MODULE 1 FAMILY AND CAREGIVER GUIDE

Read and share with your student.

TEXAS MATH SOLUTION

How to support your student as they learn about

Exploring Patterns in Linear and Quadratic Relationships

Mathematics is a connected set of ideas, and your student knows a lot. Encourage them to use the mathematics they already know when encountering new concepts in this topic.

Module Introduction

In this module your student will progress from simple functions to the more complex polynomial, rational, radical, and logarithmic functions. There are 3 topics in this module: *Extending Linear Relationships, Exploring and Analyzing Patterns,* and *Applications of Quadratics*. Your student will use what they already know about the absolute value of numbers and their understanding of the structure of equations in this module.

Academic Glossary

Each module will focus on an important term. Knowing and using these terms will help your student think, reason, and communicate their math ideas.

Term	Analyze
Definition	• To study or look closely for patterns. • To break a concept down into smaller parts to gain a better understanding of it.
Questions to Ask Your Student	• Do you see any patterns? • Have you seen something like this before? • What happens if the shape, model, or numbers change?
Related Phrases	• Examine • Evaluate • Determine • Observe • Consider • Investigate • What do you notice?

Analyze the figures shown. Can you create an expression to represent the number of blocks where *n* represents the figure number?

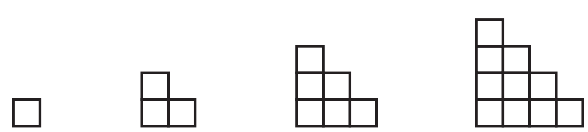

Figure 1 Figure 2 Figure 3 Figure 4

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Math Process Standards

Each module will focus on a process (or a pair of processes) that will help your student become a mathematical thinker. The "I can" statements listed below help your student to develop their mathematical learning and understanding.

Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

I can:

- work carefully and check my work.
- distinguish correct reasoning from reasoning that is flawed.
- use appropriate mathematical vocabulary when I talk with my classmates, my teacher, and others.
- specify the appropriate units of measure when I explain my reasoning.
- calculate accurately and communicate precisely to others.

Look for examples of these processes in the Topic Summaries.

The Carnegie Learning Way

Our Instructional Approach

Carnegie Learning's instructional approach is based on how people learn and real-world understandings. It is based on three key components:

ENGAGE	DEVELOP	DEMONSTRATE
Purpose: Provide an introduction that creates curiosity and uses what students already know and have experienced.	**Purpose:** Build a deep understanding of mathematics through different activities.	**Purpose:** Reflect on and evaluate what was learned.
Questions to Ask: How does this problem look like something you did in class?	**Questions to Ask:** Do you know another way to solve this problem? Does your answer make sense?	**Questions to Ask:** Is there anything you do not understand?

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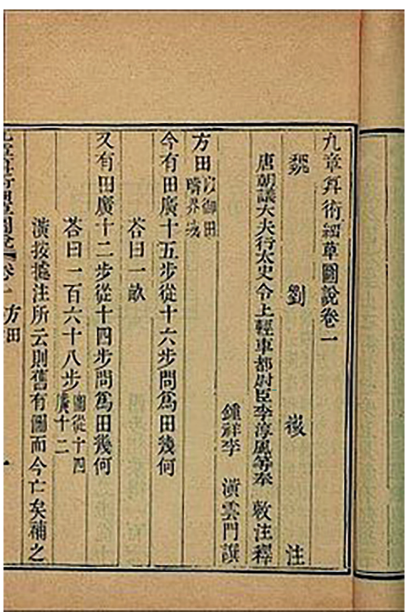
TEXAS MATH SOLUTION

Module Overview

TOPIC 1	TOPIC 2	TOPIC 3
Extending Linear Relationships	**Exploring and Analyzing Patterns**	**Applications of Quadratics**
17 Days	19 Days	15 Days
Your student will advance their ability to write and solve systems of equations, analyze solutions and use new methods such as Gaussian elimination and matrices to solve systems of linear equations in three variables.	**Your student will analyze and describe different patterns and use algebraic expressions to represent the patterns. Then they will expand their knowledge of complex numbers and use complex numbers to solve quadratic equations.**	**Your student will model real-world scenarios using quadratic functions, inequalities, and systems with a linear function or regression. They will also develop the fundamentals for inverses of functions and use inverses to further solve problems.**

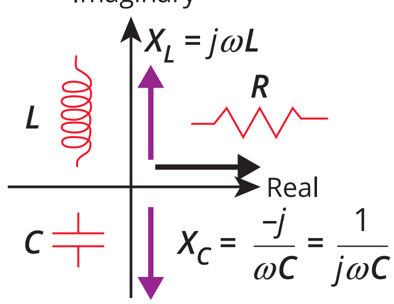
Did you know that?

The first signs of **matrix** usage was in 10th–2nd century BCE in Chinese math books.



Did you know that?

Imaginary numbers are often used in electricity, specifically alternating current (AC) electronics. AC electricity changes between positive and negative. Using imaginary and real numbers helps those working with AC electricity do calculations and avoid electrocution.



What in the world?

A simple form of **inverse functions** that we use everyday but maybe don't think about is the temperature outside! Normally we use Fahrenheit but Celsius is actually just the inverse of it!

Fahrenheit To Celsius

$C = \frac{5}{9}(F - 32)$

Celsius To Fahrenheit

$F = \frac{9}{5}C + 32$

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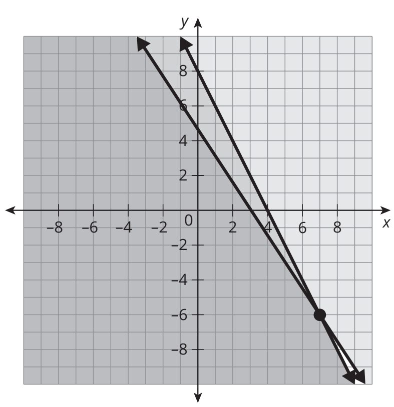
Topic 1: Extending Linear Relationships

Key Terms

- Gaussian elimination
- solution of a system of linear inequalities
- linear programming
- matrix (matrices)
- dimensions
- square matrix
- matrix element
- matrix multiplication
- identity matrix
- multiplicative inverse of a matrix
- matrix equation
- coefficient matrix
- variable matrix
- constant matrix
- absolute value
- reflection
- line of reflection
- argument of a function
- linear absolute value equation
- linear absolute value inequality
- equivalent compound inequality

The **solution of a system of inequalities** is the intersection of the solutions to each inequality.

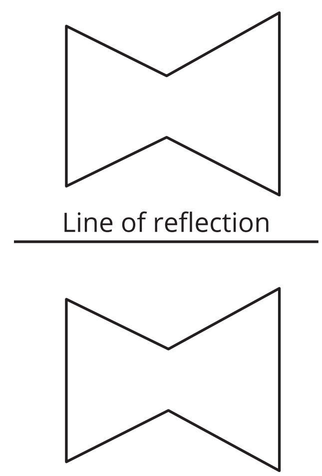
Every point that both solutions share satisfies all inequalities in the system.

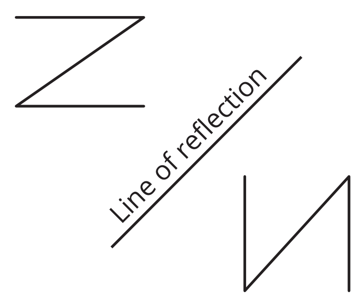


A **matrix (plural matrices)** is an array of numbers composed of rows and columns. A matrix is usually designated by a capital letter.

$$A = \begin{bmatrix} 3 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 & 2 \\ 0 & 1 & 3 & 1 & 2 \\ 2 & 1 & 0 & 1 & 1 \end{bmatrix}$$

A **line of reflection** is the line upon which an object is flipped or mirrored across.





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Students begin this topic by reviewing what they know about systems of linear equations. They use this knowledge to solve systems involving a linear and a quadratic equation and systems of three linear equations in three variables. Students also use systems of linear inequalities and **linear programming** to model solutions to real-world situations.

Gaussian Elimination

To solve a system of three linear equations using substitution, the first step is to solve for one variable in one of the equations. Then substitute this expression for that variable in the other two equations. The two new equations will then have only two unknown variables and can be solved using either substitution or linear combinations.

The goal of **Gaussian elimination** is to use linear combinations to isolate one variable for each equation. When using this method, you can:

- swap the positions of two equations.
- multiply an equation by a nonzero constant.
- add one equation to the multiple of another.

For example, you can solve the system $\begin{cases} x + 5y - 6z = 24 \\ -x - 4y + 5z = -21 \\ 5x - 4y + 2z = 21 \end{cases}$ using Gaussian elimination.

Add the first and second equation and replace the second equation.

$$\begin{array}{r} x + 5y - 6z = 24 \\ -x - 4y + 5z = -21 \\ \hline y - z = 3 \end{array} \rightarrow \begin{cases} x + 5y - 6z = 24 \\ y - z = 3 \\ 5x - 4y + 2z = 21 \end{cases}$$

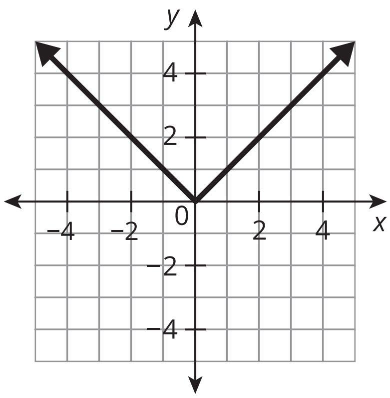
Multiply the first equation by −5 and add it to the third equation. Replace the third equation.

$$\begin{array}{r} -5x - 25y + 30z = -120 \\ 5x - 4y + 2z = 21 \\ \hline -29y + 32z = -99 \end{array} \rightarrow \begin{cases} x + 5y - 6z = 24 \\ y - z = 3 \\ -29y + 32z = -99 \end{cases}$$

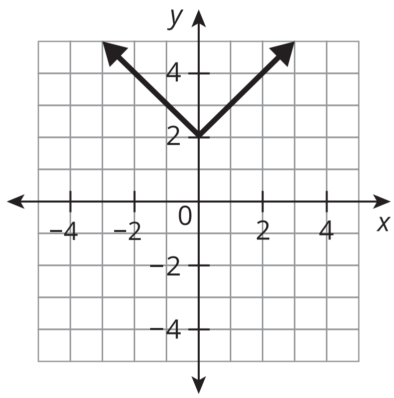
Transformations of a Linear Absolute Value Function

The general shape of the absolute value function $f(x) = |x|$ is a V shape which is the combination of the graphs $f(x) = x$ and $f(x) = -x$. For the basic function there are some major transformations that can be applied to the function $f(x)$. Vertical translations shift the function up and down, horizontal translations shift the function left and right, vertical dilations stretch or compress the graph along the y-axis, and horizontal dilations stretch or compress the function along the x-axis. The function may also be reflected across the x- or y-axis. These transformations are all applied in the transformational notation $g(x) = Af(B(x - C)) + D$.

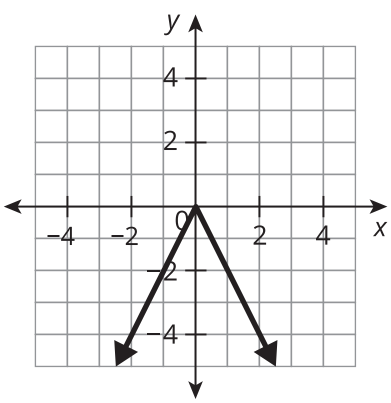
$f(x) = |x|$



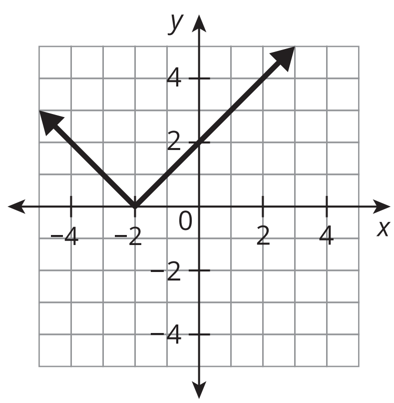
$y = |x| + 2$



$f(x) = -2|x|$



$f(x) = |x - (-2)|$



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Linear Absolute Value Equations and Inequalities

Students need to know that solving **linear absolute value equations** and inequalities uses the same methods they have learned in earlier lessons. There are just more answers to account for or there can even be a range of numbers that make the inequality true.

$$|5x - 4| = 21$$

$$+(5x - 4) = 21 \qquad -(5x - 4) = 21$$
$$5x - 4 = 21 \qquad 5x - 4 = -21$$
$$5x - 4 + 4 = 21 + 4 \qquad 5x - 4 + 4 = -21 + 4$$
$$5x = 25 \qquad 5x = -17$$
$$\frac{5x}{5} = \frac{25}{5} \qquad \frac{5x}{5} = \frac{(-17)}{5}$$
$$x = 5 \qquad x = -3\frac{2}{5}$$

Absolute Value Inequality	Equivalent Compound Inequality
$\|ax + b\| < c$	$-c < ax + b < c$
$\|ax + b\| \le c$	$-c \le ax + b \le c$
$\|ax + b\| > c$	$ax + b < -c$ or $ax + b > c$
$\|ax + b\| \ge c$	$ax + b \le -c$ or $ax + b \ge c$

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Topic 2: Exploring and Analyzing Patterns

Key Terms		
• relation • function • function notation • standard form of a quadratic function • factored form of a quadratic function • vertex form of a quadratic function	• concavity of a parabola • Quadratic Formula • the number *i* • imaginary roots • imaginary zeros • complex numbers • real part of a complex number	• imaginary part of a complex number • imaginary numbers • pure imaginary number • Fundamental Theorem of Algebra
The **Quadratic Formula** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ can be used to calculate the solutions to any quadratic equation of the form $ax^2 + bx + c = 0$, where a, b, and c represent real numbers and $a \neq 0$.	**The number *i*** is a number such that $i^2 = -1$.	The **Fundamental Theorem of Algebra** states that any polynomial equatlon of degree n must have exactly n complex roots or solutions; also, every polynomial function of degree n must have exactly n complex zeros. However, any root or zero may be a multiple root or zero. The polynomial equation $x^5 + x^2 - 6 = 0$ has 5 complex roots because the polynomial $x^5 + x^2 - 6$ has a degree of 5.
Follow the link to access the Mathematics Glossary: https://www.carnegielearning.com/texas-help/students-caregivers/		

In this topic, students first explore patterns in different forms and write algebraic expressions to represent patterns and make predictions. Then they will review the different forms of quadratic **functions** and how they are used to find key properties of quadratic functions and learn how to solve quadratic equations. Finally they will expand their knowledge of the real numbers to include complex and **imaginary numbers**, making it possible to find solutions to quadratic equations that would not be possible without using imaginary numbers.

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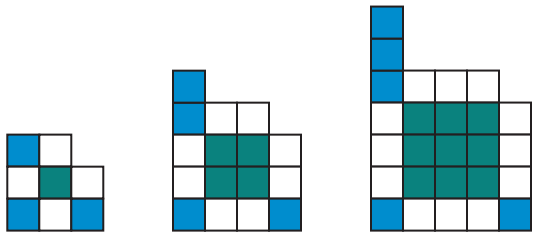
TEXAS MATH SOLUTION

Creating Algebraic Expressions From Patterns

Students need to be able to recognize patterns and understand how to represent them by writing an expression or mathematical rule to make predictions. They are presented with different shape patterns, story problems or sometimes just numbers and learn ways to identify the patterns and how to represent them.



There are expressions for calculating how many different colored blocks are in each figure as well as an expression for the total number of blocks in each figure.

Forms of Quadratics

Students represent algebraic expressions in different forms and use algebra and graphs to determine whether they are equivalent. They review linear, exponential, and quadratic functions in many forms. Students also learn to write quadratic equations given any three points. They are introduced to the complex number system and solve quadratic equations with **imaginary roots**.

standard form (general form) of a quadratic function

A quadratic function written in the form $f(x) = ax^2 + bx + c$, where $a \neq 0$, is in standard form, or general form.

The function $f(x) = -5x^2 - 10x + 1$ is written in standard form.

factored form of a quadratic function

A quadratic function written in factored form is in the form $f(x) = a(x - r_1)(x - r_2)$, where $a \neq 0$.

The function $h(x) = x^2 - 8x + 12$ written in factored form is $h(x) = (x - 6)(x - 2)$.

vertex form of a quadratic function

A quadratic function written in vertex form is in the form $f(x) = a(x - h)^2 + k$, where $a \neq 0$.

The quadratic equation $y = 2(x - 5)^2 + 10$ is written in vertex form. The vertex of the graph is the point (5, 10).

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Solving Quadratic Equations

Students will use three major ways to solve quadratic equations depending on the situation or the format of a question. Factoring and using the zero product property should be the first method to use. The second option is a method called "completing the square" where the student rewrites an expression in vertex form to help solve the equation. The third option is to use the Quadratic Formula, which can be used in any situation as long as the equation is written in standard form.

Solving by Factoring

$$x^2 - 4x + 3 = 0$$
$$(x - 3)(x - 1) = 0$$

$(x - 3) = 0$	or	$(x - 1) = 0$
$x - 3 + 3 = 0 + 3$	or	$x - 1 + 1 = 0 + 1$
$x = 3$	or	$x = 1$

Solving by Completing the Square

$$x^2 - 4x + 2 = 0$$

$$x^2 - 4x + 2 - 2 = 0 - 2$$
$$x^2 - 4x = -2$$

$$x^2 - 4x + ? = -2 + ?$$

$$x^2 - 4x + 4 = -2 + 4$$
$$x^2 - 4x + 4 = 2$$

$$(x - 2)^2 = 2$$

Once the function is in this vertex form students can solve by using square roots.

$$\sqrt{(x - 2)^2} = \pm\sqrt{2}$$

$$x - 2 = \pm\sqrt{2}$$

$$x - 2 = \sqrt{2} \text{ and } x - 2 = -\sqrt{2}$$

$$x = 2 + \sqrt{2} \text{ and } x = 2 - \sqrt{2}$$

$$x \approx 3.41 \text{ and } x \approx 0.59$$

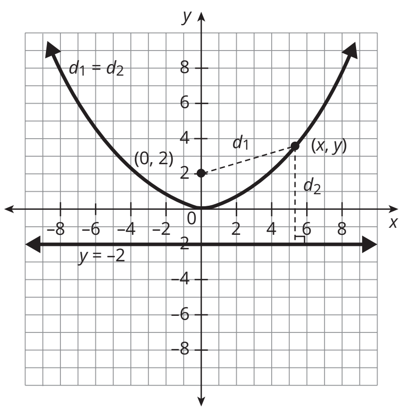
Topic 3: Applications of Quadratics

Key Terms

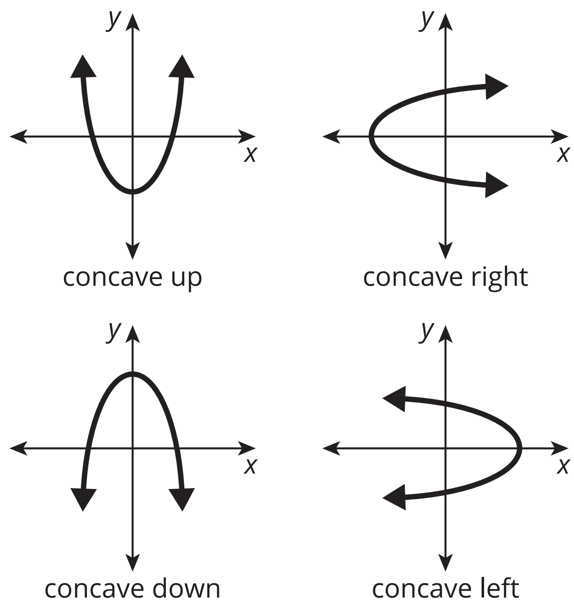
- restrict the domain
- one-to-one function
- inverse of a function
- conic section
- locus of points
- parabola
- focus
- directrix
- vertex of a parabola
- concavity
- general form of a parabola
- standard form of a parabola

The **focus** of a parabola is a point such that all points on the parabola are equidistant from the focus and the directrix.

The focus of the parabola shown is the point (0, 2). The directrix. of the parabola shown is the line $y = -2$. All points on the parabola are equidistant from the focus and directrix.



The **concavity of a parabola** describes the orientation of the curvature of the parabola.



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This topic gives students an opportunity to review what they have learned in Algebra 1 and build on that foundation as they model and solve problems involving quadratics. Students model real-world problems using quadratic inequalities, systems of quadratic functions and linear functions. They use technology to graph systems and determine the solutions. Students use familiar strategies to complete a quadratic regression to determine the curve of best fit. They determine the inverses of linear and quadratic equations. Finally, students explore **parabolas** as a **conic section** and write the general and standard equations.

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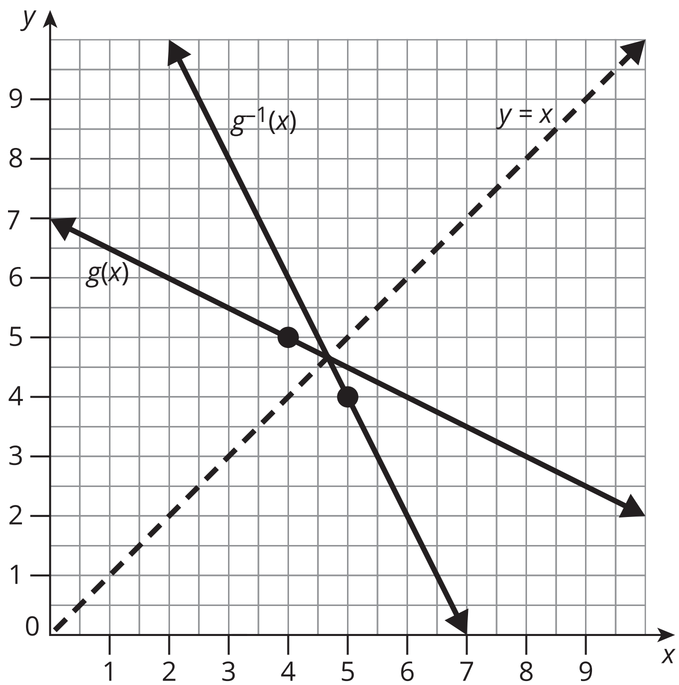
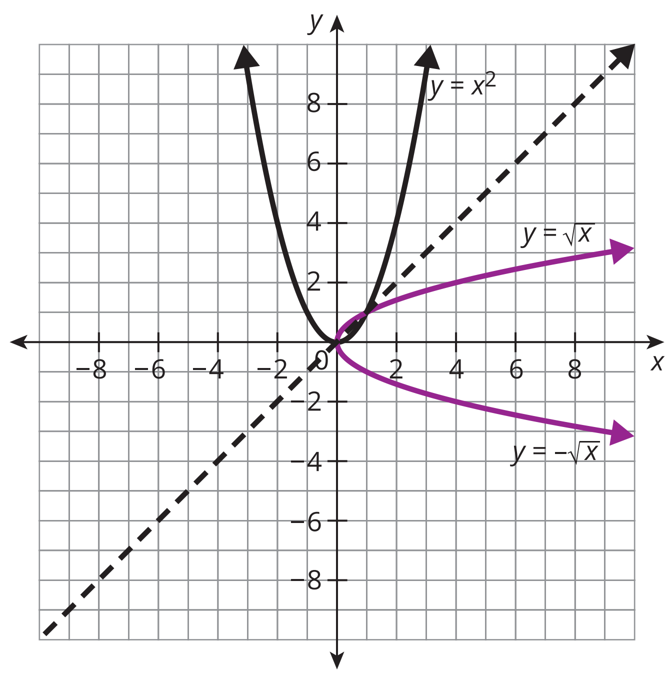
Inverse of a Function

Inverses of functions are when inputs (x-values) and outputs (y-values) are switched so that a new function is created and can be graphed. Inverses can be easily seen on a graph as they are a reflection across the diagonal line $y = x$.





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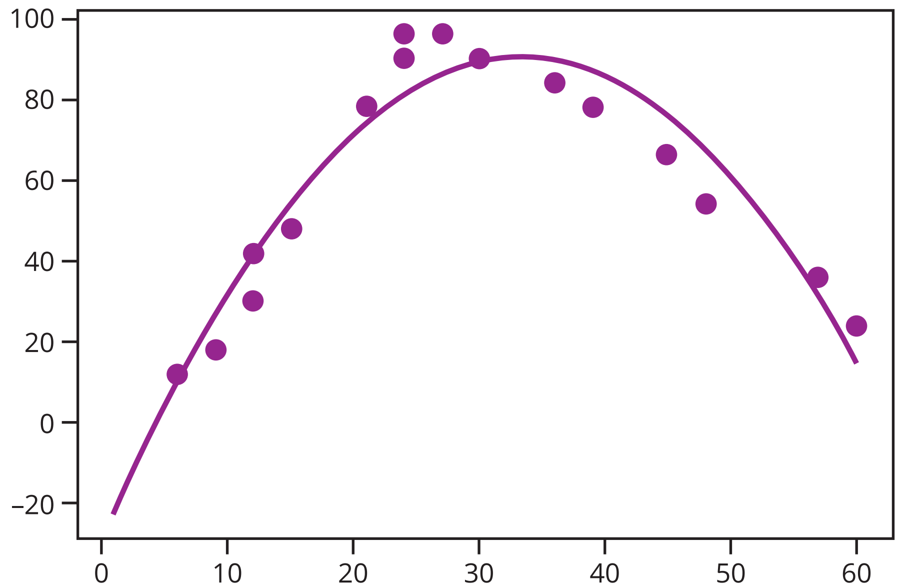
TEXAS MATH SOLUTION

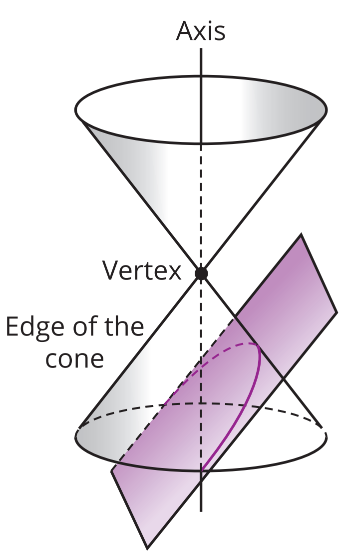
Quadratic Regression

Quadratic regression is similar to linear regression which was covered in Algebra 1, but the difference here is that the problem situations model quadratic functions. Students can create these quadratic regression equations, determine how well the equation models the data, and make predictions with the model.



Conic Sections

Parabolas can also be described as a conic section. By definition, a parabola is the set of all points in a plane that are equidistant to two other specific points called a focus and **directrix**. This concept builds upon what students had previously learned in their geometry class.



The parabola is the arc-shaped portion where the outside edge of the cone is intersecting with the plane.

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MATH PROCESS STANDARDS

How do the activities in *Exploring and Analyzing Patterns* promote student expertise in the math process standards?

NOTE: This is an example of the math process standard:

Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

- I can distinguish correct reasoning from reasoning that is flawed.

Have your student refer to page 2 for more "I can" statements.

Meredith

$x^2 - 7x - 8 = 3$

$a = 1,\ b = -7,\ c = -8$

$x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-8)}}{2(1)}$

$x = \dfrac{7 \pm \sqrt{49 + 32}}{2}$

$x = \dfrac{7 \pm \sqrt{81}}{2}$

$x = \dfrac{7 \pm 9}{2}$

$x = \dfrac{7 + 9}{2}$ or $x = \dfrac{7 - 9}{2}$

$x = \dfrac{16}{2} = 8$ or $x = \dfrac{-2}{2} = -1$

The roots are 8 and −1.

What should the equation always be set equal to before you are able to solve and determine the *a*-, *b*- and *c*-values?

[*The equation should be set equal to zero.*]

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Discuss important dates throughout this module such as assessments, assignments, or class events with your student. Use the table to record these dates and reference them as your student progresses through the module.

Important Dates	
Date	Reason

Using the link below, visit the Texas Math Solution Support Center for students and caregivers to access additional resources such as:

- Mathematics Glossaries
- Videos
- Topic Materials
- A Letter to Families and Caregivers

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