Taking Apart Numbers and Shapes

1

Writing Equivalent Expressions Using the Distributive Property

MATERIALS

None

Lesson Overview

Students divide area models in different ways to see that the sum of the areas of the smaller regions equals the area of the whole model. They then rewrite the product of two factors as a factor times the sum of two or more terms, leading to the formalization of the Distributive Property.

Grade 6

Expressions, Equations, and Relationships

(7) The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:

(D) generate equivalent expressions using the properties of operations: inverse, identity, commutative, associative, and distributive properties.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G.

Essential Ideas

- The area of a rectangle is the product of its length and width.
- You can illustrate the Distributive Property using an area model of a rectangle with side lengths a and $(b + c)$.
- The Distributive Property states that for any numbers a, b, and c, $a(b + c) = ab + ac$.
- You can rewrite equivalent expressions using properties.

Lesson Structure and Pacing: 1 Day

Engage

Getting Started: Break It Down to Build It Up

Students divide area models for the product 5×27 in two different ways. They calculate the areas of the subdivided parts before determining the area of the whole model.

Develop

Activity 1.1: Connecting Area Models and the Distributive Property

Students rewrite the product of two factors as a factor times the sum of two or more terms, leading to the formalization of the Distributive Property. They decompose factors and products into equivalent representations.

Demonstrate

Talk the Talk: The Floor Is Yours

Students design the floor plan in a gymnasium for different after-school activities. They represent their model using the Distributive Property and then explain their rationale.

Getting Started: Break It Down to Build It Up

Facilitation Notes

In this activity, students divide area models for the product 5×27 in two different ways. They calculate the areas of the subdivided parts before determining the area of the whole model.

Ask a student to read the situation aloud. Have students complete Question 1 individually. Share responses as a class.

As students work, look for

- Whether students use a vertical, horizontal, or slanted line to divide the area model.
- Splitting 27 into numbers that make the computation of area easier.
- Correct dimensions for each of the smaller regions in the area model.

Questions to ask

- What is an area model?
- Did you split the length to obtain specific values that add up to 27? If so, explain your thinking.

Misconceptions

Students may decide to make a diagonal line to split the area. While correct, discuss that their decision makes two trapezoids, or two triangles, instead of rectangles, and it is much more efficient to use rectangles. Also, rectangles are required to model the Distributive Property.

Have students complete Questions 2 and 3 individually. Share responses as a class.

Questions to ask

- What was the same about each of your area calculations? Why is that the case?
- Why does everyone get the same total area even though they split the walkway differently?

Summary

You can divide an area model into smaller regions. The sum of the areas of each region is the total area of the model.

DEVELOP

Activity 1.1
Connecting Area Models and the Distributive Property

Facilitation Notes

In this activity, students rewrite the product of two factors as a factor times the sum of two or more terms, leading to the formalization of the Distributive Property. Students decompose factors and products into equivalent representations.

Ask a student to read the introduction. Have students complete Question 1 with a partner or group. Share responses as a class.

Questions to ask

- When is the use of parentheses necessary? What do they indicate?
- Why is 5 repeated in both parentheses?
- How does this expression relate to your calculations?
- Did anyone split the rectangle up into three regions? How does that change the way you write the corresponding equation using the Distributive Property?
- Which combination of values is the most efficient one to use?

Ask a student to read the definition of the Distributive Property aloud. Ask students to work with a partner or in groups to complete Question 2. Share responses as a class.

Questions to ask

- What does it mean to distribute something?
- Explain what it is meant by "multiplication over addition."
- The multiplication symbol is not shown but rather implied. Where is multiplication implied in $a(b + c) = ab + ac$?

Read and discuss the Worked Example as a class. Have students answer Questions 3 through 5 with their partner or group, and share responses as a class.

Questions to ask

- What is the purpose of showing the arrows in the example?
- Draw a diagram to represent this expression.
- Could you write the expression as $(2 + 15)4$? Explain your thinking?
- How did you decide which addend to put in the parentheses?
- What are the factors in this equation?

- Why did you choose the sum of those two values to represent the factor?
- Are some sums a better choice than others? Explain your thinking.
- What was the error in each false statement?
- How do you read each true statement?
- What is important to keep in mind when you are distributing a factor over a quantity?

Differentiation strategies

- To scaffold support for Question 3, suggest that students use arrows similar to those used in the Worked Example. Then, have them work backwards from the partial products.
- For advanced learners, demonstrate how the Distributive Property can help with mental math by expressing one factor as the sum of the tens and ones values. For example: $3 \times 28 = 3(20 + 8) = 60 + 24 = 84$.

Summary

You can use the Distributive Property to rewrite the product of two factors as a factor times the sum of two or more terms.

Talk the Talk: The Floor Is Yours

DEMONSTRATE

Facilitation Notes

In this activity, students design the floor plan in a gymnasium for different after-school activities. They represent their model using the Distributive Property and then explain their rationale.

Have students work with a partner or in groups to complete Question 1. Share responses as a class.

Questions to ask

- What rationale did you use to split the gym floor to accommodate the three activities?
- How is the width of the gym floor reflected in your equation?
- How is the area of each activity reflected in your equation?
- A typical volleyball net spans about 36 feet from pole to pole. Did you leave enough room to set up the net? If not, how could you change your diagram and the corresponding equation?

- How does the use of the Distributive Property change when you are multiplying a single factor by the sum of three numbers?
- Does the sum of the areas of each of the three regions total 4200 square feet?

Summary

The Distributive Property allows you to represent expressions in different ways.

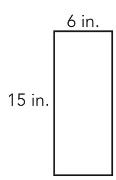
Taking Apart Numbers and Shapes 1

Writing Equivalent Expressions Using the Distributive Property

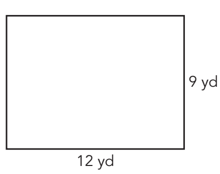
REVIEW

Calculate the area of each rectangle. Show your work.

1.



2.



LEARNING GOALS

- Write, read, and evaluate equivalent numeric expressions.
- Identify the adjacent side lengths of a rectangle as factors of the area value.
- Identify parts of an expression, such as the product and the factors.
- Write equivalent numeric expressions for the area of a rectangle by decomposing one side length into the sum of two or more numbers.
- Apply the Distributive Property to rewrite the product of two factors.

KEY TERMS

- numeric expression
- equation
- Distributive Property

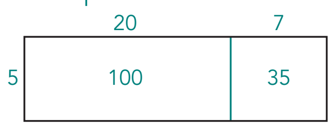
You know how to add, subtract, multiply, and divide numbers using different strategies. Taking apart numbers before you perform a mathematical operation can highlight important information or make calculations easier. How can taking apart numbers help you to express number sentences in different ways?

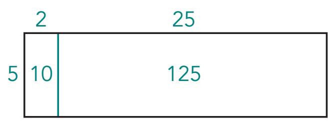
Warm Up Answers

1. 90 square inches
2. 108 square yards

Answers

1. Sample answer shown.





2. Sample answer shown above.
3. 135 square feet

Getting Started

Break It Down to Build It Up

Callie is building a rectangular walkway up to her house. The width of the walkway is 5 feet and the length is 27 feet. She needs to calculate the area of the walkway to determine the amount of materials needed to build it.

1. Mark and label 2 different ways you could divide an area model to determine the area of the walkway.

2. Determine the areas of each of the subdivided parts of your models.

3. What is the total area of the walkway?

ACTIVITY 1.1

Connecting Area Models and the Distributive Property

The numeric expression of 5×27 represents the area of the walkway from the Getting Started. A **numeric expression** is a mathematical phrase that contains numbers and operations.

The equation $5 \times 27 = 135$ shows that the expression 5×27 is equal to the expression 135.

An **equation** is a mathematical sentence that uses an equals sign to show that two or more quantities are the same as one another.

1. Reflect on the different ways you can rewrite the product of 5 and 27. Select one of your area models to complete the example.

How did you split the side length of 27? $5 \times 27 = 5(\_\_\_\_ + \_\_\_\_)$

What are the factors of each smaller region? $= (5 \cdot \_\_\_\_) + (5 \cdot \_\_\_\_)$

What is the area of each smaller region? $= \_\_\_\_ + \_\_\_\_$

What is the total area? $= \_\_\_\_$

Answers

1. Sample answer.

$$\begin{aligned} 5 \times 27 &= 5(25 + 2) \\ &= (5 \cdot 25) + (5 \cdot 2) \\ &= 125 + 10 \\ &= 135 \end{aligned}$$

Answers

2. Sample answer. The area of the whole rectangle is equal to $a(b + c)$, because a is the width and $b + c$ is the length. The area of the smaller rectangle is $a \cdot b$ and the area of the larger rectangle is $a \cdot c$. The sum of those areas, $ab + ac$, is equal to the area of the whole rectangle, $a(b + c)$.

3a. $7(3 + 10) = 21 + 70$

3b. $3(12 + 15) = 36 + 45$

3c. $8(2 + 7) = 16 + 56$

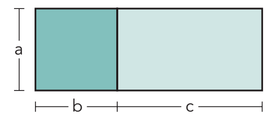
3d. $5(6 + 9) = 30 + 45$

ELL Tip

Help students differentiate between the terms *expression* and *equation*. Read the following statements aloud. Have students use thumbs-up or thumbs-down to indicate understanding.

You just used the *Distributive Property!*

The **Distributive Property**, when applied for multiplication, states that for any numbers a, b, and c, the equation $a(b + c) = ab + ac$ is true.



2. Explain the Distributive Property using the area model shown.

WORKED EXAMPLE

Consider this example of the Distributive Property.

$4(2 + 15) = 4 \cdot 2 + 4 \cdot 15$

You can read and describe the expression 4(2 + 15) in different ways. For example, you can say:

- four times the quantity of two plus fifteen,
- four times the sum of two and fifteen, or
- the product of four and the sum of two and fifteen.

You can describe the expression 4(2 + 15) as a product of two factors. The quantity (2 + 15) is both a single factor and a sum of two terms.

3. Fill in the missing addend in each box that makes the equation true.

a. $7\ (\_\_\_\_ + 10) = 21 + 70$ **b.** $3\ (\_\_\_\_ + 15) = 36 + 45$

c. $8\ (2 + \_\_\_\_) = 16 + 56$ **d.** $5\ (6 + \_\_\_\_) = 30 + 45$

Write an example or counterexample to illustrate.

- All equations have equal signs in them.
- All expressions are equations.
- All equations include expressions.

4. Rewrite a factor as the sum of two terms in each expression and use the Distributive Property to verify each product.

a. $4 \times 17 = 68$ b. $9 \times 34 = 306$

c. $3 \times 29 = 87$

5. Identify each statement as true or false. If the statement is false, show how you could rewrite it to make it a true statement.

a. True False $3(2 + 4) = 3 \cdot 2 + 4$

b. True False $6(10 + 5) = 6 \cdot 10 + 6 \cdot 5$

c. True False $7(20 + 8) = 7 + 20 \cdot 8$

d. True False $4(5 + 10) = 20 + 10$

e. True False $2(6 + 11) = 12 + 22$

Answers

4a. Sample answer.

$$4(10 + 7) = 68$$
$$40 + 28 = 68$$
$$68 = 68$$

4b. Sample answer.

$$9 \times 34 = 306$$
$$9(30 + 4) = 306$$
$$270 + 36 = 306$$
$$306 = 306$$

4c. Sample answer.

$$3 \times 29 = 87$$
$$3(20 + 9) = 87$$
$$60 + 27 = 87$$
$$87 = 87$$

5a. False; $3(2 + 4) = 3 \cdot 2 + 3 \cdot 4$

5b. True

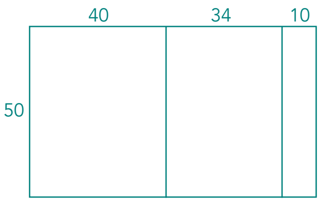
5c. False; $7(20 + 8) = 7 \cdot 20 + 7 \cdot 8$

5d. False; $4(5 + 10) = 20 + 40$

5e. True

Answers

1. Sample answer.



$50(40 + 34 + 10) =$
$50 \cdot 40 + 50 \cdot 34 +$
$50 \cdot 10$
$= 2000 + 1700 + 500$
$= 4200$

I divided the length of the gym into three parts to create three areas of different sizes for each activity.

- I made the area for playing volleyball the largest, 50 feet by 40 feet.
- I made the area for playing dodgeball, 50 feet by 34 feet, close to the same size as the volleyball area, but a bit smaller.
- I made the smallest area of the gym, 50 feet by 10 feet, for playing board games or reading since those are activities that require less movement.

NOTES

TALK the TALK

The Floor Is Yours

You can apply the Distributive Property to solve real-world problems.

Consider the situation.

Tyler is setting up the gym floor for an after-school program. He wants to include a rectangular area for playing volleyball and another for dodgeball. He also wants to have an area for kids who like to play board games or just sit and read. The gym floor is already 50 feet by 84 feet, or 4200 square feet.

1. Create a diagram to show how you would split up the gym floor. Represent your diagram using the Distributive Property and write an explanation for the areas assigned to each activity.