form as: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$.	2A.7C 2A.7D **2A.7E**	3
3	**Closing Time** The Closure Property	Students review the four basic operations over the set of natural numbers, whole numbers, integers, rational numbers, and irrational numbers and determine which operations are closed and not closed over which sets of numbers. They determine that integers and polynomials are not closed under division. The concept of polynomials being closed under an operation is defined, and students prove that polynomials are closed under subtraction, addition, and multiplication. They compare polynomials and use multiple representations to analyze and compare polynomial functions.	• When an operation is performed on any number or expression in a set and the result is in the same set, it is said to be closed under that operation. • Polynomials are closed under addition, subtraction, and multiplication. • Polynomials are not closed under division.	2A.7B 2A.7C	**0**
End of Topic Assessment					1
Learning Individually with Skills Practice or MATHia *Schedule these days strategically throughout the topic to support student learning.*					2

Topic 2: Polynomial Models

ELPS: 1.A, 1.D, 1.E, 1.F, 1.G, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.G, 4.K, 5.E

Topic Pacing: 6 days

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
1	**Not a Case of Mistaken Identity** Exploring Polynomial Identities	Students identify polynomial identities and then use them to perform calculations involving large numbers without a calculator. Euclid's Formula is stated and used to generate Pythagorean triples. Students then verify more complex polynomial identities using algebraic properties. Finally, they revisit Pythagorean triples and recognize that Euclid's Formula cannot generate every Pythagorean triple.	• Polynomial identities such as $(a + b)^2 = a^2 + 2ab + b^2$ can be used to help perform calculations with large numbers. • *Euclid's Formula* can be used to generate Pythagorean triples given positive integers r and s, where $r > s$: $(r^2 + s^2)^2 = (r^2 - s^2)^2 + (2rs)^2$.	2A.7B	**1**
2	**Elegant Simplicity** Pascal's Triangle and the Binomial Theorem	Students analyze Pascal's Triangle and then use observable patterns to create additional rows. They explore a use of Pascal's Triangle when raising a binomial to a positive integer and use it to expand several binomials. The combination formula is given and using technology and Pascal's Triangle, they calculate combinations. The Binomial Theorem is provided and students use it to expand binomials with coefficients equal to 1 and with coefficients other than 1. Students reason about the number of terms in the expanded form of a binomial raised to a positive integer for $(x + y)^n$ and $(x - y)^n$ and answer questions related to the signs of the terms in each expanded form as well as the number of distinct coefficients for the terms. Students use the Binomial Theorem to determine specific terms for binomials written in the form $(x + y)^n$ and $(x - y)^n$ with the same value for n.	• The *Binomial Theorem* states that it is possible to extend any power of $(a + b)$ into a sum of the form: $(a + b)^n = \binom{n}{0}a^nb^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n-1}a^1b^{n-1} + \binom{n}{n}a^0b^n$. • The formula for a combination of k objects from a set of n objects for $n \geq k$ is: $\binom{n}{k} = {}_nC_k = \frac{n!}{k!\,(n-k)!}$.	2A.7B	1
3	**Modeling Gig** Modeling with Polynomial Functions and Data	Students use technology to determine the best polynomial regression equation for various real-world situations—traffic patterns in a downtown area, the federal minimum wage, monthly precipitation, and inflation. They analyze the coefficient of determination to decide which regression equation best describes the data. Students then use regression equations to make predictions and answer questions related to each scenario.	• A *regression equation* is a function that models the relationship between two variables in a scatterplot. • The *coefficient of determination*, or r^2, measures the strength of the relationship between the original data and their regression equation. The value ranges from 0 to 1 with a value of 1 indicating a perfect fit between the regression equation and the original data. • Regression equations can be used to make predictions about future events.	2A.4E 2A.8A 2A.8B **2A.8C**	**1**
End of Topic Assessment					1
Learning Individually with Skills Practice or MATHia *Schedule these days strategically throughout the topic to support student learning.*					2

4 Extending Beyond Polynomials

Module Pacing: 32 Days

Topic 1: Rational Functions

ELPS: 1.A, 1.D, 1.E, 1.G, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.G, 4.K, 5.E

Topic Pacing: 16 days

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
1	**Can't Touch This** Introduction to Rational Functions	Students investigate rational functions. Given the function $g(x) = \frac{1}{x}$, they complete a table of values, graph the function, and analyze its graph. Students are introduced to vertical asymptotes and how they relate to the denominator of the function. Students then identify rational functions from a list of functions and determine which of the functions will have a vertical asymptote. They also solve a real-world problem using a rational function.	• A *rational function* is any function that can be written as the ratio of two polynomials. It can be written in the form $f(x) = \frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomial functions, and $Q(x) \neq 0$. • A *vertical asymptote* is a vertical line that a function gets closer and closer to, but never intersects. The function $g(x) = \frac{1}{x}$ has a vertical asymptote at $x = 0$, a horizontal asymptote at $y = 0$, and a domain of all real numbers except $x \neq 0$.	**2A.2A** 2A.6H 2A.6K **2A.6L** 2A.7I	2
2	**Sooooo . . . Close** Transformations of Rational Functions	Students analyze transformations of rational functions in which the numerator is a constant value. They explore the effects a negative *A*-value, a *C*-value, and a *D*-value have on rational functions as they match graphs and their equations. Students sketch graphs of rational functions given equations or key characteristics, and relate the *A*-, *B*-, *C*-, and *D*-value to the transformations of their graphs, the vertical and horizontal asymptotes, and the domain and range of the function.	• Translations of a rational function $f(x)$ are given in the form $g(x) = Af(B(x - C)) + D$, where a negative *A*-value reflects $f(x)$ vertically, the *D*-value translates $f(x)$ vertically, and a *C*-value translates $f(x)$ horizontally. • The *A*-value vertically dilates the function. This does not affect the domain or range of a rational function. • The *B*-value horizontially dilates the function. This does not affect the domain or range of a rational function. • The *C*-value affects the vertical asymptote. The vertical asymptote affects the domain. • The *D*-value affects the horizontal asymptote. The horizontal asymptote affects the range. • Vertical asymptotes of a rational function can be determined by identifying values of *x* for which the denominator equals 0.	2A.6G 2A.6K	1
Mid-Topic Assessment					1

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
3	**Must Be a Rational Explanation** Operations with Rational Expressions	Students perform the basic operations of addition, subtraction, multiplication, and division on rational expressions. They analyze examples that show similarities between the process of operating with rational numbers to operating with rational expressions. When adding and subtracting rational expressions, an emphasis is placed on determining a common denominator and using patterns to rewrite the rational expressions with the common denominator. Next, students multiply rational expressions with an emphasis to divide out common factors as a first step, and then divide rational expressions. In all cases, students identify any restrictions on the domain.	• The processes of adding, subtracting, multiplying, and dividing rational expressions are similar to the processes for rational numbers. • To determine the least common denominator of algebraic expressions, first factor the expressions and divide out common factors. • The domain restrictions for a rational expression must be based upon the original expressions. • Rational expressions are closed under the operations of addition, subtraction, multiplication, and division.	2A.7C **2A.7E** **2A.7F**	3
4	**Thunder. Thun- Thun-Thunder.** Solving Problems with Rational Equations	Students use rational functions to solve real-world problems. They analyze different methods to solve rational equations. Students sort rational equations according to their structure and identify the solution method they plan to use based on the structure of the rational equation.	• Rational functions can be used to model real-world problems. • A *rational equation* is an equation that contains one or more rational expressions. Rational equations can be proportions. • The structure of an equation often determines the most efficient method to solve the equation.	2A.6H **2A.6I** 2A.6J	2
5	**16 Tons and What Do You Get?** Solving Work, Mixture, Distance, and Cost Problems	Students use rational equations to model and solve work problems, mixture problems, distance problems, and cost problems.	• Rational functions can be used to model real-world problems. • A work problem is a type of problem that involves the rates of several workers and the time it takes to complete a job. • A mixture problem is a type of problem that involves the combination of two or more liquids and the concentrations of those liquids. • A distance problem is a type of problem that involves distance, rate, and time. • A cost problem is a type of problem that involves the cost of ownership of an item over time.	2A.6H **2A.6I**	2
End of Topic Assessment					1
Learning Individually with Skills Practice or MATHia *Schedule these days strategically throughout the topic to support student learning.*					4

Topic 2: Radical Functions

ELPS: 1.A, 1.C, 1.D, 1.E, 1.G, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.D, 4.G, 4.K, 5.E

Topic Pacing: 16 days

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
1	**Strike That, Invert It** Inverses of Power Functions	Students use patty paper to explore the effects of switching the x-axis and y-axis of graphs of power functions. They determine that the resulting graph represents the inverse of the original graph, and then identify whether the inverse graph is also a function. Students are then introduced to the term *invertible function* and the notation $f^{1}(x)$ to represent the inverse function of $f(x)$. They explain why the Horizontal Line Test can be used to determine whether a function is invertible.	• A function is the set of all ordered pairs (x, y), or $(x, f(x))$, where for every value of x there is one and only one value of y, or $f(x)$. • The *inverse of a function* is the set of all ordered pairs (y, x), or $(f(x), x)$. • If the inverse of a function is also a function, the function is said to be an *invertible function*, and its inverse is written as $f^{-1}(x)$. • A *Horizontal Line Test* is a visual method to determine whether a function has an inverse that is also a function.	**2A.2A** 2A.2B **2A.2C**	1
2	**Such a Rad Lesson** Radical Functions	Students continue to investigate power functions and their inverses. Through a Worked Example, they learn to determine the inverse of a function algebraically. The terms *square root function* and *cube root function* are defined. Through investigations of $f(x) = x^2$ and $f(x) = x^3$, students realize that the inverses of odd-degree power functions are functions, while even-degree power functions need restricted domains for their inverses to be functions. They relate key characteristics of functions and their inverses and are introduced to the general term *radical function*. Next, students use a composition of functions to determine whether two given functions are inverses. Finally, they solve problems in context using radical functions.	• The *square root function* is the inverse of the power function $f(x) = x^2$, when the domain of the power function is restricted to values greater than or equal to 0. • The *cube root function* is the inverse of the power function $f(x) = x^3$. • *Radical functions* are inverses of power functions with exponents greater than or equal to 2. • For two functions f and g, the *composition of functions* uses the output of one as the input of the other. It is expressed as $f(g(x))$ or $g(f(x))$. If $f(g(x)) = g(f(x)) = x$, then $f(x)$ and $g(x)$ are inverse functions. • Radical functions may be used to model and solve real-world problems.	**2A.2A** 2A.2B **2A.2C** 2A.2D 2A.7I	3
Mid-Topic Assessment					1
3	**Making Waves** Transformations of Radical Functions	Students use the context of building a logo to investigate the effects of transforming the square root function and the cube root function. They write equations of functions and graph them. Students also identify the domain of functions and compare their key characteristics. After completing a structured activity involving various transformations, students create their own logo design with corresponding equations and their domains.	• Transformations of radical functions can be described by the transformation function form, $g(x) = Af(B(x - C)) + D$. • Transformations can be used to model graphic designs.	**2A.2A** **2A.4C** 2A.6A 2A.7I	**1**

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
4	**Keepin' It Real** Rewriting Radical Expressions	Students analyze tables and graphs for different values of n in the expression $\sqrt[n]{x^n}$. They then identify the function family associated with each graph and write the corresponding equation. Students conclude the expression $\sqrt[n]{x^n}$ can be written as $\lvert x \rvert$ when n is even or as x when n is odd when extracting a variable from a radical. Next, they rewrite radical expressions of the form $\sqrt[n]{x^a}$ as powers with rational exponents in the form $x^{\frac{a}{n}}$. Students learn to rewrite radical expressions by extracting roots and performing operations; they then use these skills to perform operations on radical expressions. Finally, students complete a graphic organizer by writing pairs of radicals whose sum, difference, product, and quotient are each equivalent to a given expression.	• To extract a variable from a radical, the expression $\sqrt[n]{x^n}$ can be written as $\lvert x \rvert$, when n is even, or x when n is odd. • A radical expression $\sqrt[n]{x^a}$ can be rewritten as an exponential expression $x^{\frac{a}{n}}$ for all real values of x if the index is n is odd, and for all real values of x greater than or equal to 0 if the index n is even. • The root of a product is equal to the product of its roots, $\sqrt[p]{a^m b^n} = \sqrt[p]{a^m} \cdot \sqrt[p]{b^n}$. • The root of a quotient is equal to the quotient of its roots, $\sqrt[p]{\frac{a^m}{b^n}} = \frac{\sqrt[p]{a^m}}{\sqrt[p]{b^n}}$. • When multiplying or dividing radical expressions, multiply or divide numbers and variables separately and then extract roots. • Radical expressions with the same degree and same radicand are like terms. • When adding or subtracting radical expressions, add or subtract the coefficients of like terms.	2A.7G	2
5	**Into the Unknown** Solving Radical Equations	Students analyze examples to determine strategies to solve radical equations. They consider the possibility of extraneous solutions that may result from raising both sides of an equation to a power during the solution process. Students solve several radical equations in both real-world and mathematical problems.	• Strategies to solve equations, such as using the Properties of Equality and isolating the term containing the unknown, can be applied to solve radical equations. • Raising both sides of the equation to a power when solving a radical equation may introduce *extraneous solutions*. To identify extraneous solutions, you must substitute each solution into the original equation to determine whether it results in a true statement. • Radical equations can be used to model real-world problems.	**2A.4F** 2A.4G 2A.6B **2A.7H**	2
End of Topic Assessment					1
Learning Individually with Skills Practice or MATHia *Schedule these days strategically throughout the topic to support student learning.*					5

5 Inverting Functions

Module Pacing: 35 Days

Topic 1: Exponential and Logarithmic Functions

ELPS: 1.A, 1.B, 1.D, 1.E, 1.G, 1.H, 2.A, 2.C, 2.D, 2.F, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.E, 3.F, 3.H, 3.J, 4.A, 4.B, 4.C, 4.G, 4.H, 4.I, 4.K, 5.A, 5.C, 5.D, 5.E

Topic Pacing: 20 days

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
1	**Half-Life** Comparing Linear and Exponential Functions	Students use exponential functions to model real-world situations. They use tables and graphs to compare linear and exponential growth. Students analyze a Worked Example that demonstrates how to rewrite a geometric sequence with function notation. They also write functions, analyze how altering a context affects the function, and use functions to respond to questions in context.	• A geometric sequence with a positive common ratio that is not 1 can be written as an exponential function using the properties of powers. • Over time, an exponential function with a *b*-value greater than 1 always exceeds a linear function with an *m*-value greater than zero. • A *half-life* refers to the amount of time it takes a substance to decay to half of its original amount.	2A.5B	2
2	**Pert and Nert** Properties of Exponential Graphs	Students match exponential equations and their graphs, sort them by whether they are increasing or decreasing, and then compare the key characteristics of increasing and decreasing exponential functions. They use the compound interest formula to derive the value of the constant *e*. Students then apply the constant *e* in the formula for interest compounded continuously and population growth.	• For basic exponential growth functions, $f(x) = b^x$, b is a value greater than 1. For basic exponential decay functions, $f(x) = b^x$, b is a value between 0 and 1. • The compound interest formula is $A = P \cdot \left(1 + \frac{r}{k}\right)^{kt}$, where A represents the value, P represents the principal amount, r represents the interest rate, and k represents the frequency of compounding in time t. • The *natural base* $e \approx 2.7182818\ldots$ is an irrational number, also known as Euler's number. • The formula for compound interest with continuous compounding is $A = Pe^{rt}$, where P represents the principal, r represents the interest rate, and t represents time in years. • The formula for population growth is $N(t) = N_0e^{rt}$, where N_0 represents the initial population, r represents the rate of growth, and t represents time in years.	**2A.2A** 2A.5B	2

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
3	**Return of the Inverse** Logarithmic Functions	Students are introduced to logarithms and rewrite exponential equations as logarithmic equations. Students then explore logarithmic functions as inverses of exponential functions and compare their key characteristics. They address the special cases of the common logarithm and the natural logarithm. Students solve problems that can be modeled by a logarithmic function. They also explore how the graph of an exponential function appears when a logarithmic scale is applied to the *y*-axis.	• The exponential equation $y = b^x$ can be written as the logarithmic equation $x = \log_b y$. • All exponential functions are invertible. The inverse of an exponential function is a *logarithmic function*. • A *common logarithm* is a logarithm with base 10 and is usually written as "log x" without a base specified. • A *natural logarithm* is a logarithm with base e and is usually written as $\ln x$. • Logarithmic functions can be used to model real-world situations, such as the intensity of earthquakes.	**2A.2A** 2A.2B **2A.2C** 2A.5B 2A.5C 2A.7I	2
Mid-Topic Assessment					1
4	**I Like to Move It** Transformations of Exponential and Logarithmic Functions	Students apply the general transformation function $g(x) = Af(B(x - C)) + D$ to exponential and logarithmic functions. They sketch the graphs of single transformations and multiple transformations and identify any effects that the transformations have on the domain, range, and asymptotes of the functions. They write exponential or logarithmic equations for transformed functions in terms of the original function or by using a transformation equation. Students generalize the effect that a transformation on a function has on its inverse.	• In the transformation function form $g(x) = Af(B(x - C)) + D$, the D-value translates the function $f(x)$ vertically, the C-value translates $f(x)$ horizontally, the A-value vertically stretches or compresses $f(x)$, and the B-value horizontally stretches or compresses $f(x)$. • Reflections of a basic exponential function do not affect the domain or horizontal asymptote. • Reflections of a basic logarithmic function do not affect the range or vertical asymptote. • Vertical translations affect the range and the horizontal asymptote of exponential functions, while horizontal translations affect the domain and the vertical asymptote of logarithmic functions. • Transformations can be described through graphs, tables, key characteristics, writing an equation in terms of the original function, or by using a transformation equation. • A horizontal translation on a function produces a vertical translation on its inverse, while a vertical translation on a function produces a horizontal translation on its inverse. • A vertical dilation on a function produces a horizontal dilation the same number of units on its inverse, and a horizontal dilation on a function produces a vertical dilation the same number of units on its inverse.	2A.2B **2A.5A**	2

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
5	**Money, Heat, and Climate Change** Modeling Using Exponential Functions	Students model two savings scenarios, one with an exponential function $f(x)$ and one using a constant function $g(x)$. They then create a third function $h(x) = f(x) + g(x)$, graph all three functions on the same graph, and explain how they are related. For each of two new data sets, students create a scatterplot, write a regression equation, use the function to calculate output values, and interpret the reasonableness of a prediction based upon the scenario. For the first scenario, students are told to use an exponential function to model the scenario; in the second scenario, students must decide if the scenario is best modeled by a linear or exponential function. The lesson concludes with students making a list of contexts from this module and generalizing what they have in common that identifies them as best modeled by exponential functions. They also describe the shape of a scatterplot representing an exponential function and sketch possible graphs of exponential functions.	• An exponential function and a constant function can be added to create a third function that is the sum of the two functions, resulting in a graph that is a vertical translation of the original exponential function. • Technology can be used to determine exponential regression equations to model real-world situations. The regression equation can then be used to make predictions. • Sometimes referring to the scenario or obtaining further information may be required to determine whether a scatterplot is best modeled by a linear or exponential function.	2A.8B	2
6	**Drive Responsibly** Choosing a Function to Model BAC	Students analyze a context involving the blood alcohol content (BAC) of a driver and the driver's relative probability of causing an accident. Given BAC levels and their corresponding relative probability, they calculate the likelihood that each given driver causes an accident. Students are then given data from a study connecting BAC and the relative probability of causing an accident. They apply the relationship from the data to create a model predicting the likelihood of a person causing an accident based on their BAC. They summarize their learning by writing an article for a newsletter about the seriousness of drinking and driving. The lesson concludes with students connecting their process in this lesson to the steps in the mathematical modeling process.	• Determining and using a regression equation is sometimes a step in the process of solving a more complex mathematical problem, rather than the final solution. • The mathematical modeling process is an effective structure to solve complex mathematical problems.	2A.8B	2
End of Topic Assessment					1
Learning Individually with Skills Practice or MATHia *Schedule these days strategically throughout the topic to support student learning.*					6

Topic 2: Exponential and Logarithmic Equations

ELPS: 1.A, 1.B, 1.D, 1.E, 1.G, 1.H, 2.A, 2.B, 2.C, 2.D, 2.G, 2H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.E, 3.F, 3.G, 3.I, 4.A, 4.B, 4.C, 4.E, 4.G, 4.K, 5.E, 5.F, 5.G **Topic Pacing:** 15 days

Some of the content of this topic goes beyond the scope of the course standards. The content is included to enhance students' understanding of mathematics and to provide opportunities for extension.

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
1	**All the Pieces of the Puzzle** Logarithmic Expressions	Students convert between exponential and logarithmic forms of an equation, and then use this relationship to solve for an unknown base, exponent, or argument in a logarithmic equation. They estimate the values of logarithms that are not integers using a number line as a guide. Students use *always, sometimes,* and *never* to create statements regarding possible values for the base, argument, and logarithmic expression and relationships among them.	• The value of a *logarithmic expression* is equal to the value of the exponent in the corresponding exponential expression. • For an exponential equation $a^c = b$ and its corresponding logarithmic equation $\log_a b = c$, the variables have the same restrictions. The base, a, must be greater than 0 but not equal to 1, the argument, b, must be greater than zero, and the value of the exponent/logarithm, c, has no restrictions. • A simple logarithmic equation can be solved by converting it to an exponential equation. To solve for an argument in a logarithmic equation, calculate the resulting expression. To solve for an exponent in a logarithmic equation, use like bases. To solve for a base in a logarithmic equation, use common exponents. • You can estimate the value of a logarithm using the relationship that exists between logarithms and exponents. • For a fixed base greater than 1, as the value of the argument increases, the value of the logarithm increases. For a fixed argument, when the value of the base is greater than 1 and increasing, the value of the logarithm is decreasing.	2A.5C **2A.5D**	2

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
2	**Mad Props** Properties of Logarithms	Students develop rules and properties of logarithms based on their prior knowledge of various exponent rules and properties. They use properties of logarithms to write a single logarithm in expanded form, and vice versa. Students also write algebraic expressions to represent logarithmic expressions. They then summarize how exponential and logarithmic properties are related.	• Logarithms by definition are exponents, so they have properties that are similar to those of exponents and powers. • The *Zero Property of Logarithms* states: "The logarithm of 1, with any base, is always equal to 0." • The *Logarithm with Same Base and Argument* Rule states: "When the base and argument are equal, the logarithm is always equal to 1." • The *Product Rule of Logarithms* states: "The logarithm of a product is equal to the sum of the logarithms of the factors." • The *Quotient Rule of Logarithms* states: "The logarithm of a quotient is equal to the difference of the logarithms of the dividend and divisor." • The *Power Rule of Logarithms* states: "The logarithm of a power is equal to the product of the exponent and the logarithm of the base of the power."	2A.5C P.5G	1
3	**More Than One Way to Crack an Egg** Solving Exponential Equations	Students model a real-world context with an exponential function. Through this context, they encounter exponential equations that cannot be solved using the common base method. Students are introduced to the Change of Base Formula and the strategy of taking the logarithm of both sides of an exponential equation to solve for non-integer exponents or logarithms. They then use the strategy of their choice to solve exponential equations.	• The *Change of Base Formula* allows you to calculate an exact value for a logarithm by rewriting it in terms of a different base. By writing the logarithm using base 10 or base *e*, technology can be used to evaluate the expressions. • The Change of Base Formula states that $\log_b c = \frac{\log_a c}{\log_a b}$, where $a, b, c > 0$ and $a, b \neq 1$. • One method to solve an exponential equation is to write the equation in logarithmic form and then apply the Change of Base Formula. • Another method to solve an exponential equation is to take the common logarithm or natural logarithm of both sides of the exponential equation and then use the rules of logarithms to solve for x.	2A.5C **2A.5D** P.5G P.5H P.5I	2
Mid-Topic Assessment					1

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
4	**Logging On** Solving Logarithmic Equations	Students solve logarithmic equations for the base, argument, or exponent by rewriting them as exponential equations or using the Change of Base Formula. They use formulas to model real-world contexts and use properties of logarithms to solve equations containing multiple logarithms. Students consider the structure of logarithmic and exponential equations to determine the most efficient solution path.	• Logarithmic equations can be used to model real-world contexts. • When solving equations with more than one logarithmic expression, the rules of logarithms must first be applied to rewrite the equation as a single logarithm. • The structure of an exponential equation or logarithmic equation determines the most efficient solution strategy.	2A.5C **2A.5D** P.5G P.5H P.5I	2
5	**What's the Use?** Application of Exponential and Logarithmic Equations	Students write and solve exponential and logarithmic equations that model real-world situations, and use technology to write an exponential regression equation. They also address the fact that rounding too early in a series of calculations has a great effect on the level of accuracy of the solution.	• Exponential and logarithmic equations are used to model situations in the real world. • Rounding too early in a series of calculations involving exponential or logarithmic equations has a great effect on the level of accuracy of the solution.	2A.5B **2A.5D** 2A.5E P.5G P.5H P.5I	2
End of Topic Assessment					1
Learning Individually with Skills Practice or MATHia *Schedule these days strategically throughout the topic to support student learning.*					4

Topic 3: Applications of Exponential Functions

ELPS: 1.A, 1.D, 1.E, 1.G, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.G, 4.J, 4.K, 5.E

Topic Pacing: 0 days

Some of the content of this topic goes beyond the scope of the course standards. The content is included to enhance students' understanding of mathematics and to provide opportunities for extension.

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
1	**Series Are Sums** Geometric Series	Students develop two formulas to determine the sum of the terms of a geometric sequence or geometric series. They make sense of why both formulas work, compare the application of the formulas, and analyze an algebraic proof that demonstrates their equivalence. Students apply the formulas to solve problems presented as the addition of the terms of a geometric sequence, as a geometric series expressed in summation notation, and as data provided in table form. They also apply the formulas in the real-world context of credit card debt.	• A *geometric series* is the sum of the terms of a geometric sequence. • The formula to compute any geometric series is $S_n = g_n(r) - g_1$, where g_n is the last term, r is the common ratio, and g_1 is the first term. • Another formula to calculate the sum of a geometric series is $S_n = \frac{g_n(r) - g_1}{r - 1}$, where n is the number of terms, r is the common ratio, and g_1 is the first term. • Geometric series can be used to model real-world situations.	2A.5B P.5A	**0**
2	**Paint By Numbers** Art and Transformations	Students use their knowledge of seven basic functions (linear, linear absolute value, quadratic, cubic, exponential, radical, and logarithm), the basic equation for a circle, and transformations to sketch a figure on a coordinate plane. They then reverse the process and write the equations and restricted domains that could be used to create a given figure. Finally, students create a picture using at least five functions or equations of their choice.	Basic functions and equations, along with their transformations and restricted domains, can be used to create graphics on the coordinate plane.	**2A.2A** **2A.4C** **2A.5A** 2A.6A 2A.6C P.2F	**0**
3	**This Is the Title of This Lesson** Fractals	Students are introduced to the terms *self-similar, fractal,* and *iterative process*. They investigate the Sierpinski Triangle, the Menger Sponge, the Koch Snowflake, and the Sierpinski Carpet as examples of fractals. Students describe the growth patterns of the fractals in terms of geometric sequences, and explain how the geometric sequences behave as the number of stages approaches infinity.	• A fractal is a complex geometric shape that is formed by an iterative process. Fractals are infinite and self-similar across different scales. • The Sierpinski Triangle, the Menger Sponge, the Koch Snowflake, and the Sierpinski Carpet are examples of fractals with characteristics that can be described by geometric sequences.	2A.5B P.5E AQR.2H	**0**
End of Topic Assessment					**0**
Learning Individually with Skills Practice or MATHia *Schedule these days strategically throughout the topic to support student learning.*					0

Total Days: 150

Learning Together: 91

Learning Individually: 43

Assessments: 16