TEXAS MATH SOLUTION

Accelerated Grade 6

Teacher's Implementation Guide

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with Kelly Edenfield, Josh Fisher,
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Our Manifesto

WE BELIEVE that quality math education is important for all students, to help them develop into creative problem solvers, critical thinkers, life-long learners and more capable adults.

WE BELIEVE that math education is about more than memorizing equations or performing on tests—it's about delivering the deep conceptual learning that supports ongoing growth and future development.

WE BELIEVE all students learn math best when teachers believe in them, expect them to participate, and encourage them to own their learning.

WE BELIEVE teachers are fundamental to student success and need powerful, flexible resources and support to build dynamic cultures of collaborative learning.

WE BELIEVE our learning solutions and services can help accomplish this, and that by working together with educators and communities we serve, we guide the way to better math learning.

LONG + LIVE + MATH

At Carnegie Learning, we choose the path that has been proven most effective by research and classroom experience. We call that path the Carnegie Learning Way. Follow this code to take a look inside.

Acknowledgments

Middle School Math Solution Authors

- Sandy Bartle Finocchi, Chief Mathematics Officer
- Amy Jones Lewis, Senior Director of Instructional Design
- Kelly Edenfield, Instructional Designer
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"Mathematics is so much more than rules and algorithms. It is learning to reason, to make connections, and to make sense of the world. We believe in Learning by Doing™—students need to actively engage with the content if they are to benefit from it. Your classroom environment will determine what type of discourse, questioning, and sharing will take place. Students deserve a safe place to talk, to make mistakes, and to build deep understanding of mathematics. My hope is that these instructional materials help you shift the mathematical authority in your class to your students. Be mindful to facilitate conversations that enhance trust and reduce fear."

Sandy Bartle Finocchi, Chief Mathematics Officer

"My hope is that you know that your students are capable of thinking like mathematicians. This book is designed to give them the opportunity to struggle with challenging tasks, to talk about math with their classmates, and to make and fix mistakes. I hope that you use this book to build this capacity in your students—to ask the necessary questions to uncover what students already know and connect it to what they are learning, to encourage creative thinking, and to give just enough support to keep students on the right path."

Amy Jones Lewis, Senior Director of Instructional Design

"At Carnegie Learning we have created an organization whose mission and culture is defined by student success. Our passion is creating products that make sense of the world of mathematics and ignite a passion in students. Our hope is that students will enjoy our resources as much as we enjoyed creating them."

Barry Malkin, CEO

The Carnegie Learning Way

At Carnegie Learning, we choose the path that has been proven most effective by research and classroom experience. We call that path the **Carnegie Learning Way.**

Our Instructional Approach

Carnegie Learning's instructional approach is a culmination of the collective knowledge of our researchers, instructional designers, cognitive learning scientists, and master practitioners. It's based on both a scientific understanding of how people learn and a real-world understanding of how to apply that science to mathematics instructional materials. At its core, our instructional approach is based on three simple, key components:

ENGAGE

Activate student thinking by tapping into prior knowledge and real-world experiences. Provide an introduction that generates curiosity and plants the seeds for deeper learning.

DEVELOP

Build a deep understanding of mathematics through a variety of activities—real-world problems, sorting activities, Worked Examples, and peer analysis—in an environment where collaboration, conversations, and questioning are routine practices.

DEMONSTRATE

Reflect on and evaluate what was learned. Ongoing formative assessment underlies the entire learning experience, driving real-time adjustments, next steps, insights, and measurements.

Our Research

Carnegie Learning has been deeply immersed in research ever since it was founded by cognitive and computer scientists from Carnegie Mellon University. Our research extends far beyond our own walls, playing an active role in the constantly evolving field of cognitive and learning science. Our internal researchers collaborate with a variety of independent research organizations, tirelessly working to understand more about how people learn, and how learning is best facilitated. We supplement this information with feedback and data from our own products, teachers, and students, to continuously evaluate and elevate our instructional approach and its delivery.

Our Support

We're all in. In addition to our books and software, implementing Carnegie Learning in your classroom means you get access to an entire ecosystem of ongoing classroom support, including:

Professional Learning: Our team of Master Math Practitioners is always there for you, from implementation to math academies to a variety of other options to help you hone your teaching practice.

Texas Support Center: We've customized a Support Center just for you and your students. The Texas Support Center provides articles and videos to help you implement the Texas Math Solution, from the basics to get you started to more targeted support to guide you as you scaffold instruction for all learners in your classroom. Visit www.CarnegieLearning.com/texas-help to explore online and to access content that you can also share with your students and their caregivers.

MyCL: This is the central hub that gives you access to all of the products and resources that you and your students will need. Visit MyCL at www.CarnegieLearning.com/login.

LONG + LIVE + MATH: When you join this community of like-minded math educators, suddenly you're not alone. You're part of a collective, with access to special content, events, meetups, book clubs, and more. Because it's a community, it's constantly evolving! Visit www.longlivemath.com to get started.

Scan this code to visit the Texas Support Center and look for references throughout the Front Matter to learn more about the robust resources you will find in the Support Center.

Our Blend of Learning

Carnegie Learning combines consumable textbooks, MATHia® (our intelligent 1-on-1 math tutoring software), and transformative professional learning and data analysis services into a comprehensive and cohesive learning solution.

A key aspect of this blend is its combination of two forms of learning:

Learning Together: With our consumable textbooks, students work in groups, not only to develop math skills, but to learn how to collaborate, create, communicate and problem-solve.

Learning Individually: Through MATHia, students receive 1-to-1 adaptive math coaching, providing a personalized learning path and ongoing formative assessment.

Carnegie Learning's blend also strikes the right balance in other ways:

CONCEPTUAL UNDERSTANDING	+	PROCEDURAL UNDERSTANDING
LEARNING TOGETHER	+	LEARNING INDIVIDUALLY
CREATIVE EXPLORATION	+	SKILL MASTERY
ACCESSIBILITY	+	RIGOR

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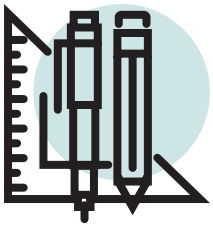
Instructional Design

In a word, every single piece of Carnegie Learning's Texas Math Solution is **intentional.** Our instructional designers work alongside our master math practitioners, cognitive scientists, and researchers to intentionally design, draft, debate, test, and revise every piece, incorporating the latest in learning science.

Intentional Mathematics Design



Carnegie Learning's Texas Math Solution is thoroughly and thoughtfully designed to ensure students build the foundation they'll need to experience ongoing growth in mathematics.

Mathematical Coherence: The arc of mathematics develops coherently, building understanding by linking together within and across grades, so students can learn concepts more deeply and apply what they've learned to more complex problems going forward.

Mathematical Process Standards: Carnegie Learning is organized around the Mathematical Process Standards to encourage experimentation, creativity, and false starts, which is critical if we expect students to tackle difficult problems in the real world, and persevere when they struggle.

Multiple Representations: Carnegie Learning recognizes the importance of connecting multiple representations of mathematical concepts. Lessons present content visually, algebraically, numerically, and verbally.

Transfer: Carnegie Learning focuses on developing transfer. Doing A and moving on isn't the goal; being able to do A and then do B, C, and D, transferring what you know from A, is the goal.

Texas Math Solution Year at a Glance

This Year at a Glance highlights the sequence of topics and the number of blended instructional days (1 day is 45 minutes) allocated for Accelerated Grade 6 in the Texas Math Solution. The pacing information also includes time for assessments, providing you with an instructional map that covers 180 days of the school year. As you set out at the beginning of the year, we encourage you to still modify this plan as necessary.

Want More Support Designing Your Long Term Plan?

You can find this Year at a Glance and additional guidance on planning intentionally and flexibly on the Texas Support Center at www.CarnegieLearning.com/texas-help.

Texas Accelerated Grade 6: Year at a Glance

*1 Day Pacing = 45 min. Session

Module	Topic	Pacing	TEKS
Process Standards are embedded in every module: 6.1A, 6.1B, 6.1C, 6.1D, 6.1E, 6.1F, 6.1G			
1 Composing & Decomposing	1: Factors and Multiples	5	6.7A, 6.7D
	2: Positive Rational Numbers	7	6.2D, 6.2E, 6.3A, 6.3B, 6.3E, 6.4F, 6.5C
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2 Relating Quantities	1: Ratios	15	6.4A, 6.4B, 6.4C, 6.4E, 6.5A, 6.5D, 6.6C
	2: Percents	7	6.2D, 6.4E, 6.4F, 6.4G, 6.5B, 6.5C
	3: Unit Rates and Conversions	8	6.4B, 6.4D, 6.4H, 6.5A, 7.4A, 7.4B, 7.4E
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3 Moving Beyond Positive Quantities	1: Signed Numbers and the Four Quadrants	11	6.2A, 6.2B, 6.2C, 6.2D, 6.11A, 7.2A
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	2: Fractional Rates	5	7.4B, 7.4C, 7.4D, 7.4E
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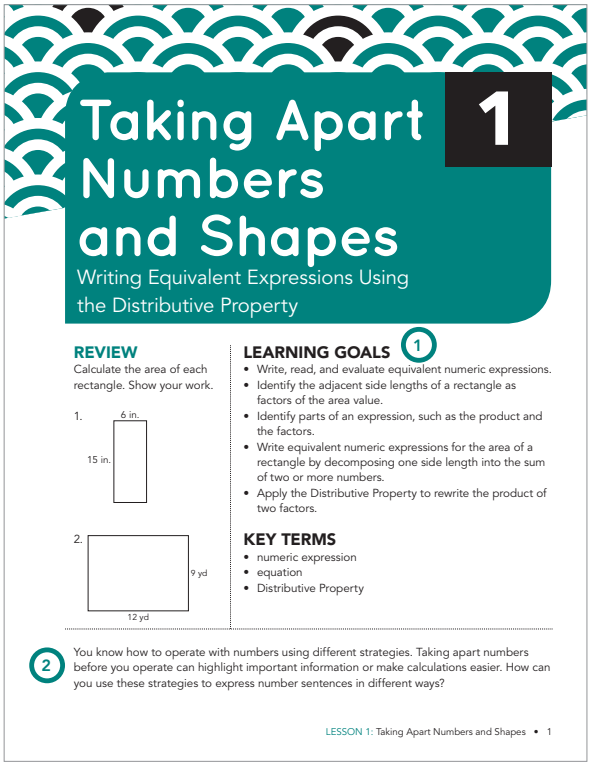
6 Describing Variability of Quantitites	1: The Statistical Process	7	6.12A, 6.12B, 6.12D, 6.13A, 6.13B
	2: Numerical Summaries of Data	7	6.12A, 6.12B, 6.12C, 6.12D, 6.13A
		14	
	Total Days:	180	

Lesson Structure

Each lesson of the Texas Math Solution has the same structure. This consistency allows both you and your students to track your progress through each lesson. Key features of each lesson are noted.

Lesson Structure



Taking Apart Numbers and Shapes 1

Writing Equivalent Expressions Using the Distributive Property

REVIEW

Calculate the area of each rectangle. Show your work.

1. 6 in., 15 in.

2. 9 yd, 12 yd

LEARNING GOALS (1)

- Write, read, and evaluate equivalent numeric expressions.
- Identify the adjacent side lengths of a rectangle as factors of the area value.
- Identify parts of an expression, such as the product and the factors.
- Write equivalent numeric expressions for the area of a rectangle by decomposing one side length into the sum of two or more numbers.
- Apply the Distributive Property to rewrite the product of two factors.

KEY TERMS

- numeric expression
- equation
- Distributive Property

(2) You know how to operate with numbers using different strategies. Taking apart numbers before you operate can highlight important information or make calculations easier. How can you use these strategies to express number sentences in different ways?

LESSON 1: Taking Apart Numbers and Shapes • 1

1. Learning Goals
Learning goals are stated for each lesson to help you take ownership of the learning objectives.

2. Connection
Each lesson begins with a statement connecting what you have learned with a question to ponder.

Return to this question at the end of this lesson to gauge your understanding.

Lesson Structure • 15

ENGAGE

Establishing Mathematical Goals to Focus Learning

Create a classroom climate of collaboration and establish the learning process as a partnership between you and students.

Communicate continuously with students about the learning goals of the lesson to encourage self-monitoring of their learning.

Visit the Texas Support Center for additional guidance on how to foster a classroom environment that promotes collaboration and communication.

Activating Student Thinking

Your students enter each class with varying degrees of experience and mathematical success. The focus of the Getting Started is to tap into prior knowledge and real-world experiences, to generate curiosity, and to plant seeds for deeper learning.

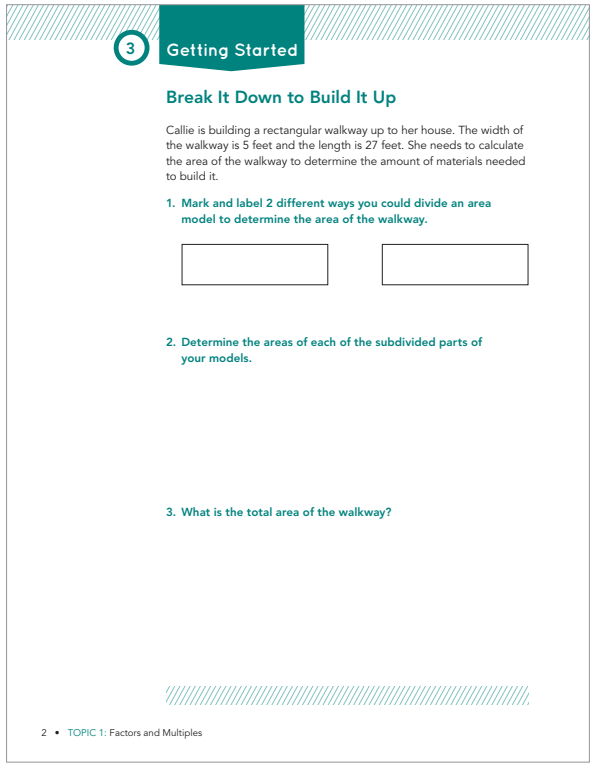
Pay particular attention to the strategies students use, for these strategies reveal underlying thought processes and present opportunities for connections as students proceed through the lesson.

Supporting English Language Learners

Visit the Texas Support Center for facilitation strategies to support students at varying levels of language proficiency as they complete the Getting Started activities in each lesson.

> Mathematics is the science of patterns. So, we encourage students throughout this course to notice, test, and interpret patterns in a variety of ways—to put their "mental tentacles" to work in every lesson, every activity. Our hope is that this book encourages you to do the same for your students, and create an environment in your math classroom where productive and persistent learners develop and thrive.
>
> Josh Fisher, Instructional Designer

3. Getting Started
Each lesson begins with a Getting Started. When working on the Getting Started, use what you know about the world, what you have learned previously, or your intuition. The goal is just to get you thinking and ready for what's to come.



3 Getting Started

Break It Down to Build It Up

Callie is building a rectangular walkway up to her house. The width of the walkway is 5 feet and the length is 27 feet. She needs to calculate the area of the walkway to determine the amount of materials needed to build it.

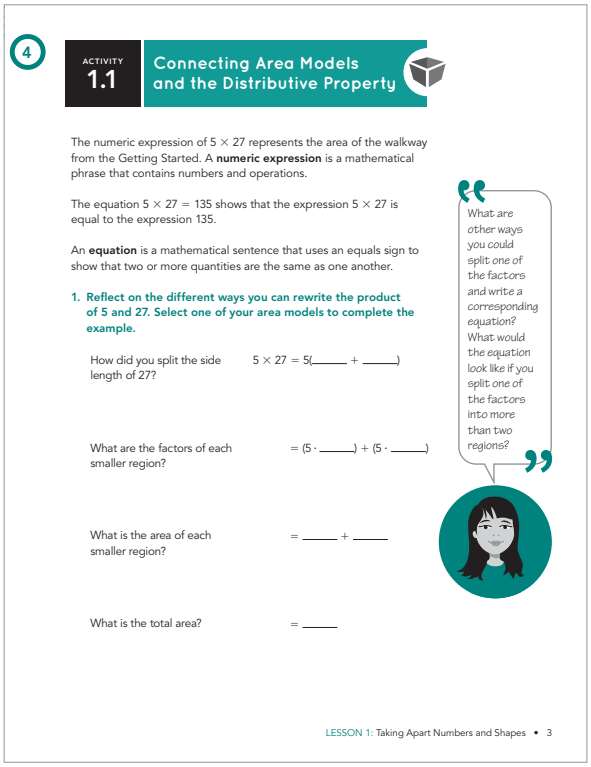
1. **Mark and label 2 different ways you could divide an area model to determine the area of the walkway.**

2. **Determine the areas of each of the subdivided parts of your models.**

3. **What is the total area of the walkway?**

2 • TOPIC 1: Factors and Multiples

16 • Lesson Structure



4

ACTIVITY 1.1

Connecting Area Models and the Distributive Property

The numeric expression of 5×27 represents the area of the walkway from the Getting Started. A **numeric expression** is a mathematical phrase that contains numbers and operations.

The equation $5 \times 27 = 135$ shows that the expression 5×27 is equal to the expression 135.

An **equation** is a mathematical sentence that uses an equals sign to show that two or more quantities are the same as one another.

1. Reflect on the different ways you can rewrite the product of 5 and 27. Select one of your area models to complete the example.

How did you split the side length of 27?

$5 \times 27 = 5(\_\_\_\_ + \_\_\_\_)$

What are the factors of each smaller region?

$= (5 \cdot \_\_\_\_) + (5 \cdot \_\_\_\_)$

What is the area of each smaller region?

$= \_\_\_\_ + \_\_\_\_$

What is the total area?

$= \_\_\_\_$

What are other ways you could split one of the factors and write a corresponding equation? What would the equation look like if you split one of the factors into more than two regions?

LESSON 1: Taking Apart Numbers and Shapes • 3

4. Activities
You are going to build a deep understanding of mathematics through a variety of activities in an environment where collaboration and conversations are important and expected.

You will learn how to solve new problems, but you will also learn why those strategies work and how they are connected to other strategies you already know.

Remember:

- It's not just about answer-getting. The process is important.
- Making mistakes is a critical part of learning, so take risks.
- There is often more than one way to solve a problem.

Activities may include real-world problems, sorting activities, Worked Examples, or analyzing sample student work.

Be prepared to share your solutions and methods with your classmates.

Lesson Structure • 17

DEVELOP

Aligning Teaching to Learning

Students learn when they are actively engaged in a task: reasoning about the math, writing their solutions, justifying their strategies, and sharing their knowledge with peers.

Support productive struggle by allowing students time to engage with and persevere through the mathematics.

Support student-to-student discourse as well as whole-class conversations that elicit and use evidence of student thinking.

Supporting English Language Learners

Visit the Texas Support Center for facilitation strategies to support students at varying levels of language proficiency as they engage in mathematical discourse throughout each lesson.

DEMONSTRATE

Ongoing Formative Assessment Drives Instruction

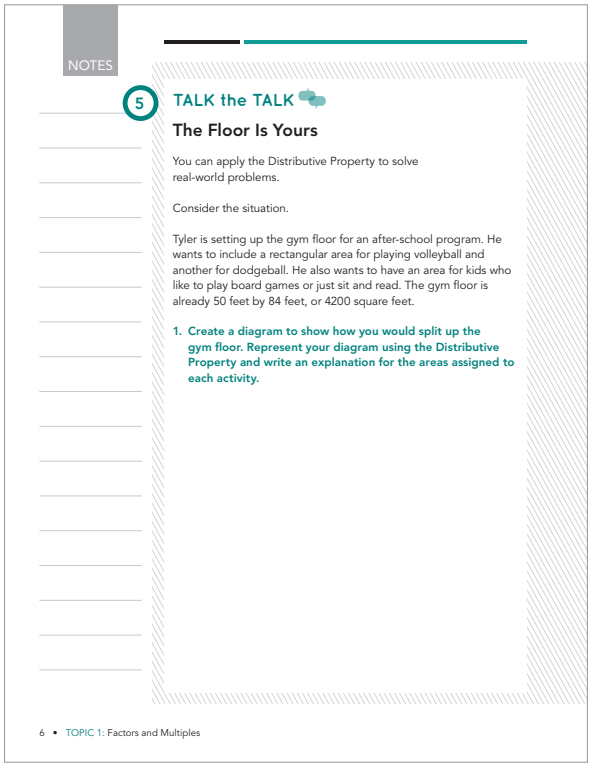
For students to take responsibility for their own learning, they need to be encouraged to self-assess. Students can use the Talk the Talk to monitor their own progress towards mastering the learning goals. Listen and review their answers and explanations and provide feedback to help them improve their understanding.

As you plan the next lesson, consider the connections you can make to build off the strengths or fill any gaps identified from this formative assessment.

5. Talk the Talk
Talk the Talk gives you an opportunity to reflect on the main ideas of the lesson.

- Be honest with yourself.
- Ask questions to clarify anything you don't understand.
- Show what you know!

Don't forget to revisit the question posed on the lesson opening page to gauge your understanding.


NOTES

5 TALK the TALK

The Floor Is Yours

You can apply the Distributive Property to solve real-world problems.

Consider the situation.

Tyler is setting up the gym floor for an after-school program. He wants to include a rectangular area for playing volleyball and another for dodgeball. He also wants to have an area for kids who like to play board games or just sit and read. The gym floor is already 50 feet by 84 feet, or 4200 square feet.

1. **Create a diagram to show how you would split up the gym floor. Represent your diagram using the Distributive Property and write an explanation for the areas assigned to each activity.**

6 • TOPIC 1: Factors and Multiples

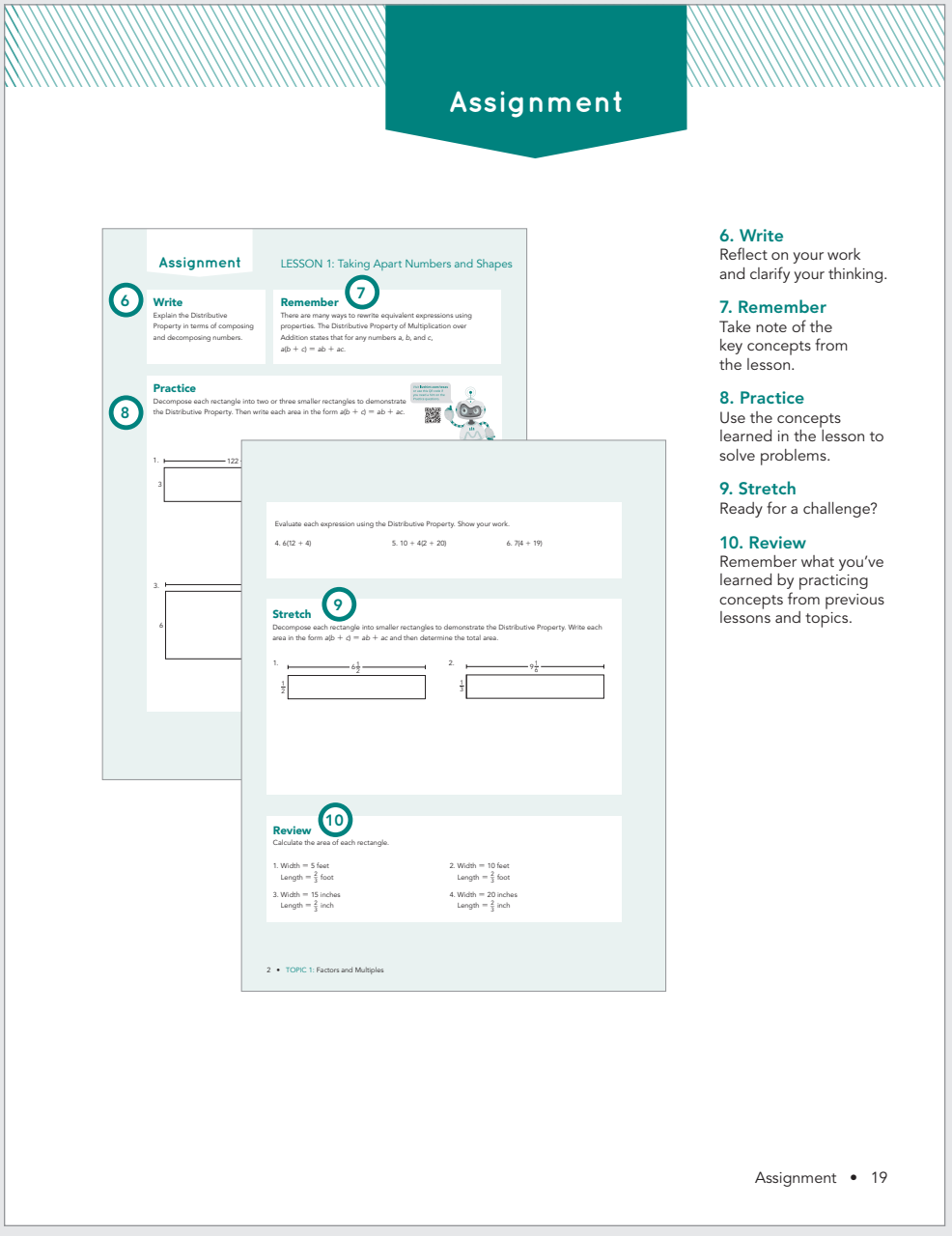
18 • Lesson Structure

Supporting English Language Learners

Visit the Texas Support Center for facilitation strategies to support students at varying levels of language proficiency as they demonstrate their understanding in the Talk the Talk activities in each lesson.

Assignment

An intentionally designed Assignment follows each lesson.



6. Write
Reflect on your work and clarify your thinking.

7. Remember
Take note of the key concepts from the lesson.

8. Practice
Use the concepts learned in the lesson to solve problems.

9. Stretch
Ready for a challenge?

10. Review
Remember what you've learned by practicing concepts from previous lessons and topics.

Assignment • 19

There is one Assignment per lesson. Lessons often span multiple days. Be thoughtful about which portion of the Assignment students can complete based on that day's progress.

The **Stretch** section is not necessarily appropriate for all learners. Assign this to students who are ready for more advanced concepts.

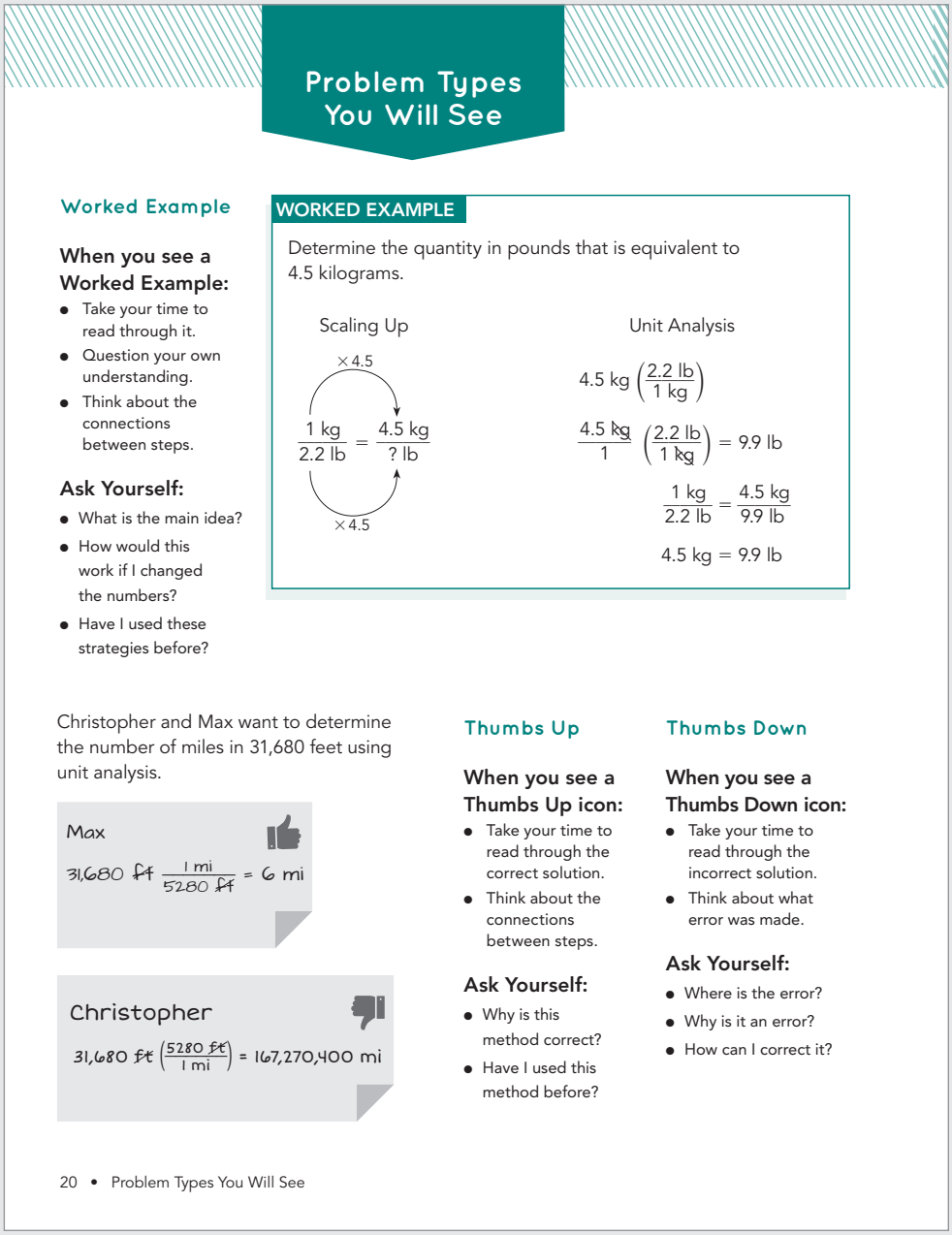
The **Review** section provides spaced practice of concepts from the previous lesson and topic and of the fluency skills important for the course.

Problem Types You Will See

Lessons include a variety of problem types to engage students in reasoning about the math.

Worked Examples

Worked Examples help students develop their skills as they question their understanding, make connections with the steps, and ultimately explain the progression of the steps towards the final outcome. They represent and mimic an internal dialog about the mathematics and the strategies, and the questions that follow them are designed to serve as a model for self-questioning and self-explanations—while making sure that students don't skip over a Worked Example without interacting with it, thinking about it, and responding to its accompanying questions. This approach aids students as they develop their desired habits of mind for being conscientious about the importance of steps and their order.



Thumbs Up / Thumbs Down

Thumbs Up problems give students the opportunity to analyze viable methods and problem-solving strategies. Questions are presented to help students consider the various strategies in-depth, and to focus on an analysis of correct responses. Because research shows that providing only positive examples is less effective for eliminating common student misconceptions than also showing negative examples, incorrect responses are provided alongside the correct responses. From the incorrect responses, students learn to determine where the error in calculation is, why the method is wrong or is being used wrong, and also how to correct the method to calculate the solution properly.

Tim and Dan love cereal, but don't want spend a lot of money. After scanning the aisle in the grocery store for the lowest prices, the boys make the following statements.

- **Tim says, "I found Sweetie Oat Puffs for $0.14 per ounce. That's the cheapest cereal in the aisle!"**
- **Dan replies, "It's not cheaper than Sugar Hoops! The unit price for that is 6.25 oz per dollar."**

Who is correct? Explain your reasoning.

Who's Correct

When you see a Who's Correct icon:

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine correct or not correct.

Ask Yourself:

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

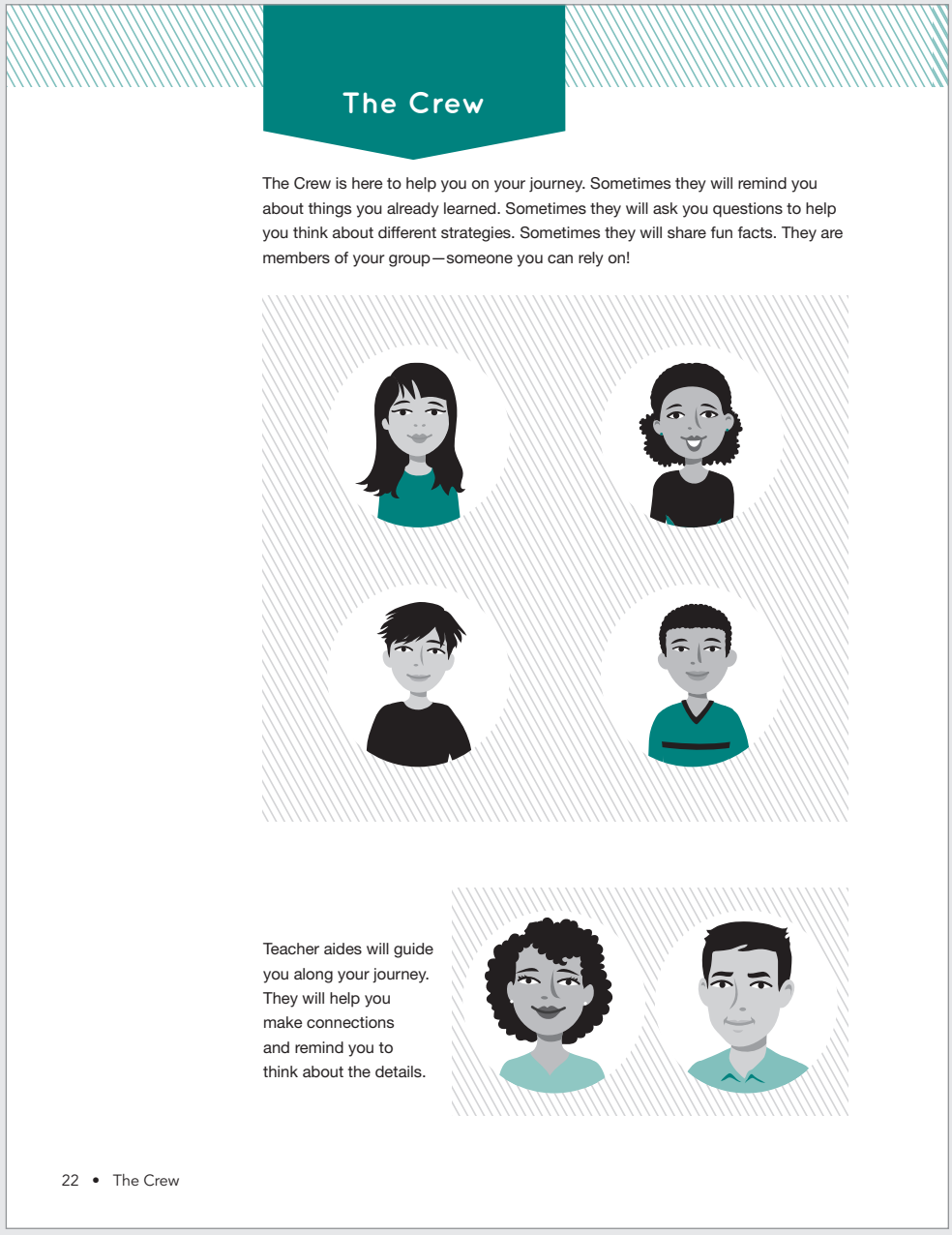
Who's Correct?

"Who's Correct?" problems are an advanced form of correct vs. incorrect responses. In this problem type, students are not told who is correct. Students have to think more deeply about what the strategies really mean, and whether each of the solutions made sense. Students will determine what is correct and what is incorrect, and then explain their reasoning. These types of problems will help students analyze their own work for errors and correctness.

Promoting Self-Reflection

The Crew

Characters are embedded throughout the Texas Math Solution to remind students to stop and think in order to promote productive reflection. The characters are used in a variety of ways: they may remind students to recall a previous mathematical concept, help students develop expertise to think through problems, and occasionally, present a fun fact.



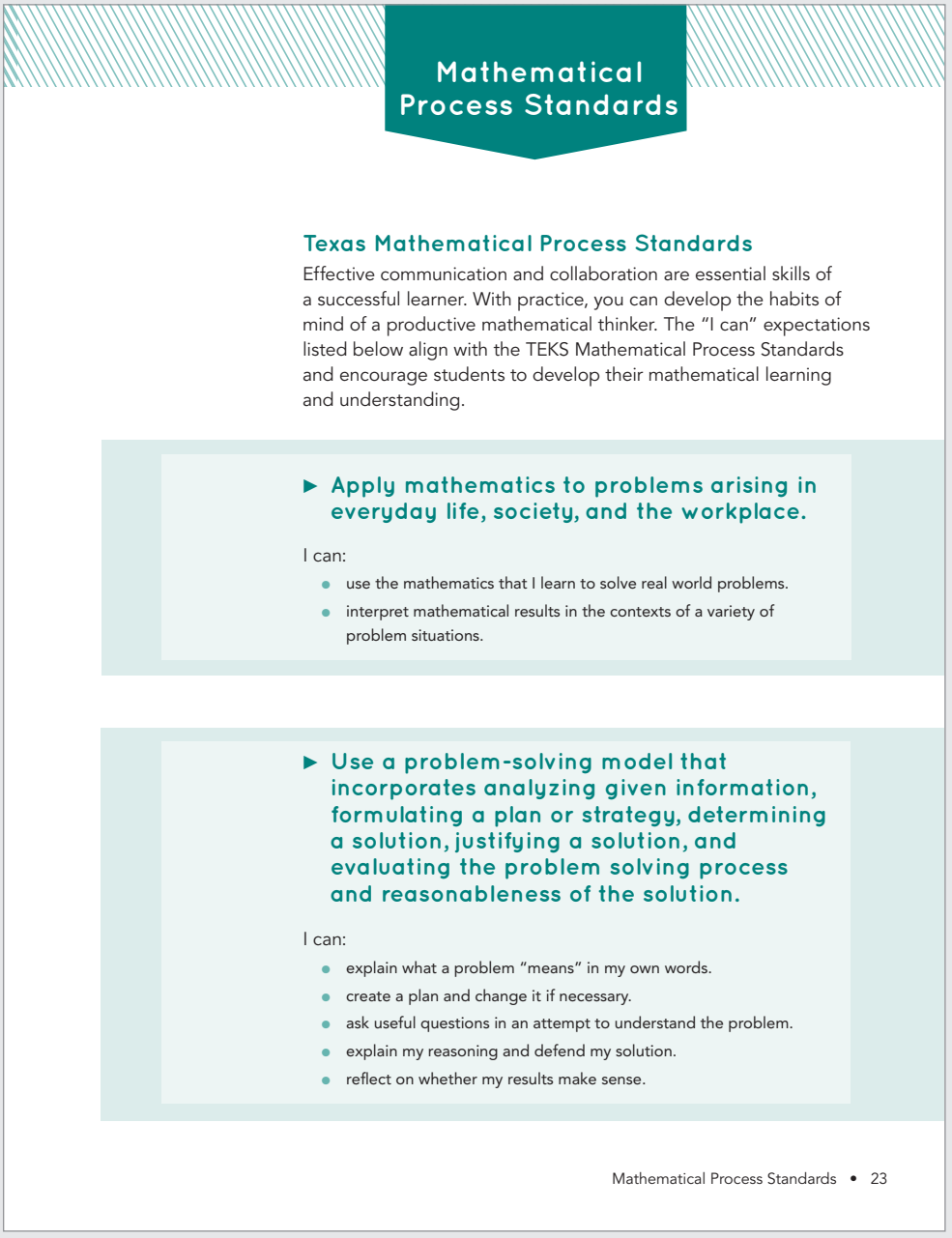
The Crew

The Crew is here to help you on your journey. Sometimes they will remind you about things you already learned. Sometimes they will ask you questions to help you think about different strategies. Sometimes they will share fun facts. They are members of your group—someone you can rely on!

Teacher aides will guide you along your journey. They will help you make connections and remind you to think about the details.

22 • The Crew

Mathematical Process Standards



Mathematical Process Standards

Texas Mathematical Process Standards

Effective communication and collaboration are essential skills of a successful learner. With practice, you can develop the habits of mind of a productive mathematical thinker. The "I can" expectations listed below align with the TEKS Mathematical Process Standards and encourage students to develop their mathematical learning and understanding.

▶ **Apply mathematics to problems arising in everyday life, society, and the workplace.**

I can:

- use the mathematics that I learn to solve real world problems.
- interpret mathematical results in the contexts of a variety of problem situations.

▶ **Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem solving process and reasonableness of the solution.**

I can:

- explain what a problem "means" in my own words.
- create a plan and change it if necessary.
- ask useful questions in an attempt to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

Mathematical Process Standards • 23

Note

Each lesson provides opportunities for students to think, reason, and communicate their mathematical understanding. However, it is your responsibility as a teacher to recognize these opportunities and incorporate these practices into your daily rituals. Expertise is a long-term goal, and students must be encouraged to apply these practices to new content throughout their school career.

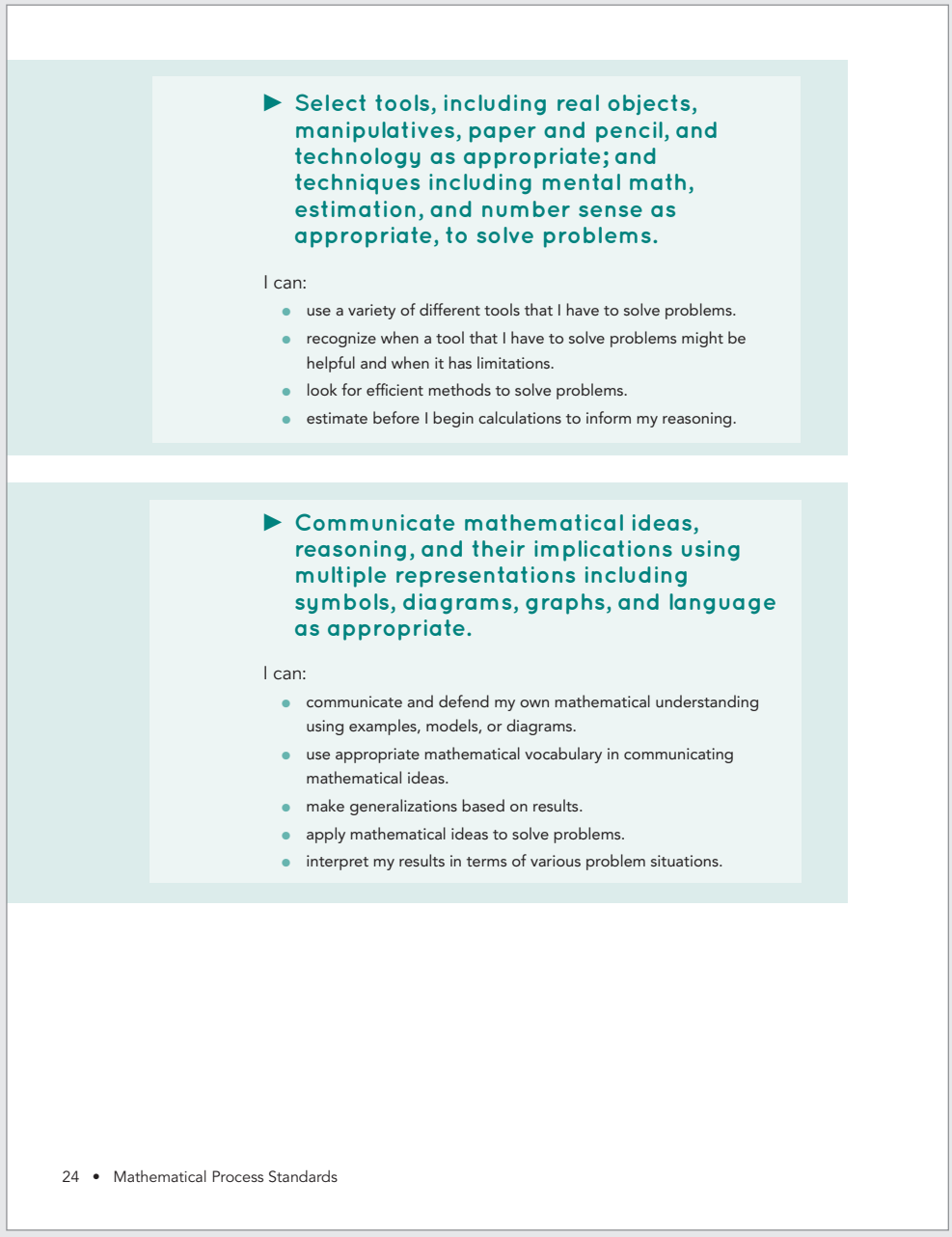
Supporting Students to Use Mathematical Tools

Visit the Texas Support Center for strategies to support students as they use mathematical tools, including formula charts and reference sheets.

Note

When you are facilitating each lesson, listen carefully and value diversity of thought, redirect students' questions with guiding questions, provide additional support with those struggling with a task, and hold students accountable for an end product. When students share their work, make your expectations clear, require that students defend and talk about their solutions, and monitor student progress by checking for understanding.

Consider having students create "I can" statements to promote their self-reflection.



▶ **Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate; and techniques including mental math, estimation, and number sense as appropriate, to solve problems.**

I can:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.

▶ **Communicate mathematical ideas, reasoning, and their implications using multiple representations including symbols, diagrams, graphs, and language as appropriate.**

I can:

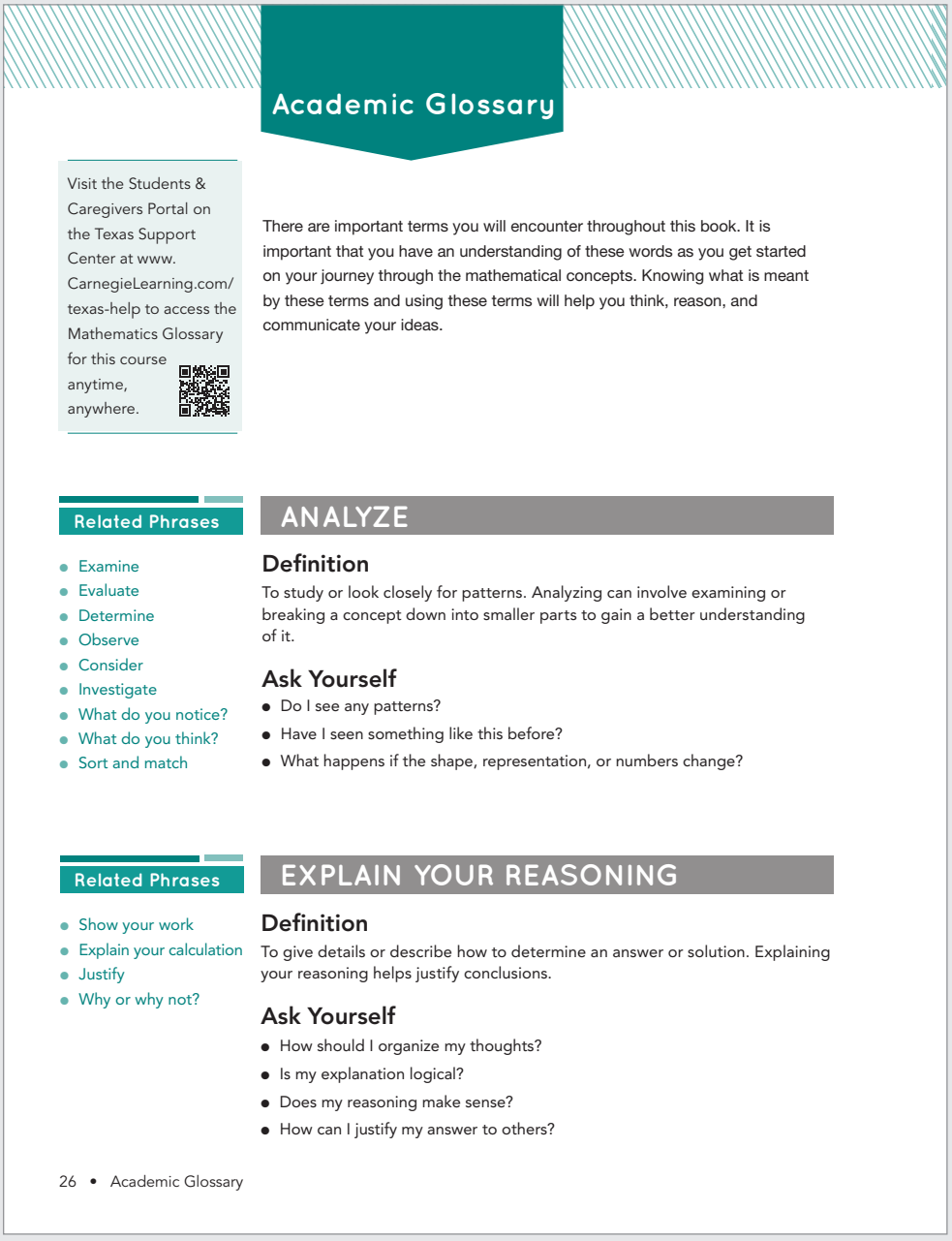
- communicate and defend my own mathematical understanding using examples, models, or diagrams.
- use appropriate mathematical vocabulary in communicating mathematical ideas.
- make generalizations based on results.
- apply mathematical ideas to solve problems.
- interpret my results in terms of various problem situations.

24 • Mathematical Process Standards

Supporting ALL Learners

Visit the Texas Support Center for facilitation strategies to support ALL students as the engage in the Mathematical Process Standards.

Academic Glossary



Academic Glossary

Visit the Students & Caregivers Portal on the Texas Support Center at www.CarnegieLearning.com/texas-help to access the Mathematics Glossary for this course anytime, anywhere.

There are important terms you will encounter throughout this book. It is important that you have an understanding of these words as you get started on your journey through the mathematical concepts. Knowing what is meant by these terms and using these terms will help you think, reason, and communicate your ideas.

Related Phrases

- Examine
- Evaluate
- Determine
- Observe
- Consider
- Investigate
- What do you notice?
- What do you think?
- Sort and match

ANALYZE

Definition

To study or look closely for patterns. Analyzing can involve examining or breaking a concept down into smaller parts to gain a better understanding of it.

Ask Yourself

- Do I see any patterns?
- Have I seen something like this before?
- What happens if the shape, representation, or numbers change?

Related Phrases

- Show your work
- Explain your calculation
- Justify
- Why or why not?

EXPLAIN YOUR REASONING

Definition

To give details or describe how to determine an answer or solution. Explaining your reasoning helps justify conclusions.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Does my reasoning make sense?
- How can I justify my answer to others?

26 • Academic Glossary

It is critical for students to possess an understanding of the language of their text. Students must learn to read for different purposes and write about what they are learning. Encourage students to become familiar with the key words and the questions they can ask themselves when they encounter these words.

It is our recommendation to be explicit about your expectations of language use and the way students write responses throughout the text. Encourage students to answer questions with complete sentences. Complete sentences help students reflect on how they arrived at a solution, make connections between topics, and consider what a solution means both mathematically as well as in context.

Supporting Students at Varying Levels of Language Proficiency

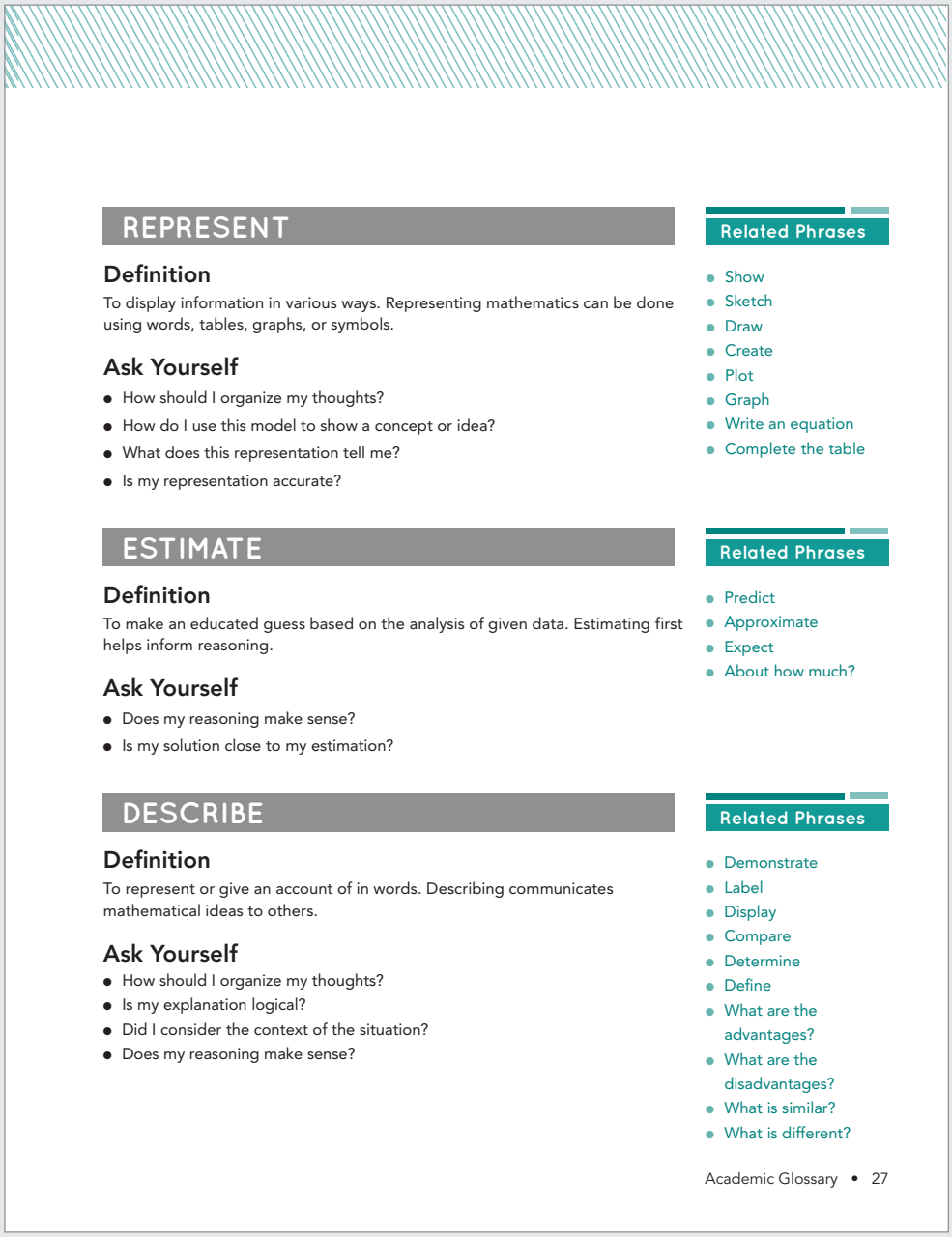
Visit the Texas Support Center for guidance on how to leverage the Academic Glossary to support students at varying levels of language proficiency.

Ask Yourself

The Ask Yourself questions help students develop the proficiency to explain to themselves the meaning of problems.

Real-World Context

Real-world contexts confirm concrete examples of mathematics. The scenarios in the lessons help students recognize and understand that quantitative relationships seen in the real world are no different than quantitative relationships in mathematics. Some problems begin with a real-world context to remind students that the quantitative relationships they already use can be formalized mathematically. Other problems will use real-world situations as an application of mathematical concepts.



REPRESENT

Definition

To display information in various ways. Representing mathematics can be done using words, tables, graphs, or symbols.

Ask Yourself

- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Is my representation accurate?

Related Phrases

- Show
- Sketch
- Draw
- Create
- Plot
- Graph
- Write an equation
- Complete the table

ESTIMATE

Definition

To make an educated guess based on the analysis of given data. Estimating first helps inform reasoning.

Ask Yourself

- Does my reasoning make sense?
- Is my solution close to my estimation?

Related Phrases

- Predict
- Approximate
- Expect
- About how much?

DESCRIBE

Definition

To represent or give an account of in words. Describing communicates mathematical ideas to others.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?

Related Phrases

- Demonstrate
- Label
- Display
- Compare
- Determine
- Define
- What are the advantages?
- What are the disadvantages?
- What is similar?
- What is different?

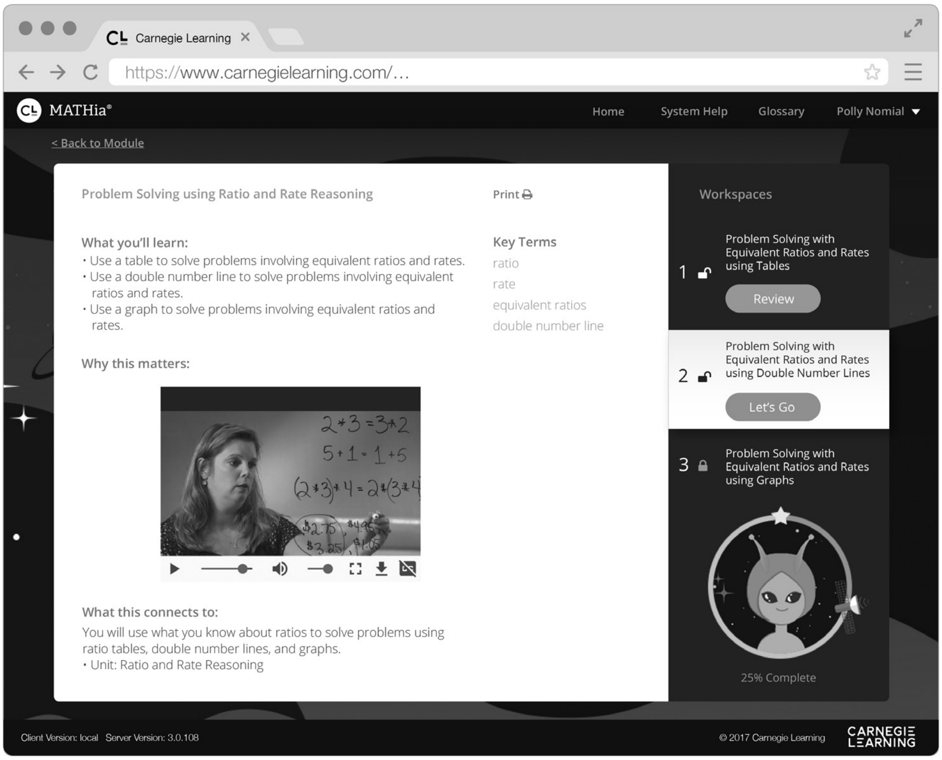
Academic Glossary • 27

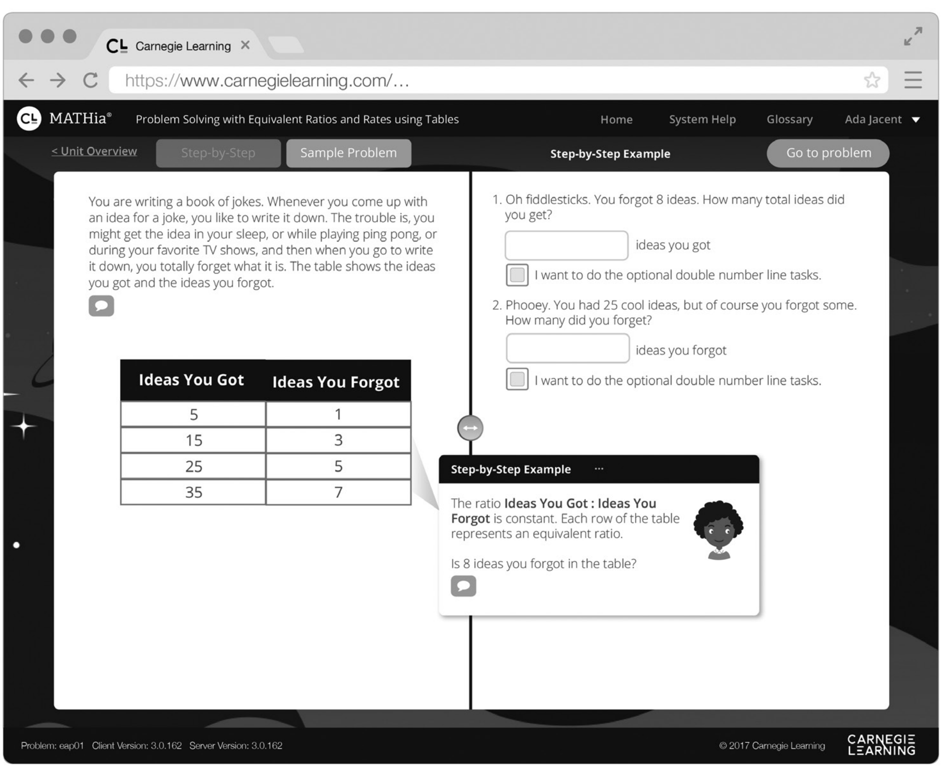
Home Connection

Encourage your students to explore the Students & Caregivers portal on the Texas Support Center to access a variety of resources to support their learning at home and elsewhere outside of the classroom.

MATHia Structure

Each unit in MATHia maximizes student learning while collecting critical data about what they do or do not know at every step. Students can access MATHia anywhere, anytime.





ENGAGE

Unit Overview

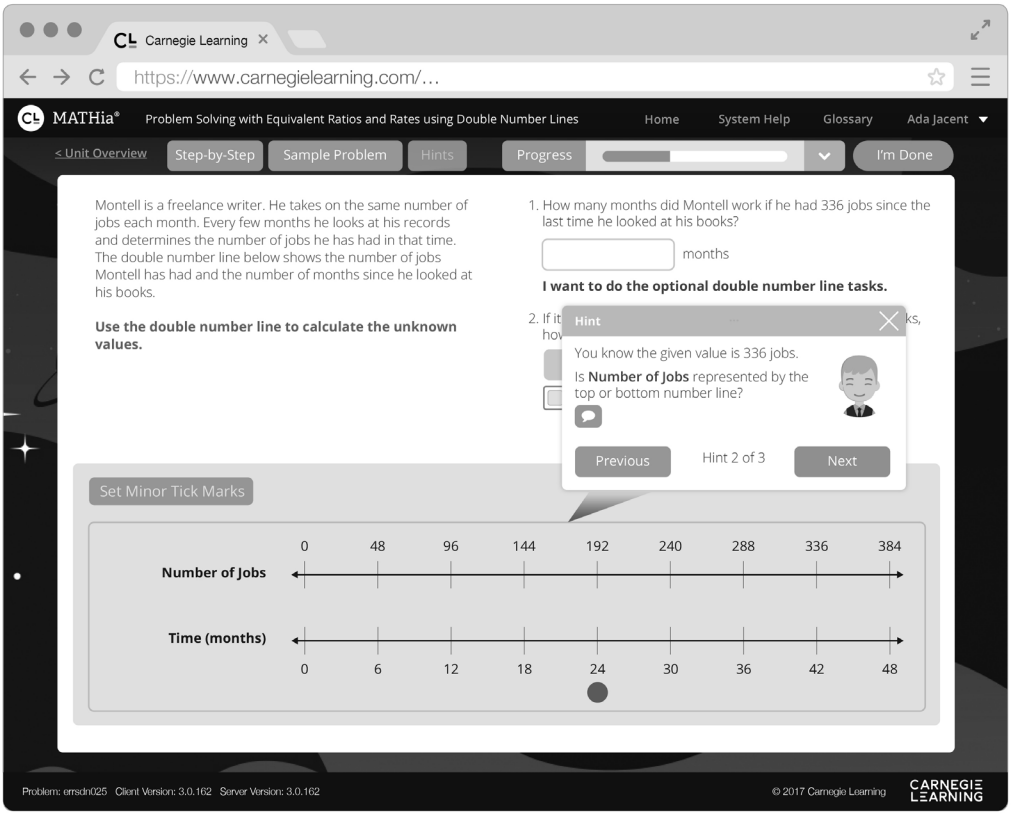
The Unit Overview page engages students in the learning experience, providing them with a clear set of learning goals, a link to the real world, and a connection back to the math they already know so they can build from it throughout the unit.

Step by Step

Step by Step demonstrates how to use the tools in a lesson by guiding students step by step through a sample math problem.

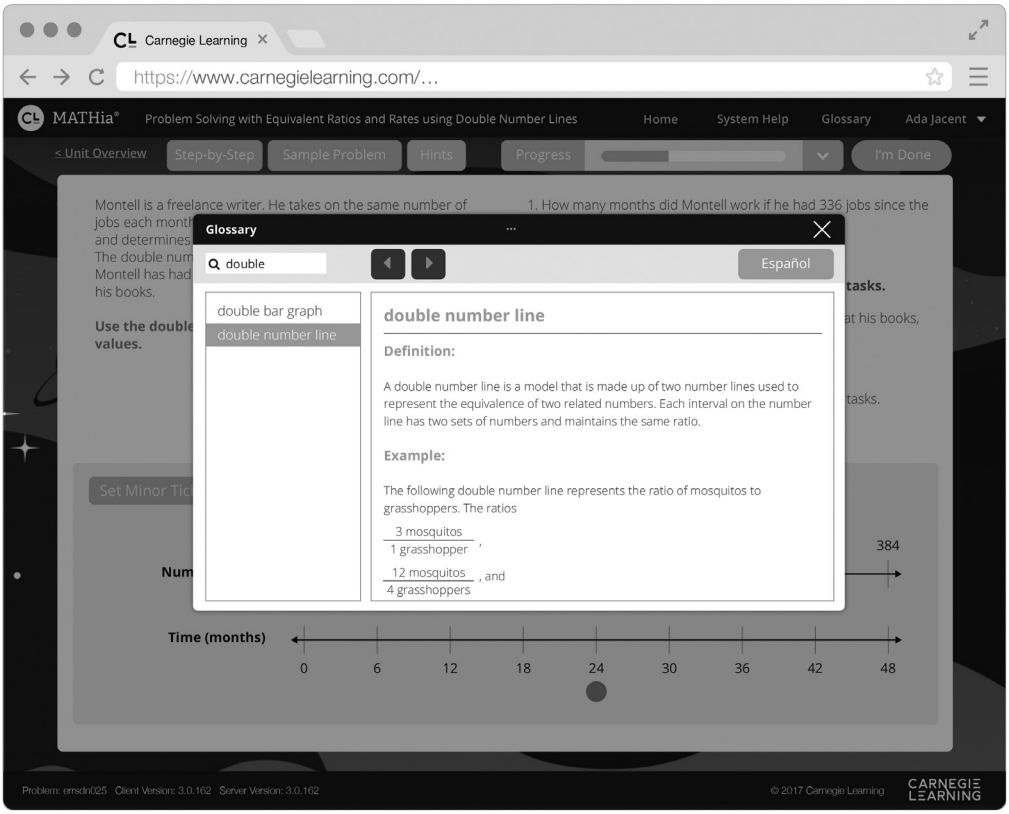
Hints

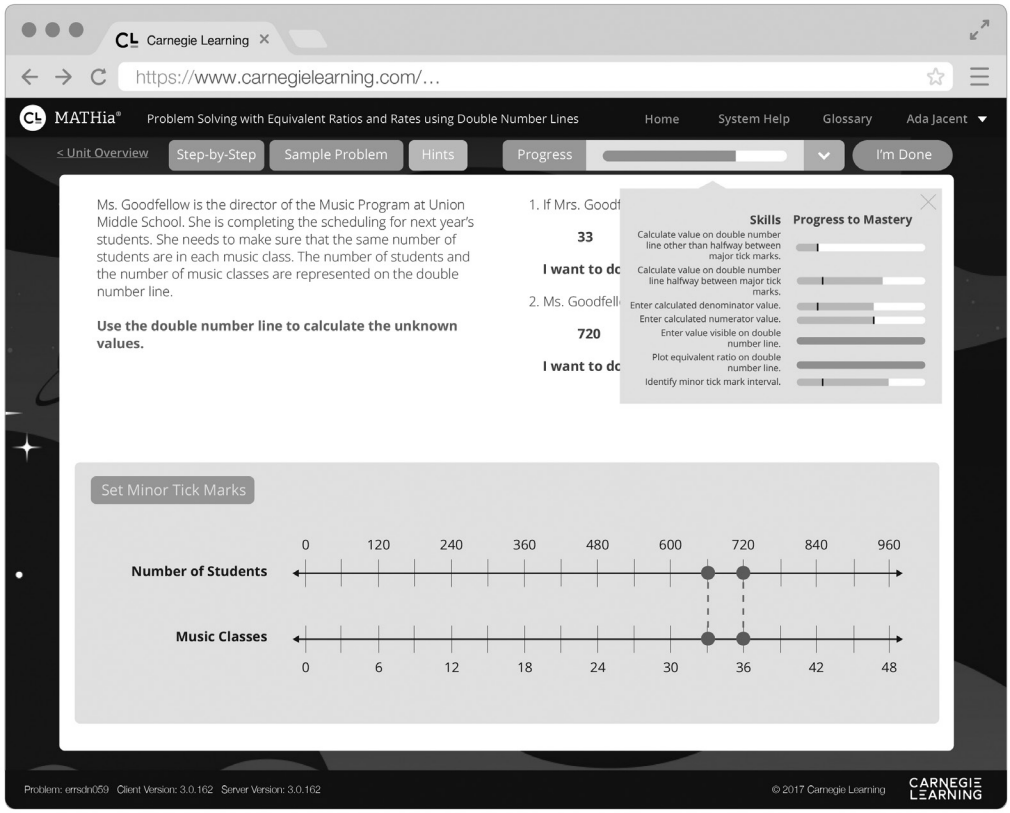
Multi-level hints are available throughout the software to help students solve the problems they are working on.



Glossary

The Glossary is available throughout the software. It contains a list of definitions and examples for key mathematical terms used throughout the curriculum.





DEVELOP AND DEMONSTRATE

Formative Assessment

The Develop and Demonstrate phases of our instructional design happen simultaneously. The reports provide the detail to interpret student performance. Facilitation and suggestions for follow-up are available via our online Resource Center.

Progress Bar

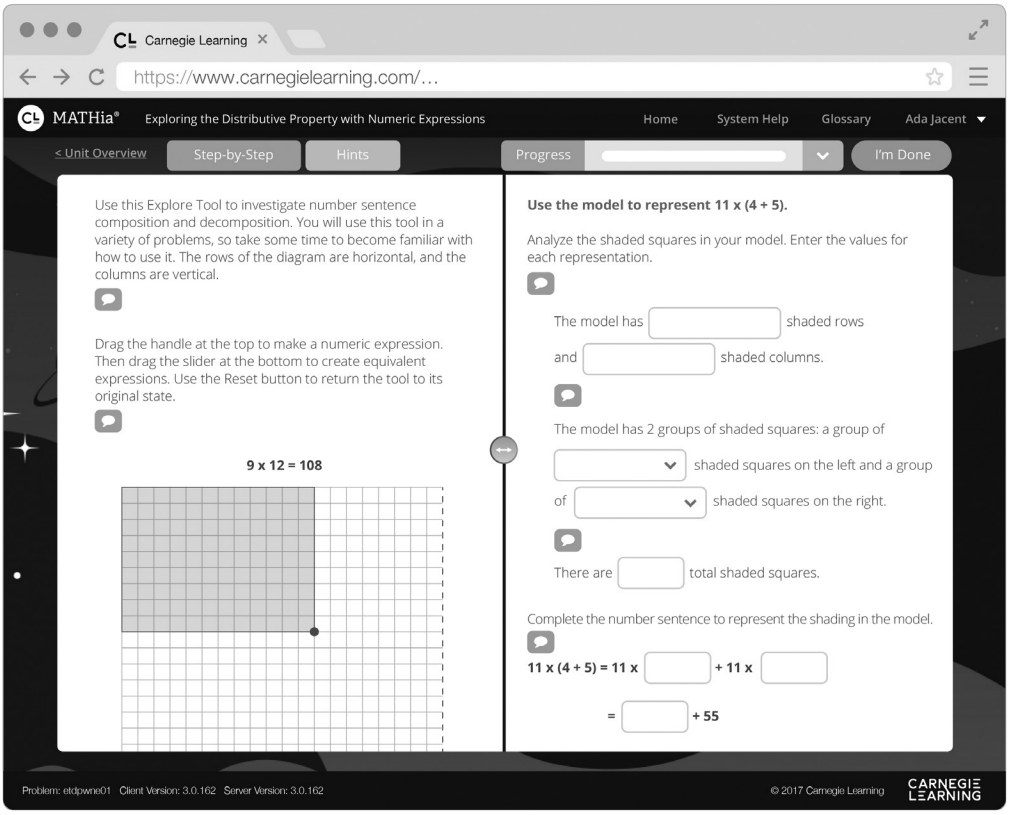
The Progress Bar shows a summary of the major skills that are being covered in a given problem-solving workspace as well as students' progress on those skills.

Problem Types in MATHia

MATHia features different instructional strategies to engage students as they develop their math skills.

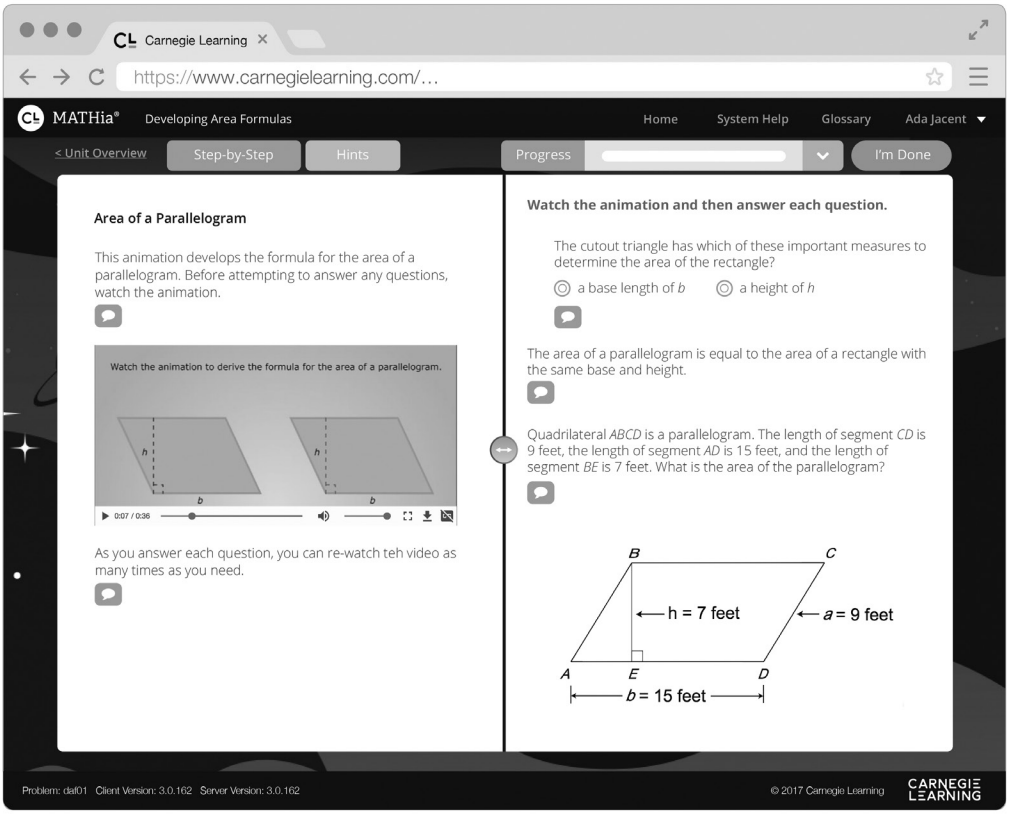
Explore Tools

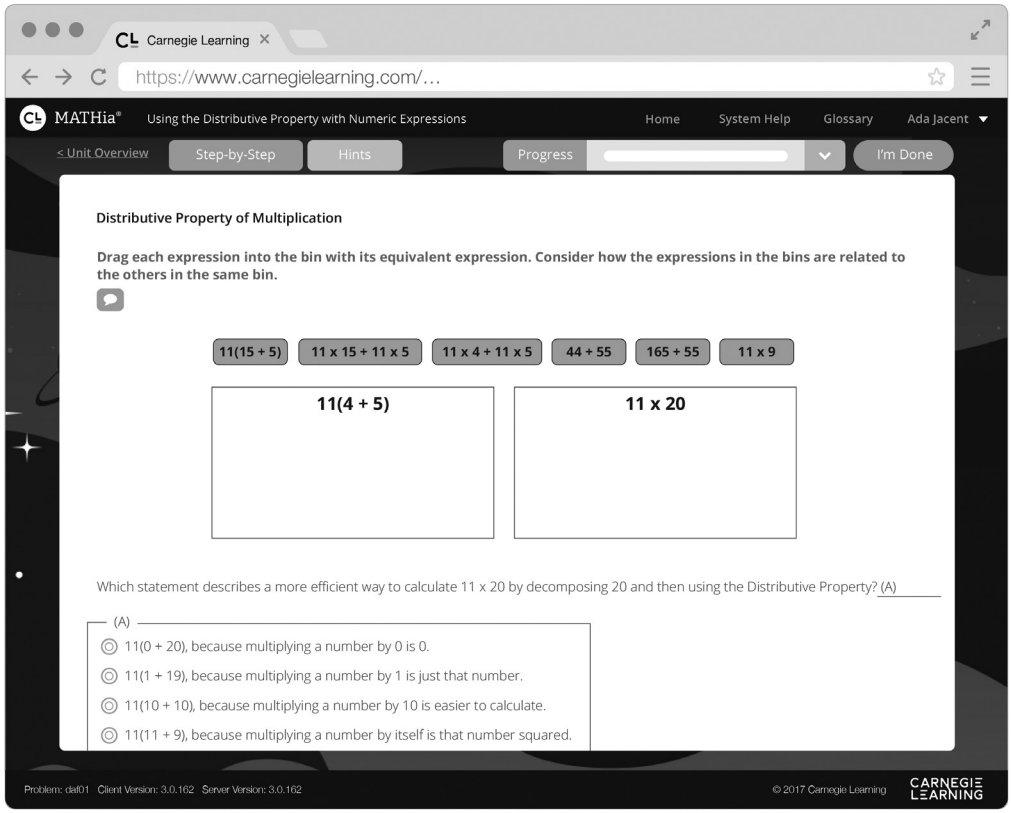
Explore Tools provide students the opportunity to investigate different mathematical concepts, search for patterns, and look for structure in ways that make sense to them. These tools also provide optional supports for students as they answer questions and solve problems.



Animations

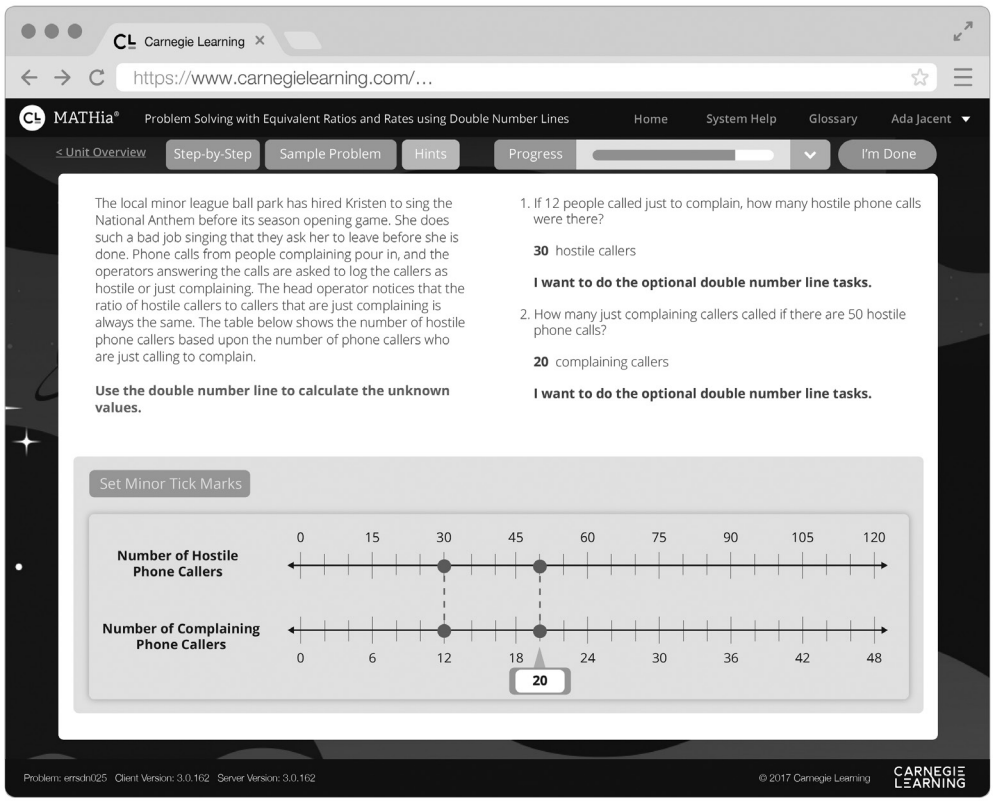
Animations provide students with an opportunity to watch, pause, and re-watch demonstrations of various mathematical concepts. They are a way to connect the visual representations of different mathematical ideas to their abstract underpinnings through visual representations and audio narrative.





Classification Tools

Classification Tools allow students to apply their mathematical understanding by categorizing answers based on similarities. These tools also provide students with the means to demonstrate proficiency in recognizing patterns in problem structure.

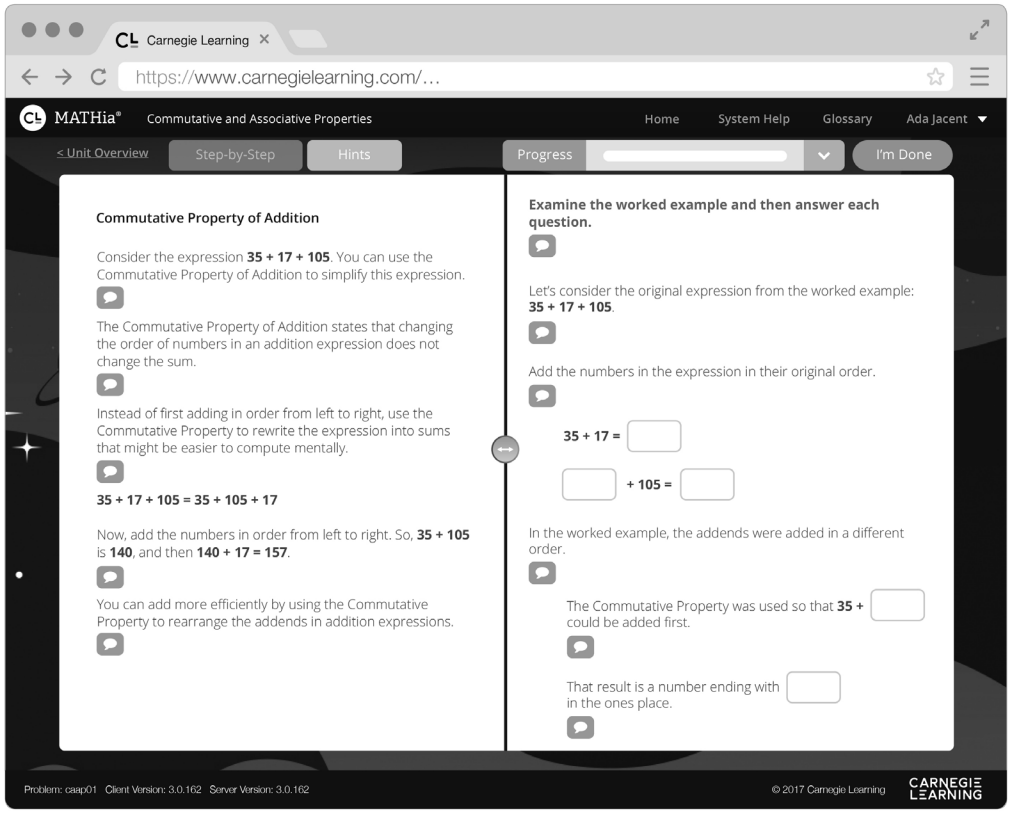


Problem-Solving Tools

Problem-Solving Tools provide students with highly individualized and self-paced instruction that adapts to their exact needs to deepen their conceptual understanding of the mathematics. Through adaptive learning technologies, they engage in reasoning and sense making.

Worked Examples

Worked Examples provide students with a tool that allows them to question their understanding, make connections with the steps, and ultimately self-explain. Analyzing Worked Examples also allows students to identify their own misconceptions, make sense of the mathematical concepts, and then ultimately to persevere in problem solving.



Teacher's Implementation Guide

The Teacher's Implementation Guide (TIG) is designed to fully support a wide-range of teachers implementing our materials: from first year teachers to 30-year veterans; from first time Carnegie Learning users to master practitioners.

One goal in developing the Teacher's Implementation Guide was to make our instructional design apparent to the users.

The lessons of each topic were written to be accessible to the full range of learners. With every instructional decision you make, keep in mind your mathematical objectives for the topic and module and the course. Plan each lesson by thinking about how you will create access for your particular group of students, maintain access and pace throughout the lesson, and assess their understanding along the way. We recommend that you do the math in each topic before implementing the activities with your specific group of students.

Visit the Texas Support Center at www.CarnegieLearning.com/texas-help for additional resources to support you anytime, anywhere.

What makes this Teacher's Implementation Guide useful?

Effective Lesson Design: Each lesson has a consistent structure for teachers and students to follow. The learning experiences are engaging and effective for students.

Pacing: Each course is designed to be taught in a 180-day school year. Pacing suggestions are provided for each lesson. Each day in the pacing guide is an equivalent to about a 45 minute instructional period.

Instructional Supports: Guiding questions are provided for teachers to use as they're circulating the room, as well as differentiation strategies, common student misconceptions, and student look fors.

Clearly Defined Mathematics: The content and instructional goals are clearly described at the module, topic, lesson, and activity levels.

The TIG is critical to understanding how the mathematics that students encounter should be realized in the classroom. The TIG describes the depth of understanding that students need to develop for each standard and a pathway for all learners to be successful. It provides differentiation strategies to support students who struggle, to extend certain activities for students who are advanced in their understanding of the content, and to support English Language Learners.

Module and Topic Overviews

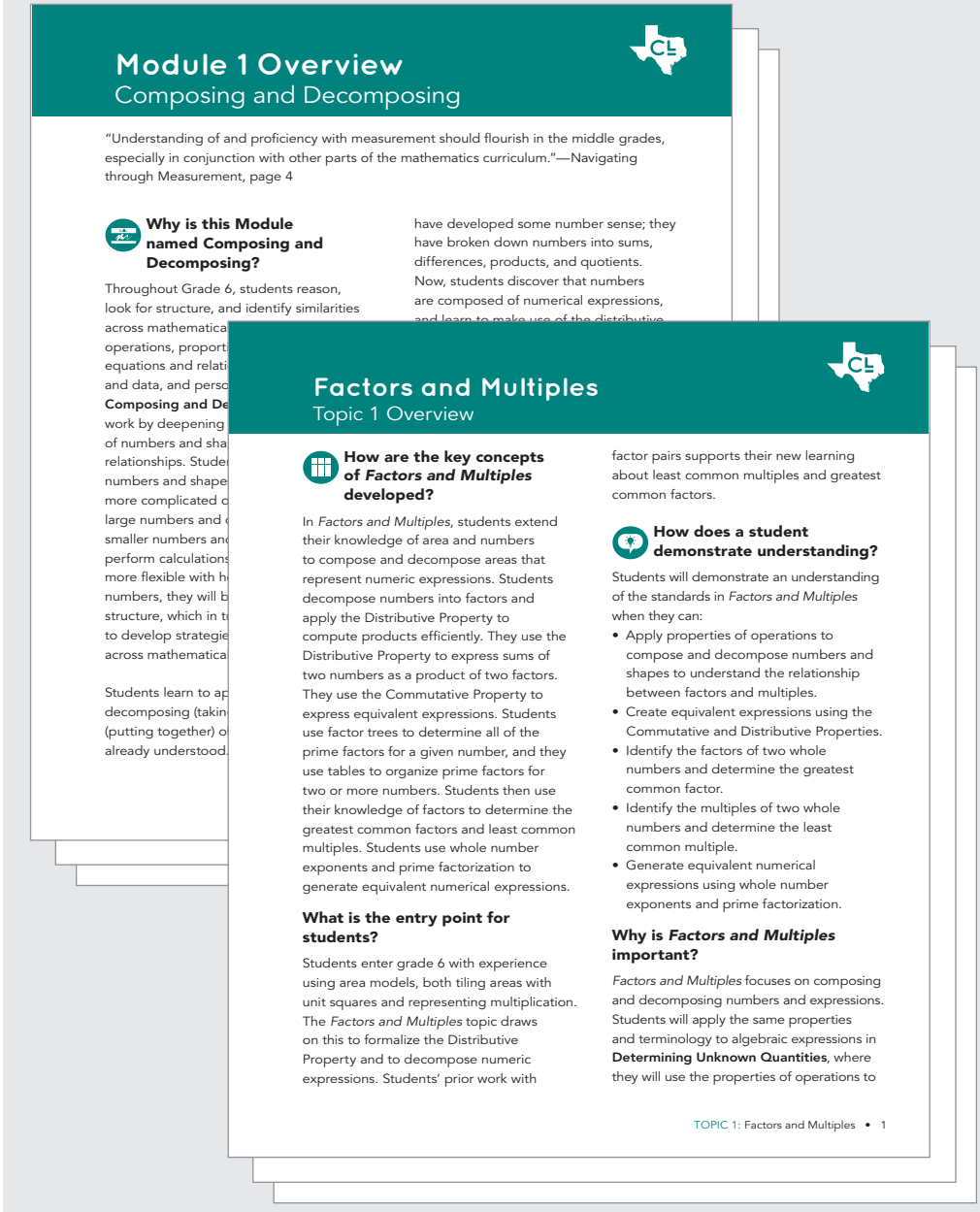
You are responsible for teaching the essential concepts associated with a particular course. You need to understand how activities within lessons build to achieve understanding within topics, and how topics build to achieve understanding throughout the course. In the Texas Math Solution, Carnegie Learning seeks to establish a shared curriculum vision with you.

Module Overview

Each module begins with an overview that describes the reasoning behind the name, the mathematics being developed, the connections to prior learning, the connections to future learning, and the pacing information.

Topic Overview

A Topic Overview describes how the topic is organized, the entry point for students, how a student will demonstrate understanding, why the mathematics is important, how the activities promote expertise in the practice standards, descriptions of the learning individually opportunities, and more detailed information to help with pacing.



Module 1 Overview
Composing and Decomposing

"Understanding of and proficiency with measurement should flourish in the middle grades, especially in conjunction with other parts of the mathematics curriculum."—Navigating through Measurement, page 4

Why is this Module named Composing and Decomposing?

Throughout Grade 6, students reason, look for structure, and identify similarities across mathematica
operations, proport
equations and relati
and data, and perso
Composing and De
work by deepening
of numbers and sha
relationships. Stude
numbers and shape
more complicated c
large numbers and
smaller numbers an
perform calculations
more flexible with h
numbers, they will b
structure, which in t
to develop strategie
across mathematica

Students learn to ap
decomposing (takin
(putting together) o
already understood.

have developed some number sense; they have broken down numbers into sums, differences, products, and quotients. Now, students discover that numbers are composed of numerical expressions,
and learn to make use of the distributive

Factors and Multiples
Topic 1 Overview

How are the key concepts of *Factors and Multiples* developed?

In *Factors and Multiples*, students extend their knowledge of area and numbers to compose and decompose areas that represent numeric expressions. Students decompose numbers into factors and apply the Distributive Property to compute products efficiently. They use the Distributive Property to express sums of two numbers as a product of two factors. They use the Commutative Property to express equivalent expressions. Students use factor trees to determine all of the prime factors for a given number, and they use tables to organize prime factors for two or more numbers. Students then use their knowledge of factors to determine the greatest common factors and least common multiples. Students use whole number exponents and prime factorization to generate equivalent numerical expressions.

What is the entry point for students?

Students enter grade 6 with experience using area models, both tiling areas with unit squares and representing multiplication. The *Factors and Multiples* topic draws on this to formalize the Distributive Property and to decompose numeric expressions. Students' prior work with factor pairs supports their new learning about least common multiples and greatest common factors.

How does a student demonstrate understanding?

Students will demonstrate an understanding of the standards in *Factors and Multiples* when they can:

- Apply properties of operations to compose and decompose numbers and shapes to understand the relationship between factors and multiples.
- Create equivalent expressions using the Commutative and Distributive Properties.
- Identify the factors of two whole numbers and determine the greatest common factor.
- Identify the multiples of two whole numbers and determine the least common multiple.
- Generate equivalent numerical expressions using whole number exponents and prime factorization.

Why is *Factors and Multiples* important?

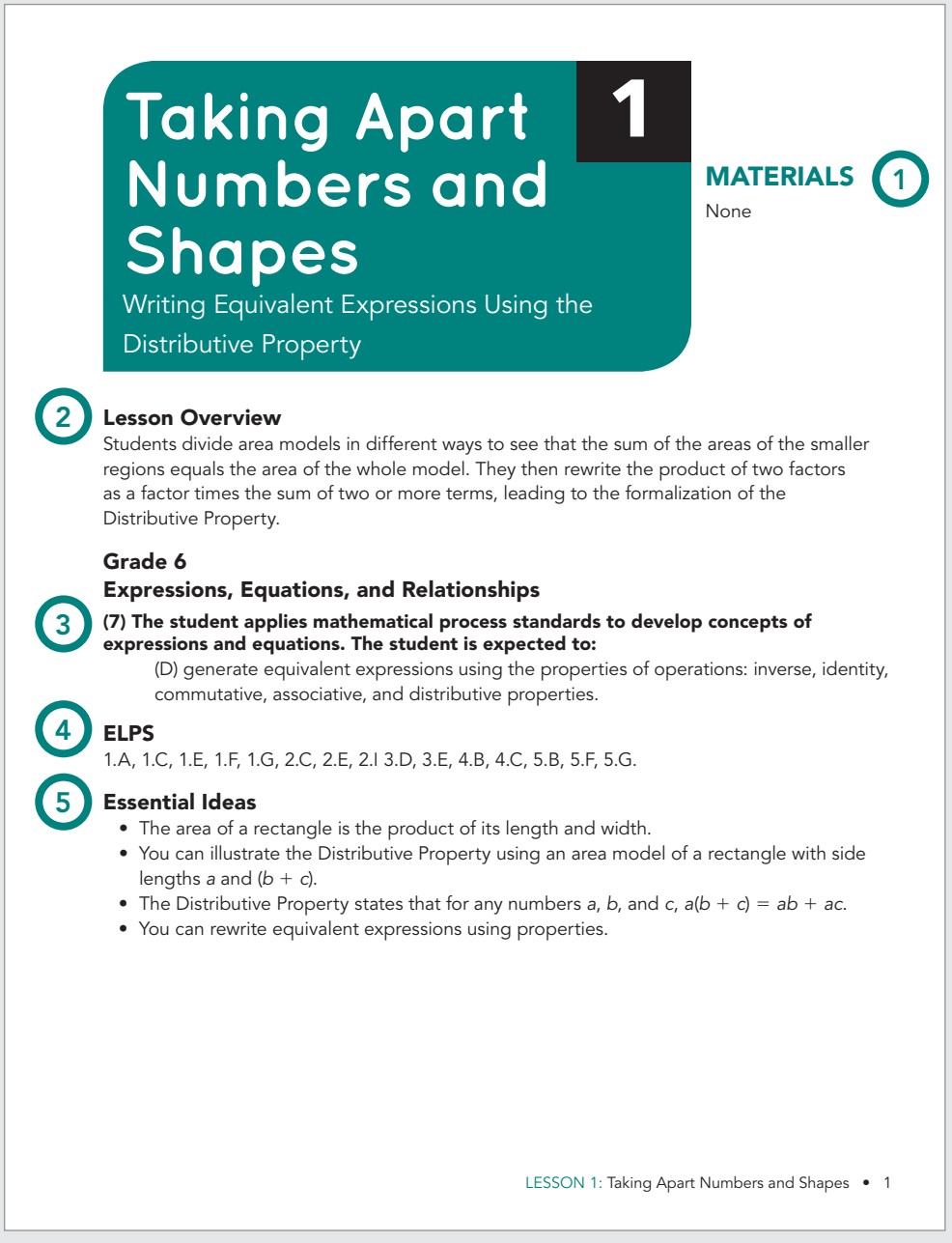
Factors and Multiples focuses on composing and decomposing numbers and expressions. Students will apply the same properties and terminology to algebraic expressions in **Determining Unknown Quantities**, where they will use the properties of operations to

TOPIC 1: Factors and Multiples • 1

"Teachers must first develop their ideas about where the curriculum program is going mathematically (curriculum vision) before deciding whether the curriculum materials will help them reach that mathematical goal (curriculum trust)" (Drake & Sherin, 2009, p. 325).

Facilitation Notes

For each lesson, you are provided with detailed facilitation notes to fully support your planning process. This valuable resource provides point-of-use support that serves as your primary resource for planning, guiding, and facilitating student learning.



Taking Apart Numbers and Shapes

1

Writing Equivalent Expressions Using the Distributive Property

MATERIALS 1

None

2 **Lesson Overview**

Students divide area models in different ways to see that the sum of the areas of the smaller regions equals the area of the whole model. They then rewrite the product of two factors as a factor times the sum of two or more terms, leading to the formalization of the Distributive Property.

Grade 6

Expressions, Equations, and Relationships

3 **(7) The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:**

(D) generate equivalent expressions using the properties of operations: inverse, identity, commutative, associative, and distributive properties.

4 **ELPS**

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G.

5 **Essential Ideas**

- The area of a rectangle is the product of its length and width.
- You can illustrate the Distributive Property using an area model of a rectangle with side lengths a and $(b + c)$.
- The Distributive Property states that for any numbers a, b, and c, $a(b + c) = ab + ac$.
- You can rewrite equivalent expressions using properties.

LESSON 1: Taking Apart Numbers and Shapes • 1

1. Materials

Materials required for the lesson are identified.

2. Lesson Overview

The Lesson Overview sets the purpose and describes the overarching mathematics of the lesson, explaining how the activities build and how the concepts are developed.

3. TEKS Addressed

The focus TEKS for each lesson are listed. Carnegie Learning recognizes that modeling is not done in isolation but instead in relationship to other TEKS. You will see these TEKS interleaved throughout the course, indicated by an asterisk(*).

4. ELPS Addressed

The English Language Proficiency Standards for each lesson are listed. As you plan, consider these ELPS and determine the instructional strategies that you will use to meet these ELPS.

5. Essential Ideas

These statements are derived from the standards and state the concepts students will develop.

6. Lesson Structure

This section highlights how the parts of the lesson fit within the instructional design: Engage, Develop, and Demonstrate. A summary of each activity is included.

7. Pacing

Lessons often span more than one 45-minute class period. Suggested pacing is provided for each lesson so that the entire course can be completed in a school year.

6

Lesson Structure and Pacing: 1 Day 7

Engage

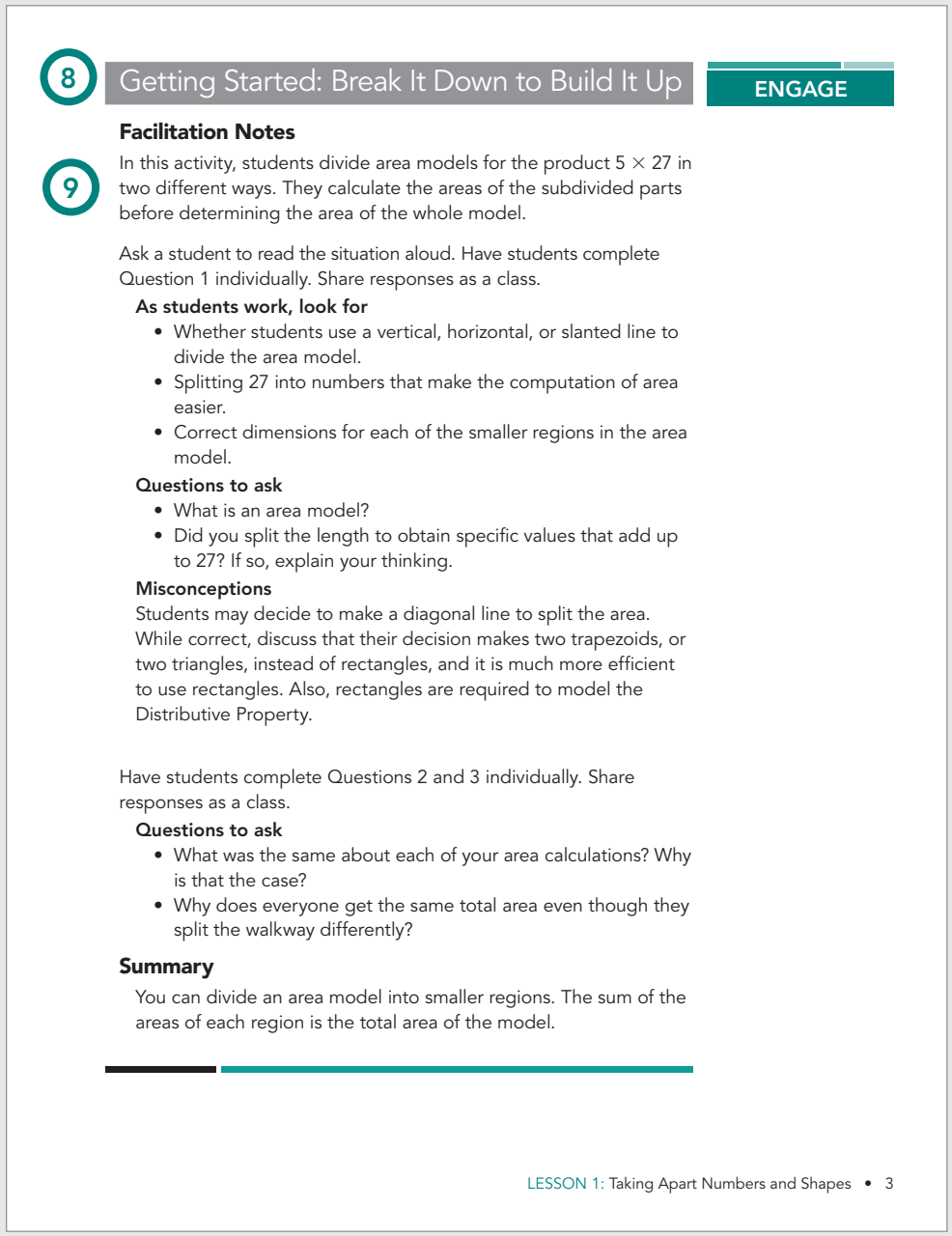
Getting Started: Break It Down to Build It Up
Students divide area models for the product 5×27 in two different ways. They calculate the areas of the subdivided parts before determining the area of the whole model.

Develop

Activity 1.1: Connecting Area Models and the Distributive Property
Students rewrite the product of two factors as a factor times the sum of two or more terms, leading to the formalization of the Distributive Property. They decompose factors and products into equivalent representations.

Demonstrate

Talk the Talk: The Floor Is Yours
Students design the floor plan in a gymnasium for different after-school activities. They represent their model using the Distributive Property and then explain their rationale.



8 Getting Started: Break It Down to Build It Up

ENGAGE

Facilitation Notes

9 In this activity, students divide area models for the product 5 × 27 in two different ways. They calculate the areas of the subdivided parts before determining the area of the whole model.

Ask a student to read the situation aloud. Have students complete Question 1 individually. Share responses as a class.

As students work, look for

- Whether students use a vertical, horizontal, or slanted line to divide the area model.
- Splitting 27 into numbers that make the computation of area easier.
- Correct dimensions for each of the smaller regions in the area model.

Questions to ask

- What is an area model?
- Did you split the length to obtain specific values that add up to 27? If so, explain your thinking.

Misconceptions

Students may decide to make a diagonal line to split the area. While correct, discuss that their decision makes two trapezoids, or two triangles, instead of rectangles, and it is much more efficient to use rectangles. Also, rectangles are required to model the Distributive Property.

Have students complete Questions 2 and 3 individually. Share responses as a class.

Questions to ask

- What was the same about each of your area calculations? Why is that the case?
- Why does everyone get the same total area even though they split the walkway differently?

Summary

You can divide an area model into smaller regions. The sum of the areas of each region is the total area of the model.

LESSON 1: Taking Apart Numbers and Shapes • 3

8. Facilitation Notes by Activity

A detailed set of guidelines walks the teacher through implementing the Getting Started, Activities, and Talk the Talk portions of the lesson. These guidelines include an activity overview, grouping strategies, guiding questions, possible student misconceptions, differentiation strategies, student look fors, and an activity summary.

9. Activity Overview

Each set of Facilitation Notes begins with an overview that highlights how students will actively engage with the task to achieve the learning goals.

> Position yourself to take full advantage of the richness of the mathematics addressed in the textbook. The Facilitation Notes provide guidance to reach each student from their current level of understanding to advance to the next stage. Place yourself in the position of the student by experiencing the textbook activities prior to class. Realize your role in the classroom—empower your students! Step back and let them do the math with confidence in their role as learner and your role as facilitator of learning.
>
> Janet Sinopoli, Instructional Designer

10. Differentiation Strategies

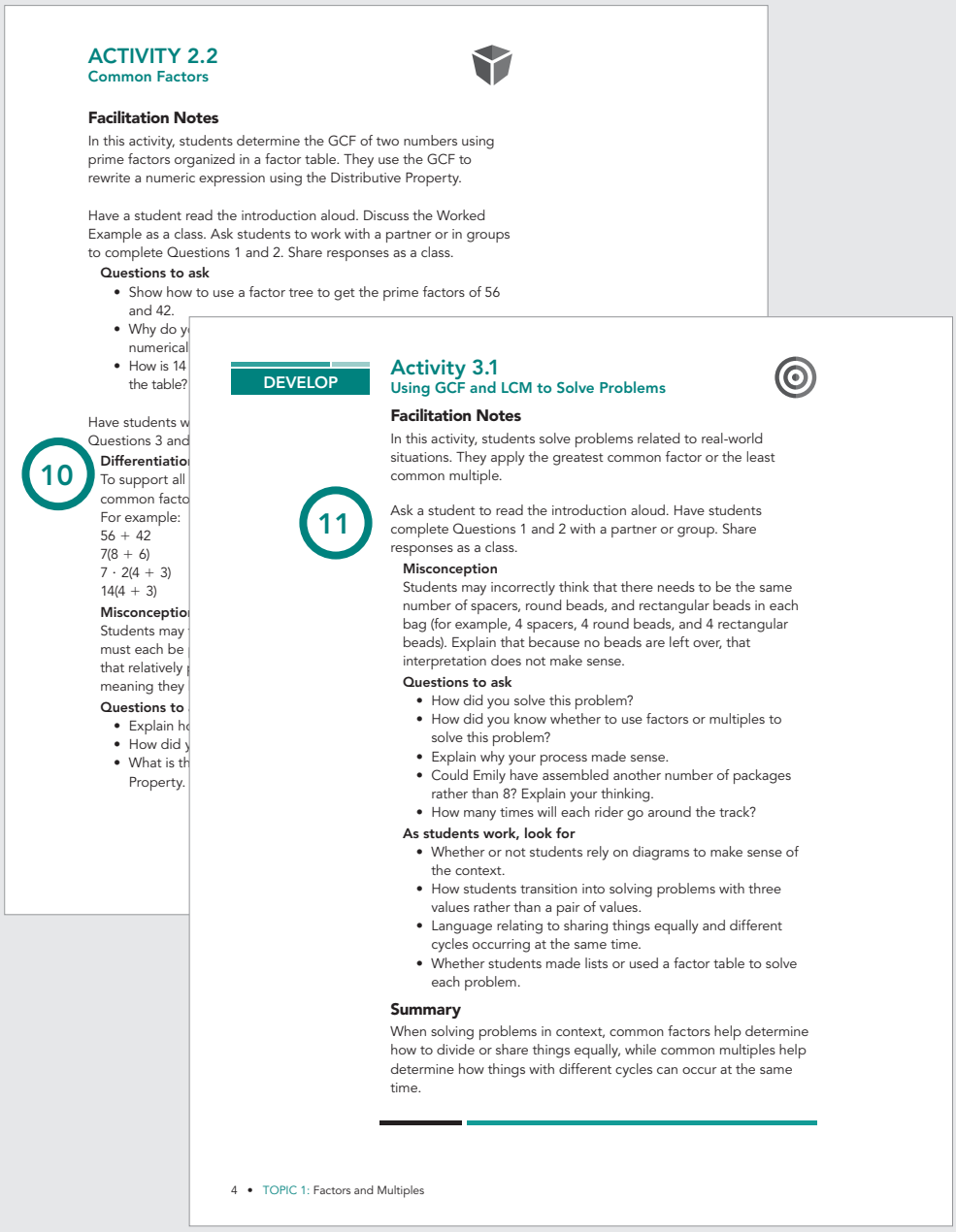
To assist all students, instructional strategies are provided that benefit the full range of learners.

11. Grouping Strategies

Suggestions appear to help chunk each activity into manageable pieces and establish the cadence of the lesson.

Learning is social. Whether students work in pairs or in groups, the critical element is that they are engaged in discussion. Carnegie Learning believes, and research supports, that student-to-student discourse is a motivating factor; it increases student learning and supports ongoing formative assessment. Additionally, it provides students with opportunities to have mathematical authority.

Working collaboratively can, when done well, encourage students to articulate their thinking (resulting in self-explanation) and also provides metacognitive feedback (by reviewing other students' approaches and receiving feedback on your own).



ACTIVITY 2.2
Common Factors

Facilitation Notes

In this activity, students determine the GCF of two numbers using prime factors organized in a factor table. They use the GCF to rewrite a numeric expression using the Distributive Property.

Have a student read the introduction aloud. Discuss the Worked Example as a class. Ask students to work with a partner or in groups to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- Show how to use a factor tree to get the prime factors of 56 and 42.
- Why do y
numerical
- How is 14
the table?

Have students w
Questions 3 and

10

Differentiatio
To support all
common facto
For example:
56 + 42
7(8 + 6)
7 · 2(4 + 3)
14(4 + 3)

Misconceptio
Students may
must each be
that relatively
meaning they

Questions to
- Explain h
- How did y
- What is th
Property.

DEVELOP

Activity 3.1
Using GCF and LCM to Solve Problems

Facilitation Notes

In this activity, students solve problems related to real-world situations. They apply the greatest common factor or the least common multiple.

11

Ask a student to read the introduction aloud. Have students complete Questions 1 and 2 with a partner or group. Share responses as a class.

Misconception
Students may incorrectly think that there needs to be the same number of spacers, round beads, and rectangular beads in each bag (for example, 4 spacers, 4 round beads, and 4 rectangular beads). Explain that because no beads are left over, that interpretation does not make sense.

Questions to ask
- How did you solve this problem?
- How did you know whether to use factors or multiples to solve this problem?
- Explain why your process made sense.
- Could Emily have assembled another number of packages rather than 8? Explain your thinking.
- How many times will each rider go around the track?

As students work, look for
- Whether or not students rely on diagrams to make sense of the context.
- How students transition into solving problems with three values rather than a pair of values.
- Language relating to sharing things equally and different cycles occurring at the same time.
- Whether students made lists or used a factor table to solve each problem.

Summary
When solving problems in context, common factors help determine how to divide or share things equally, while common multiples help determine how things with different cycles can occur at the same time.

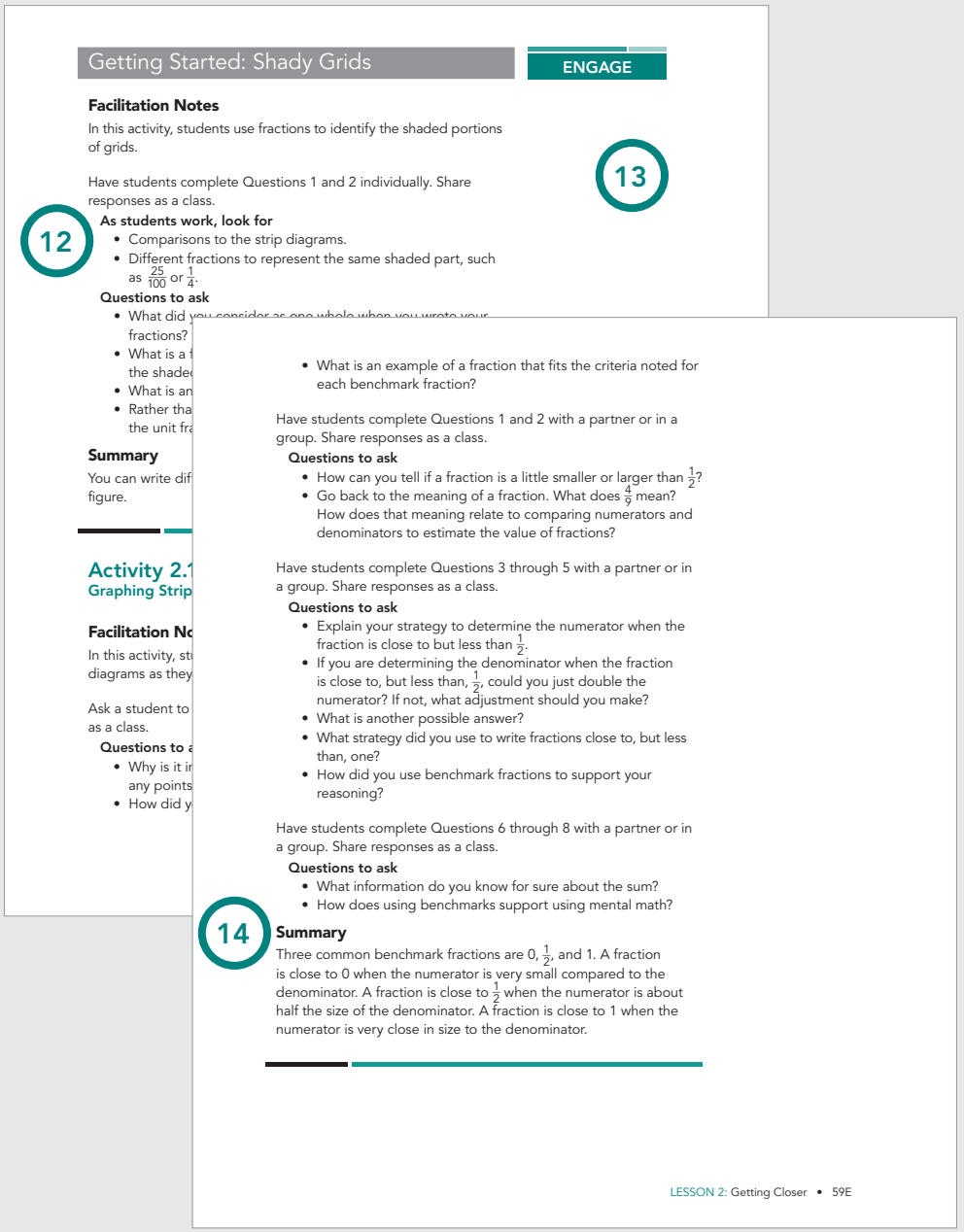
4 • TOPIC 1: Factors and Multiples

The student discussion is then transported to a classroom discussion facilitated by the teacher to guarantee all necessary mathematics is addressed, once again, with the same benefits of discussion.

Alternative Grouping Strategies

Differentiation strategies will also provide other grouping strategies, such as whole class participation and the jigsaw method, are sometimes recommended for specific activities. These are listed as Differentiation Strategies.

More information about grouping strategies is available online in the Texas Support Center at www.CarnegieLearning.com/texas-help



Getting Started: Shady Grids

ENGAGE

Facilitation Notes

In this activity, students use fractions to identify the shaded portions of grids.

Have students complete Questions 1 and 2 individually. Share responses as a class.

12

As students work, look for

- Comparisons to the strip diagrams.
- Different fractions to represent the same shaded part, such as $\frac{25}{100}$ or $\frac{1}{4}$.

Questions to ask

- What did you consider as one whole when you wrote your fractions?
- What is a f… the shade…
- What is an…
- Rather tha… the unit fra…

Summary

You can write dif… figure.

13

Activity 2.1
Graphing Strip…

Facilitation No…

In this activity, st… diagrams as they…

Ask a student to … as a class.

Questions to a…

- Why is it i… any points…
- How did y…

- What is an example of a fraction that fits the criteria noted for each benchmark fraction?

Have students complete Questions 1 and 2 with a partner or in a group. Share responses as a class.

Questions to ask

- How can you tell if a fraction is a little smaller or larger than $\frac{1}{2}$?
- Go back to the meaning of a fraction. What does $\frac{4}{9}$ mean? How does that meaning relate to comparing numerators and denominators to estimate the value of fractions?

Have students complete Questions 3 through 5 with a partner or in a group. Share responses as a class.

Questions to ask

- Explain your strategy to determine the numerator when the fraction is close to but less than $\frac{1}{2}$.
- If you are determining the denominator when the fraction is close to, but less than, $\frac{1}{2}$, could you just double the numerator? If not, what adjustment should you make?
- What is another possible answer?
- What strategy did you use to write fractions close to, but less than, one?
- How did you use benchmark fractions to support your reasoning?

Have students complete Questions 6 through 8 with a partner or in a group. Share responses as a class.

Questions to ask

- What information do you know for sure about the sum?
- How does using benchmarks support using mental math?

14

Summary

Three common benchmark fractions are 0, $\frac{1}{2}$, and 1. A fraction is close to 0 when the numerator is very small compared to the denominator. A fraction is close to $\frac{1}{2}$ when the numerator is about half the size of the denominator. A fraction is close to 1 when the numerator is very close in size to the denominator.

LESSON 2: Getting Closer • 59E

12. As Students Work, Look For

These notes provide specific language, strategies, and/or errors to look and listen for you as you circulate and monitor students working in pairs or groups. You can incorporate these ideas when students share their responses with the class.

Note

Talk the Talk helps you to assess student learning and to make decisions about helpful connections you need to make in future lessons.

13. White Space

The white space in each margin is intentional. Use this space to make additional planning notes or to reflect on the implementation of the lesson.

14. Summary

The summary brings the activity to closure. This statement encapsulates the big mathematical ideas of the particular activity.

Supporting English Learners

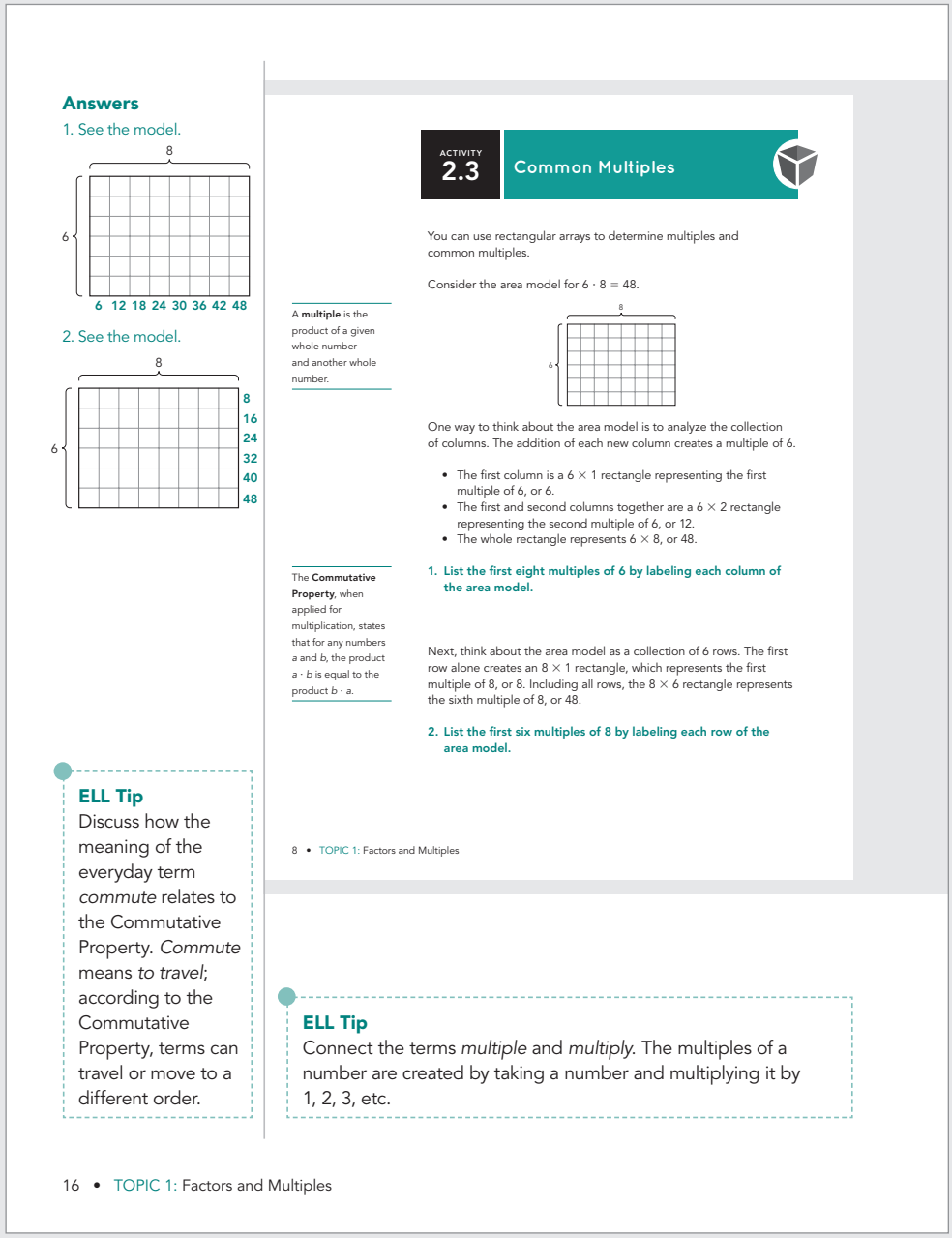
English learners often face multiple challenges in the mathematics classroom beyond language development skills, including a lack of confidence, peer-to-peer understanding, and building solid conceptual mastery. The Carnegie Learning Texas Math Solution seeks to support English Learners (ELs) as they develop skills in both mathematics and language.

Throughout instruction, EL tips are placed for teachers at point-of-use on the mini-lesson page in the TIG. They provide additional modifications to support this special population.

These tips:

- Inform teachers of potential learning obstacles specific to the lesson.
- Provide engaging activities for learning and assessment.
- Reinforce newly acquired mathematic language to gain an increasing level of comprehension of English.
- Introduce students to language needed to understand a specific context.

Students internalize new content language by using and reusing it in meaningful ways in a variety of different speaking activities that build concept and language attainment.



Answers

1. See the model.

2. See the model.

ELL Tip
Discuss how the meaning of the everyday term *commute* relates to the Commutative Property. *Commute* means *to travel*; according to the Commutative Property, terms can travel or move to a different order.

16 • TOPIC 1: Factors and Multiples

ACTIVITY 2.3 Common Multiples

You can use rectangular arrays to determine multiples and common multiples.

Consider the area model for $6 \cdot 8 = 48$.

A **multiple** is the product of a given whole number and another whole number.

One way to think about the area model is to analyze the collection of columns. The addition of each new column creates a multiple of 6.

- The first column is a 6×1 rectangle representing the first multiple of 6, or 6.
- The first and second columns together are a 6×2 rectangle representing the second multiple of 6, or 12.
- The whole rectangle represents 6×8, or 48.

1. List the first eight multiples of 6 by labeling each column of the area model.

The **Commutative Property**, when applied for multiplication, states that for any numbers a and b, the product $a \cdot b$ is equal to the product $b \cdot a$.

Next, think about the area model as a collection of 6 rows. The first row alone creates an 8×1 rectangle, which represents the first multiple of 8, or 8. Including all rows, the 8×6 rectangle represents the sixth multiple of 8, or 48.

2. List the first six multiples of 8 by labeling each row of the area model.

ELL Tip
Connect the terms *multiple* and *multiply*. The multiples of a number are created by taking a number and multiplying it by 1, 2, 3, etc.

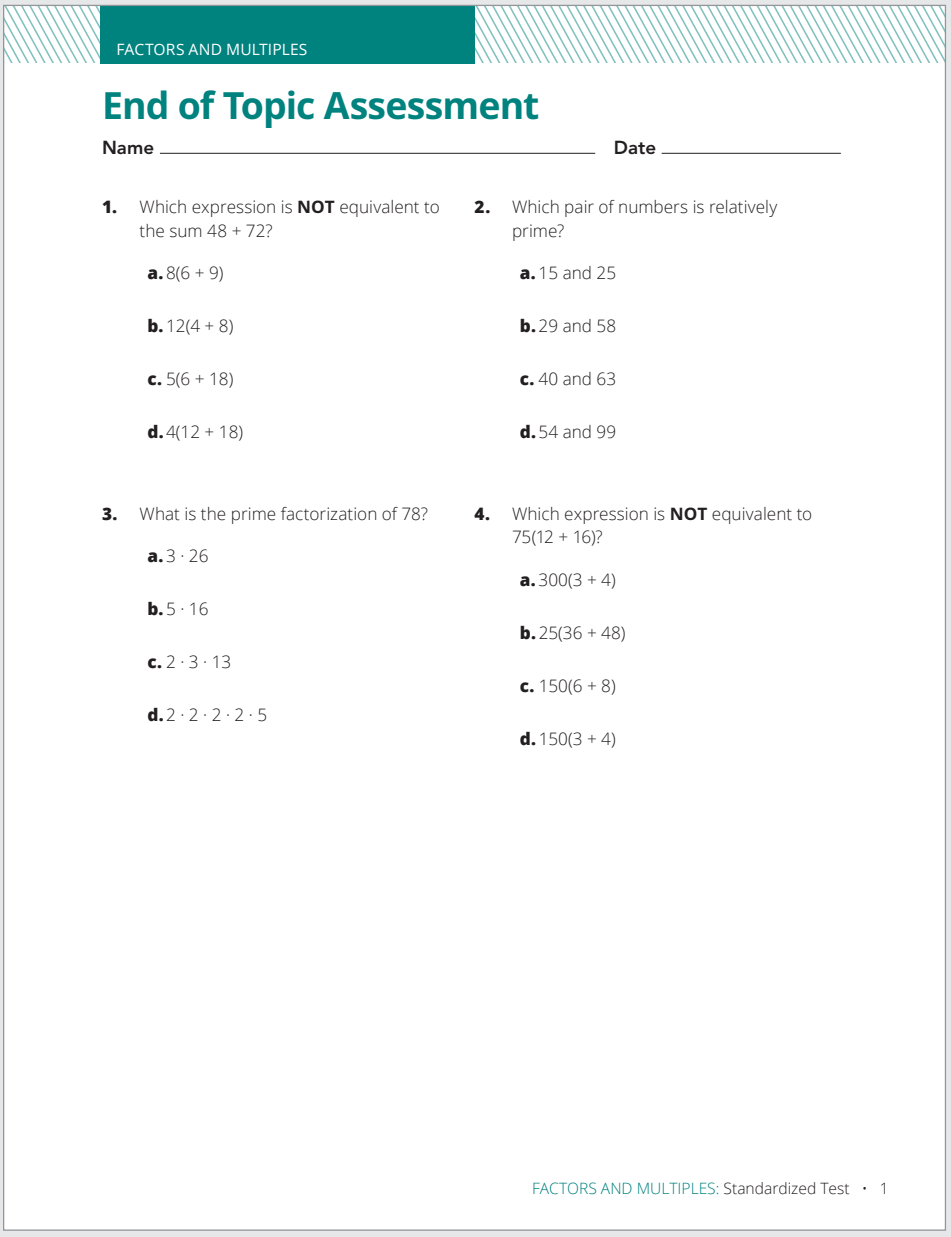
8 • TOPIC 1: Factors and Multiples

For More Support

Visit the Texas Support Center for many more resources to support you and your students who are English Learners.

Assessments

Formative assessment tools are provided throughout each lesson and workspace, providing you with ongoing feedback of student performance and encouraging students to monitor their own progress. End of Topic summative assessments are provided to measure student performance on a clearly denoted set of standards For certain Topics that extend longer than four instructional weeks, a standardized Mid-Topic Assessment is also provided.



FACTORS AND MULTIPLES

End of Topic Assessment

Name _____ **Date** _____

1. Which expression is **NOT** equivalent to the sum 48 + 72?

a. 8(6 + 9)

b. 12(4 + 8)

c. 5(6 + 18)

d. 4(12 + 18)

2. Which pair of numbers is relatively prime?

a. 15 and 25

b. 29 and 58

c. 40 and 63

d. 54 and 99

3. What is the prime factorization of 78?

a. 3 · 26

b. 5 · 16

c. 2 · 3 · 13

d. 2 · 2 · 2 · 2 · 5

4. Which expression is **NOT** equivalent to 75(12 + 16)?

a. 300(3 + 4)

b. 25(36 + 48)

c. 150(6 + 8)

d. 150(3 + 4)

FACTORS AND MULTIPLES: Standardized Test • 1

End of Topic Assessment

Multiple choice questions help students prepare for standardized tests. All items are multiple choice.

Supporting Students to Use Mathematical Tools

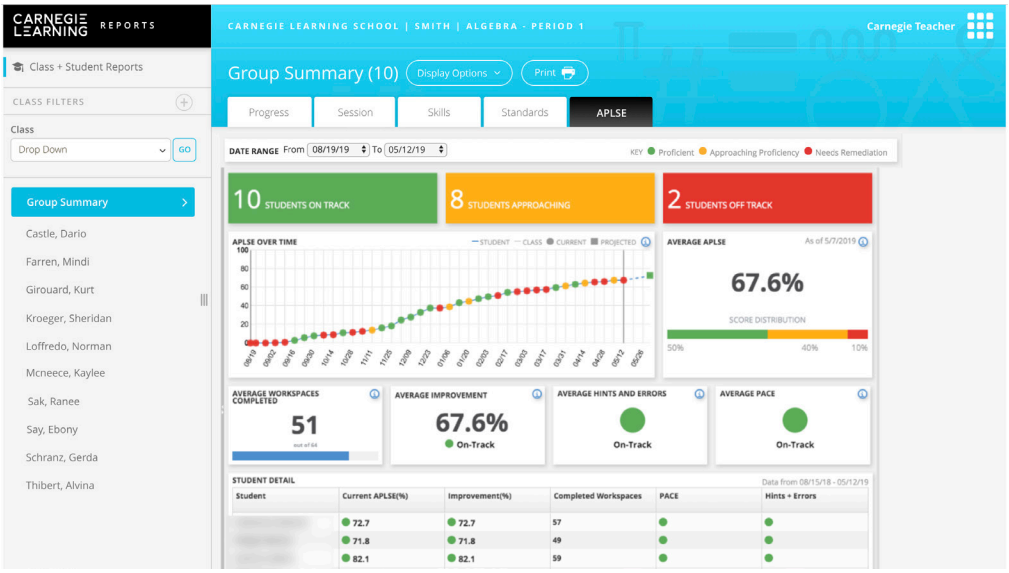
Visit the Texas Support Center for strategies to support students as they use mathematical tools, including formula charts and reference sheets.

Assessing Student Learning in MATHia

MATHia provides easy-to-use reports for you to have insight into your class and individual student's progress. Data from these reports create action—whether determining how many students are mastering standards, to grouping your students into smaller earning groups, and teacher-student conferencing.

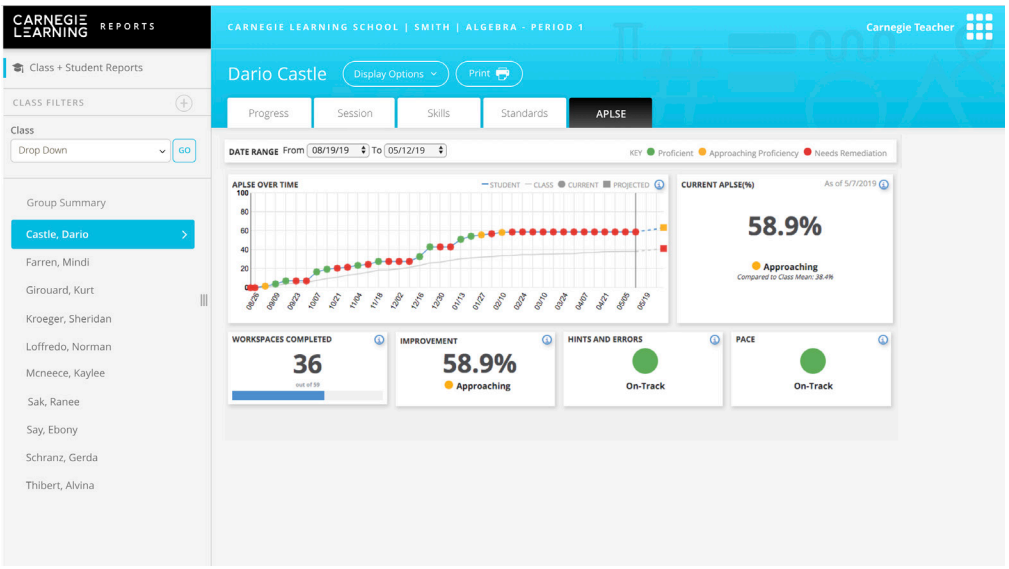
APLSE

The Adaptive Personalized Learning ScorE (APLSE) Report is a predictive report that displays class and student progress over time. The APLSE Report takes all aspects of a class or student's work into consideration and provides each class and student with an APLSE Score.



Class View

The class view of the APLSE Report provides insight into the current overall progress of the entire class as well as the current projection to year-end performance.

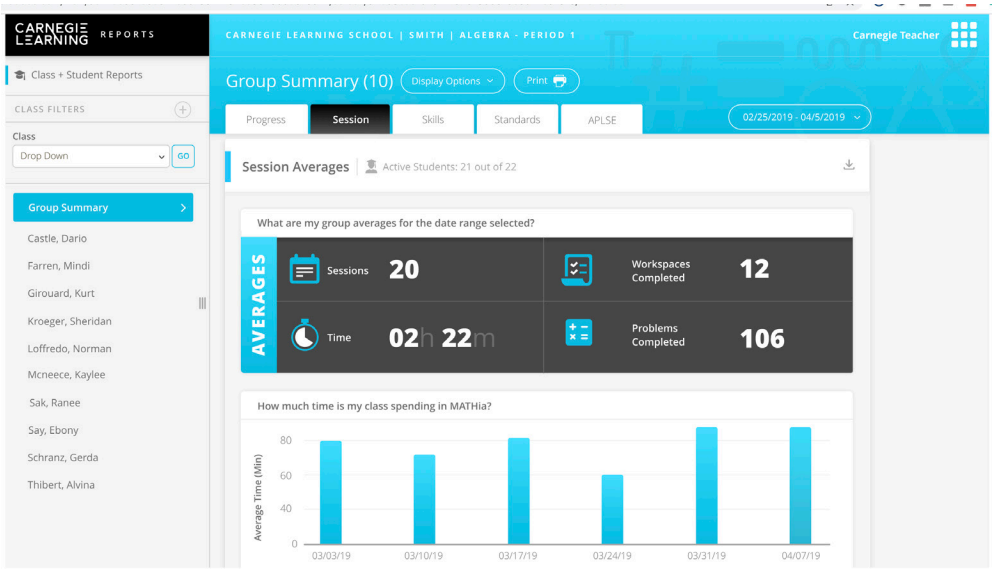


Student View

The student view of the APLSE Report displays the student's current APLSE Score, and whether or not the student is on track to complete the curriculum by the end of the class.

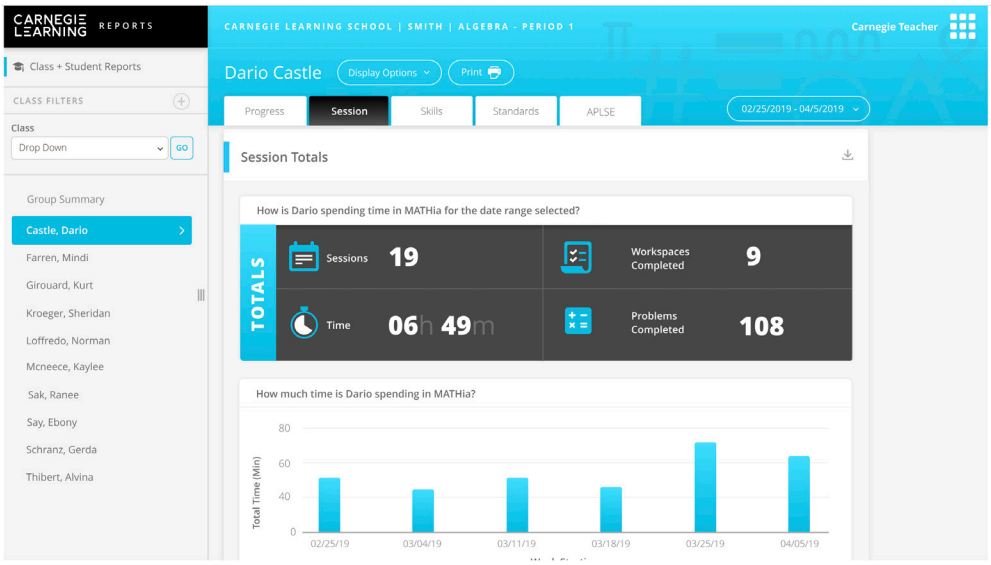
Session Report

The Session Report is designed to give you a day-to-day view of work being completed by students.



Class View

The class view of this report gives you a clear view of student work completed during a single class period, a week in the lab, or up to a five-week stretch.

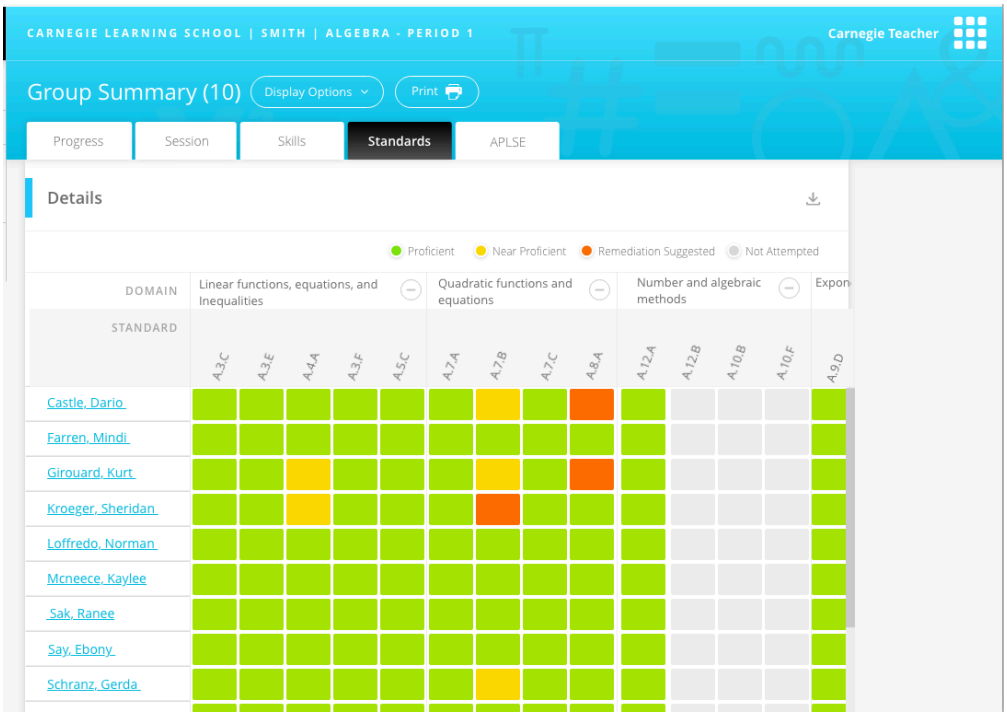


Student View

All the metrics from the Class Session Report are the same for the Student Session Report, except instead of class averages, you see actual individual student metrics for the selected date range.

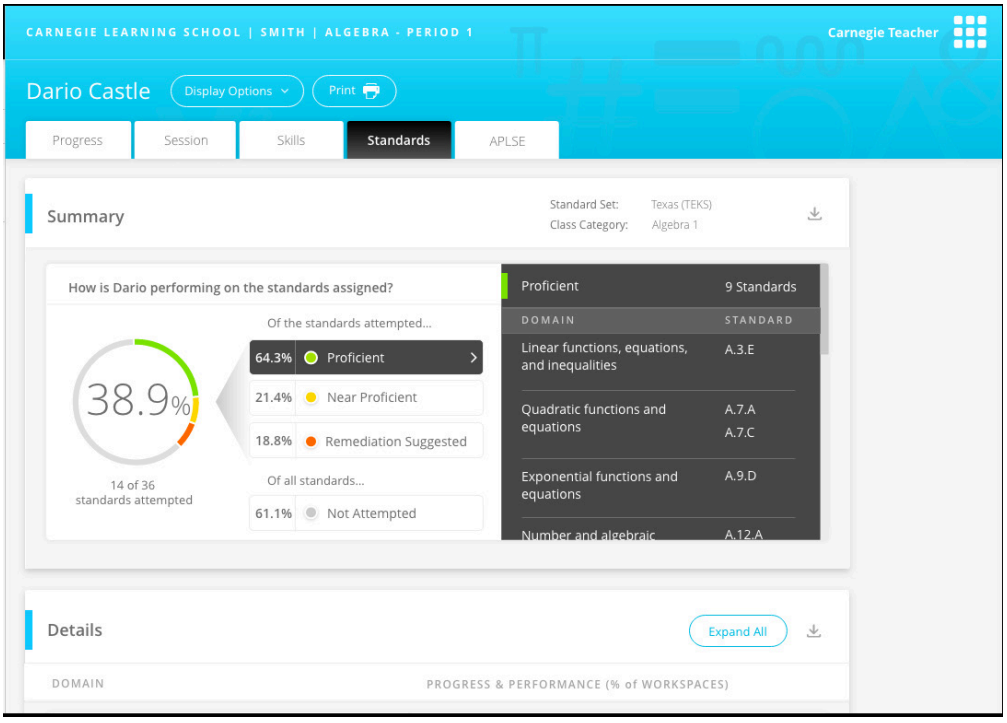
Standards Report

The Standards Report is designed to provide an easy view into how well students are mastering, or have mastered, specific standards.



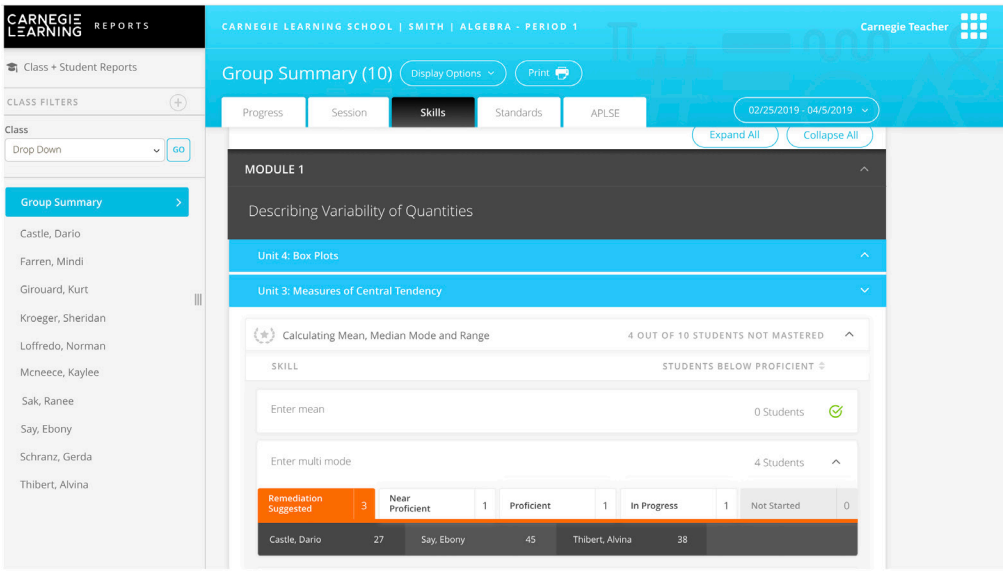
Class View

The class view of the Standards Report displays summary-level data for progress and performance on the standards assigned in the curriculum.



Student View

The Student Standards Report displays progress and performance data on the standards assigned in the curriculum.

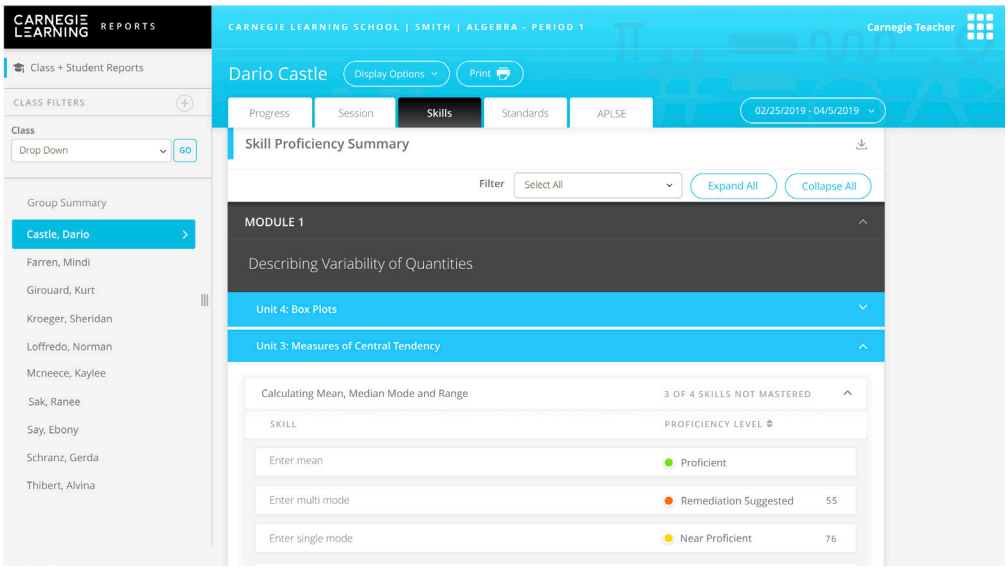


Student Detail Report

The Student Detail Report provides detailed information about the class and student progress and performance at the module, unit, and workspace levels in MATHia.

Class View

The class summary view of the Student Detail Report monitors class-level progress through the software. The data shows the current module placement for all students in the class, displaying totals for percentage of the syllabus completed, time spent on task, and completed modules, units, and workspaces.



Student View

The Student Detail Report monitors student progress and efforts in very specific content areas. The report identifies student progress across the entire syllabus, including syllabus, module, unit, and workspace completion status, total time spent in each unit, and performance scores for each completed workspace.

Reporting Scenarios

Additional reports are available. The full set of MATHia reports are located in the Teachers Toolkit in MyCL.

Each time students log into MATHia, each student's data is constantly recorded and assessed while the software is also adapting programmatically to the mastery level of each individual student. You can use our reporting system to continually assess this progress and use the results to create individualized, data-driven learning plans.

The table shown describes how MATHia reports can be used at the individual student or class level.

If you would like to …	… then, run this report:	Class or Student View
Identify current student placement in a class	Student Detail Report	Class View
Prepare for parent conferences or IEP meetings	APLSE Progress Report or Student Detail Report	Student View
Locate class-level summary data helpful for grading	APLSE Progress Report	Student View
Group students according to standards progress	Standards Report	Class View
Summarize class progress in the curriculum	Student Detail Report	Class View
View a summary of how a student is progressing in the software	Student Detail Report	Student View
Identify a student's most recent session	Session Report	Student View
Summarize student usage data	Session Report or Student Detail Report	Student View

Getting Ready

Carnegie Learning recognizes that it is the classroom teachers who make the material come alive for students, transforming the way math is taught. Implementation requires integrating learning together and learning individually.

Prepare for Learning Together

PREPARE YOURSELF

The most important first step you can take in preparing to teach with these instructional materials is to become comfortable with the mathematics.

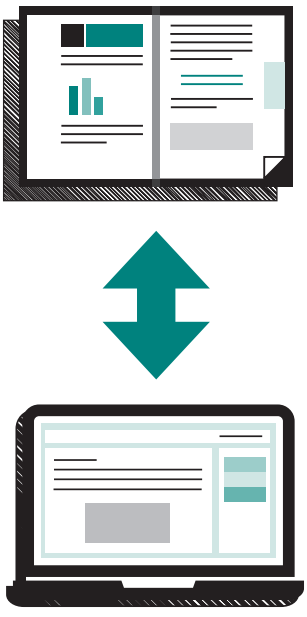
- Read through the Module 1 Overview and the Topic 1 Overview.
- Do the math of the first Topic, and consider the facilitation notes.
- Prepare team building activities to intentionally create a student-centered environment.

Prepare for Learning Individually

Plan how you will introduce students to MATHia. Explain to them the benefits of working individually and why practice is important.

- Test out the computers or tablets that your students will be using.
- Set up classes in Teacher's Toolkit.
- Assign yourself to your class so you can work through the math, too.

Prepare for Connecting the Text and MATHia



Think about strategies to help students make connections between the two learning experiences.

- Structure both environments similarly (e.g., warm-up, student work time, and closure). Provide closure around the mathematical concepts encountered each day in either environment to ensure a smooth transition. Additionally, use this time to celebrate student successes.
- As students work in the textbook, specifically ask, "Remember doing this in MATHia?" or "How would you answer this in MATHia?"
- As students work on the software, specifically ask, "How did we solve this in the textbook?" or "Does this look similar to a problem that we've done in the textbook?"

PREPARE YOUR CLASSROOM

Prepare the Environment

The classroom is often considered the third teacher. Consider how to create a learning environment that engages students and fosters a sense of ownership. The use of space in your classroom should be flexible and encourage open sharing of ideas. If you are in person, consider the following:

- Consider how your students are going to use the consumable book. It is the student's record of their learning. Many teachers have students move an entire topic to a three-ring binder as opposed to carrying the entire book.
- Arrange your desks so students can talk and collaborate with each other.
- Prepare a toolkit for groups to use as they work together and share their reasoning (read the materials list in each Topic Overview).
- Consider where you will display student work, both complete and in-progress.
- Create a word wall of key terms used in the text and MATHia.

PREPARE YOUR STUDENTS

Prepare the Learners

If you expect students to work well together, they need to understand what it means to collaborate and how it will benefit them. It is important to establish classroom guidelines and structure groups to create a community of learners.

- Facilitate team building activities and encourage students to learn each others' names.
- Set clear expectations for how the class will interact:
  - Their text is a record of their learning and is to be used as a reference for any assignments or tests you give.
  - They will be doing the thinking, talking, and writing in your classroom.
  - They will be working and sharing their strategies and reasoning with their peers.
  - Mistakes and struggles are normal and necessary.

PREPARE GUARDIANS

Prepare the Support

- Prepare a letter to send home on the first day.
- Encourage guardians to read the introduction of the student book or visit our website at **www.CarnegieLearning.com**.
- Ensure that guardians receive the Family Guide at the start of the first topic and each subsequent topic.
- Consider a Family Math Night some time within the first few weeks of the school year.
- Encourage guardians to explore the Students & Caregivers Portal on the Texas Support Center at www.CarnegieLearning.com/texas-help.

Home Connection

Research has proven time and again that family engagement greatly improves a student's likelihood of success in school.

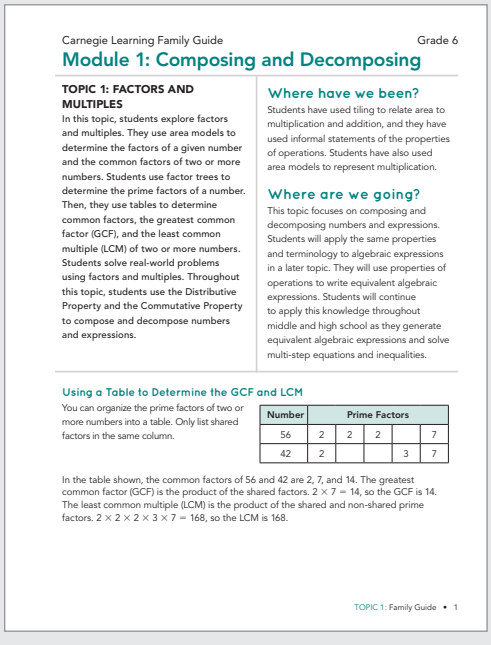
The Students & Caregivers Portal on the Texas Support Center provides:

- Getting to Know Carnegie Learning video content to provide an introduction to the instructional materials and research.
- Getting Started Guide with system requirements for MATHia.
- Articles and quick tip videos offering strategies for how guardians can support student learning. **Visit the Texas Support Center regularly to access new content and resources for students and caregivers as they learn mathematics in a variety of environments outside of the classroom.**

Family Guides

Each topic contains a Family Guide that overviews the mathematics of the topic, how that math is connected to what students already know, and how that knowledge will be used in future learning. It also may include an illustration of math from the real-world, a sample standardized test question, information to bust math myths, talking points or questions caregivers can use with their students, and a few of the key terms that students will learn.

We recognize that learning outside of the classroom is crucial to students' success at school. While we don't expect parents to be math teachers, the Family guides are designed to assist caregivers as they talk to their students about what they are learning. Our hope is that both the students and their parents will read and benefit from the guides.



Carnegie Learning Family Guide Grade 6

Module 1: Composing and Decomposing

TOPIC 1: FACTORS AND MULTIPLES

In this topic, students explore factors and multiples. They use area models to determine the factors of a given number and the common factors of two or more numbers. Students use factor trees to determine the prime factors of a number. Then, they use tables to determine common factors, the greatest common factor (GCF), and the least common multiple (LCM) of two or more numbers. Students solve real-world problems using factors and multiples. Throughout this topic, students use the Distributive Property and the Commutative Property to compose and decompose numbers and expressions.

Where have we been?

Students have used tiling to relate area to multiplication and addition, and they have used informal statements of the properties of operations. Students have also used area models to represent multiplication.

Where are we going?

This topic focuses on composing and decomposing numbers and expressions. Students will apply the same properties and terminology to algebraic expressions in a later topic. They will use properties of operations to write equivalent algebraic expressions. Students will continue to apply this knowledge throughout middle and high school as they generate equivalent algebraic expressions and solve multi-step equations and inequalities.

Using a Table to Determine the GCF and LCM

You can organize the prime factors of two or more numbers into a table. Only list shared factors in the same column.

Number	Prime Factors				
56	2	2	2		7
42	2			3	7

In the table shown, the common factors of 56 and 42 are 2, 7, and 14. The greatest common factor (GCF) is the product of the shared factors. $2 \times 7 = 14$, so the GCF is 14. The least common multiple (LCM) is the product of the shared and non-shared prime factors. $2 \times 2 \times 2 \times 3 \times 7 = 168$, so the LCM is 168.

TOPIC 1: Family Guide • 1

Myth: "I don't have the math gene."

Let's be clear about something. There isn't ***a*** gene that controls the development of mathematical thinking. Instead, there are probably ***hundreds*** of genes that contribute to it.

A recent study suggests that mathematical thinking arises from the ability to learn a language. Given the right input from the environment, children learn to speak without formal instruction. They can learn number sense and pattern recognition the same way.

To further nurture your child's mathematical growth, attend to the learning environment. You can think of it as providing a nutritious mathematical diet that includes: discussing math in the real world, offering encouragement, being available to answer questions, allowing your student to struggle with difficult concepts, and providing space for plenty of practice.

#mathmythbusted

Talking Points

Discuss With Your Student

Your student is learning to compose and decompose numbers using different techniques. You can further support your student's learning by asking questions about the work they do in class or at home.

Questions to Ask

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

Key Terms

Distributive Property
The Distributive Property states that for any numbers a, b, and c, $a(b + c) = ab + ac$.

Commutative Property
The Commutative Property states that for any numbers a and b, the product $a \times b$ is equal to the product $b \times a$.

greatest common factor (GCF)
The GCF is the largest factor two or more numbers have in common.

least common multiple (LCM)
The LCM is the smallest multiple (other than zero) that two or more numbers have in common.

2 • TOPIC 1: Factors and Multiples

We're here for you.
The Carnegie Learning Texas Support Team is available to help with any issue at texashelp@carnegielearning.com.

Monday–Friday
8:00 am–8:00 pm CT
via email, phone, or live chat.

Our expert team provides support for installations, networking, and technical issues, and can also help with general questions related to pedagogy, classroom management, content, and curricula.

You Might Be Wondering...

Why are the student books consumable?

The Student Edition contains all of the resources students need to complete the course. Students are to actively engage in this textbook, topic by topic, creating a record of their learning as they go. There is room to record answers, take notes, draw diagrams, and fix mistakes.

Why do we believe in our brand of blended: Learning Together and Learning Individually?

There has been a lot of research on the benefits of learning collaboratively. Independent practice is necessary for students to become fluent and automatic in a skill. A balance of these two pieces provides students with the opportunity to develop a deep conceptual understanding through collaboration with their peers, while demonstrating their understanding independently.

Why don't we have a Worked Example at the start of every lesson?

Throughout the Texas Math Solution, we do provide Worked Examples. Sweller and Cooper (1985) argue that Worked Examples are educationally efficient because they reduce working memory load. Ward and Sweller (1990) found that alternating between problem solving and viewing Worked Examples led to the best learning. Students often read Worked Examples with the intent to confirm that they understand the individual steps. However, the educational value of the Worked Example often lies in thinking about how the steps connect to each other and how particular steps might be added, omitted or changed, depending on context.

Where are the colorful graphics to get students' attention?

Color and visuals make for stronger student engagement, right? Not quite. Our instructional materials have little extraneous material. This approach follows from research showing that "seductive details" used to spice up the presentation of material often have a negative effect on student learning (Mayer et al., 2001; Harp & Meyer, 1998). Students may not know which elements of an instructional presentation are essential and which are intended simply to provide visual interest. So, we focus on the essential materials. While we strive to make our educational

materials attractive and engaging to students, research shows that only engagement based on the mathematical content leads to learning.

Why so many words?

For students to deeply learn the math, they need to work through it. They also need to develop their work and demonstrate that they really understand it. Math isn't just about solving equations or formulas—it's about thinking, working through ideas, and seeing how the math relates to the real world.

Notes:

> If you have questions, reach out to us for support. Our team of master practitioners have been where you are. We made mistakes and we learned from them. We want to help you. We have many professional development options. Whether we come to your school for a workshop, join you in your classroom for modeling or coaching, or you join us online for a webinar or an entire course, our goal is to make sure you feel supported and prepared to use the tasks you'll find in this book to their fullest!
>
> Kasey Bratcher, Senior VP of Professional Learning