2 Searching for Common Ground

Identifying Common Factors and Common Multiples

WARM UP

In the array of numbers shown, circle the prime numbers, cross out the composite numbers, and use a box to identify any number that is neither prime nor composite.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

LEARNING GOALS

- Identify the factors and multiples of numbers and the common factors and multiples of two whole numbers.
- Use powers and exponents to write the prime factorization of a number.
- Write and evaluate numeric expressions using the Distributive Property to model composing and decomposing the areas of rectangles.
- Rewrite the sum of two whole numbers with a common factor as a product using the Distributive Property.

KEY TERMS

- common factor
- base
- power
- exponent
- greatest common factor (GCF)
- relatively prime
- multiple
- Commutative Property
- least common multiple (LCM)

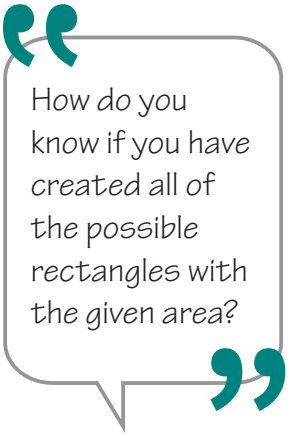
You have decomposed rectangles to determine areas and products of numbers. How can you use shapes to see relationships between pairs of numbers?

How Many Rectangles Can You Build?

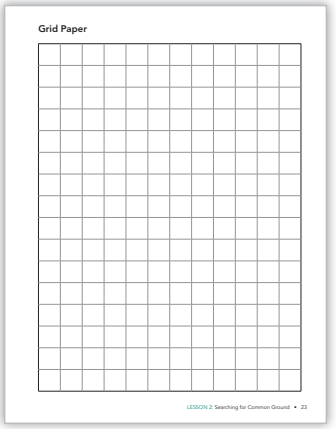
Understanding the area of rectangles is helpful when learning about factors. A rectangular area model is one way to represent multiplication. You and your partner will create area models for the numbers 12 and 16.

Number assigned to me _____

Number assigned to my partner _____



Use the grid paper located on page 13 to create and cut out as many unique rectangles as possible with the area of your assigned number.



1. **Label each rectangle with its dimensions. List the dimensions of all of the rectangles that you created for your assigned number.**

2. **How did you represent factors and factor pairs in your rectangles?**

3. **List all of the factors of 12 and 16.**

Together with your partner, combine one of your rectangles and one of your partner's rectangles to make a bigger rectangle. Use this method to create additional rectangles.

4. **Complete the table with the information for each combined rectangle that you and your partner created.**

Dimensions of Rectangle with an Area of 12	Dimensions of Rectangle with an Area of 16	Dimensions of the Combined Rectangle	Area of the Combined Rectangle as a Sum of the Smaller Rectangles	Total Area of Combined Rectangle
$l \times w_1$	$l \times w_2$	$l(w_1 + w_2)$	$A_1 + A_2$	

5. **How are the dimensions of the combined rectangle related to its total area?**

6. **For each combined rectangle you and your partner created, write a numeric expression that relates the dimensions of the combined rectangle to the sum of the areas of the smaller rectangles.**

Consider any factors shared between your number and your partner's number.

Common factors are the factors shared between the numbers.

7. **How are the common factors represented in the combined rectangles that you and your partner created?**

8. **How did you represent common factors in the numeric expressions that you and your partner wrote?**

9. **List the common factors of the two numbers.**

ACTIVITY
2.1 Prime Factors

You just determined the factors for a given number, as well as the common factors that two numbers share.

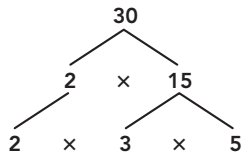
In this activity, you will learn to determine the prime factors of a given number.

A factor tree is a way to organize the prime factors of a number. Choose any factor pair to get started.

WORKED EXAMPLE

Use a factor tree to write the prime factorization for 30.

- Begin with the number 30.
- Pick any whole number factor pair of 30, other than 1 and 30.
- Draw a branch from 30 to each factor, 2 and 15.
- Since both of the factors are not prime, you are not finished.
- Use branches to write a factor pair for 15.
- Because 2, 3, and 5 are all prime, this factor tree is complete.



1. Use the factor tree to write the prime factorization of 30.

The factor tree in the Worked Example is not the only factor tree that you can create for 30.

2. How many different factor trees are there for 30?

3. Construct a factor tree and write the prime factorization for each number.

a. 24 b. 81 c. 96

You may have noticed that each prime factorization in Question 3 had repeat factors. You can represent repeated multiplication as a *power*. A **power** has two elements: the base and the exponent.

The **base** of a power is the factor multiplied by itself repeatedly, and the **exponent** of the power is the number of times you use the base as a factor.

$$2 \times 2 \times 2 \times 2 = 2^4$$

base → 2^4 ← exponent

power

4. Identify the base and exponent in each power. Then, write each power in words.

a. 7^5 b. 4^8

5. Write the prime factorization for each number in Question 3 using powers.

a. 24 b. 81 c. 96

ACTIVITY
2.2 Common Factors

Suppose you want to determine the common factors of 56 and 42, but you do not have grid paper or scissors to create rectangles. Is there another way?

WORKED EXAMPLE

One way to determine common factors is to use prime factorization. Start by writing each number as a product of its prime factors.

$56 = 2 \cdot 2 \cdot 2 \cdot 7$
$42 = 2 \cdot 3 \cdot 7$

Organize the prime factors into a table. Only list shared factors in the same column.

Number	Prime Factors				
56	2	2	2		7
42	2			3	7

The common factors of the two numbers are the numbers that are in both rows and the product of the numbers that are in both rows.

The common factors of 56 and 42 are 2, 7, and 14.

1. How do you know that 14 is a common factor of 56 and 42?

2. Why is there a space between 2 and 7 in the top row of the table?

3. **Consider the numbers 54 and 84.**

   a. **Create a table of prime factors.**

   b. **Identify all of the common factors of 54 and 84.**

   c. **Of the common factors, which factor is the largest?**

The **greatest common factor (GCF)** is the largest factor two or more numbers have in common.

4. **Rewrite each sum using the GCF and the Distributive Property.**

   a. **56 + 42**

   b. **54 + 84**

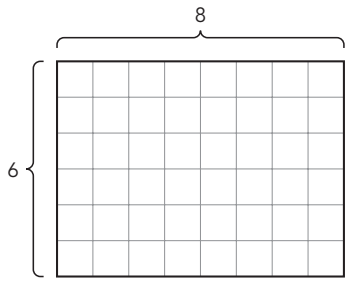
Two numbers that do not have any common factors other than 1 are **relatively prime**.

ACTIVITY
2.3 Common Multiples

You can use rectangular arrays to determine multiples and common multiples.

Consider the area model for $6 \cdot 8 = 48$.

A **multiple** is the product of a given whole number and another whole number.



One way to think about the area model is to analyze the collection of columns. The addition of each new column creates a multiple of 6.

- The first column is a 6×1 rectangle representing the first multiple of 6, or 6.
- The first and second columns together are a 6×2 rectangle representing the second multiple of 6, or 12.
- The whole rectangle represents 6×8, or 48.

The **Commutative Property**, when applied for multiplication, states that for any numbers a and b, the product $a \cdot b$ is equal to the product $b \cdot a$.

1. List the first eight multiples of 6 by labeling each column of the area model.

Next, think about the area model as a collection of 6 rows. The first row alone creates an 8×1 rectangle, which represents the first multiple of 8, or 8. Including all rows, the 8×6 rectangle represents the sixth multiple of 8, or 48.

2. List the first six multiples of 8 by labeling each row of the area model.

While 48 is a multiple shared by both 6 and 8, it is not the **least common multiple (LCM)**. The LCM is the smallest multiple (other than zero) that two or more numbers have in common.

Analyze the multiples of 6 and 8 that you labeled on the area model.

3. Identify the least common multiple of 6 and 8.

LCM(6, 8) = _____

As demonstrated by the rectangular array, for any two whole numbers *a* and *b*, a common multiple is $a \cdot b$. However, this number may not be the *least* common multiple of *a* and *b*.

4. Determine the least common multiple of 6 and 9.

a. List the first 9 multiples of 6.

b. List the first 6 multiples of 9.

c. What is the least common multiple of 6 and 9?

5. Determine the least common multiple of 7 and 8.

LCM(7, 8) = _____

6. Using prime factorization, how can you determine whether the least common multiple of two numbers is the product of the two numbers or is less than the product of the two numbers?

ACTIVITY
2.4 Using Prime Factors to Determine the LCM

Suppose you want to determine the LCM of 56 and 42 without drawing an area model. Is there another way?

WORKED EXAMPLE

Organize the prime factors into a table. Only list shared factors in the same column.

Number	Prime Factors				
56	2	2	2		7
42	2			3	7

Determine the prime factors that the numbers share.

Shared prime factors: 2, 7

Determine the prime factors that the numbers *do not* share.

Non-shared prime factors: 2, 2, 3

The least common multiple of the two numbers is the product of their shared prime factors and non-shared prime factors.

$2 \cdot 2 \cdot 2 \cdot 3 \cdot 7 = 168$

LCM(56, 42) = 168

1. For each pair of numbers, determine their product. Then, use a factor table to determine their least common multiple and their greatest common factor.

a. 12 and 10

b. 9 and 15

c. 9 and 10

d. 5 and 9

2. Write a sentence to describe the relationship between the product, GCF, and LCM.

NOTES

TALK the TALK

Bringing It Back Around

You have composed and decomposed numbers using factors and multiples.

Use the relationship between factors and multiples to answer each question.

1. **Consider the sum 36 + 24.**

   a. **Express the sum 36 + 24 as many ways as possible as the product $a(b + c)$.**

   b. **How can you use factors to determine whether you have listed all possible products $a(b + c)$ that are equivalent to 36 + 24?**

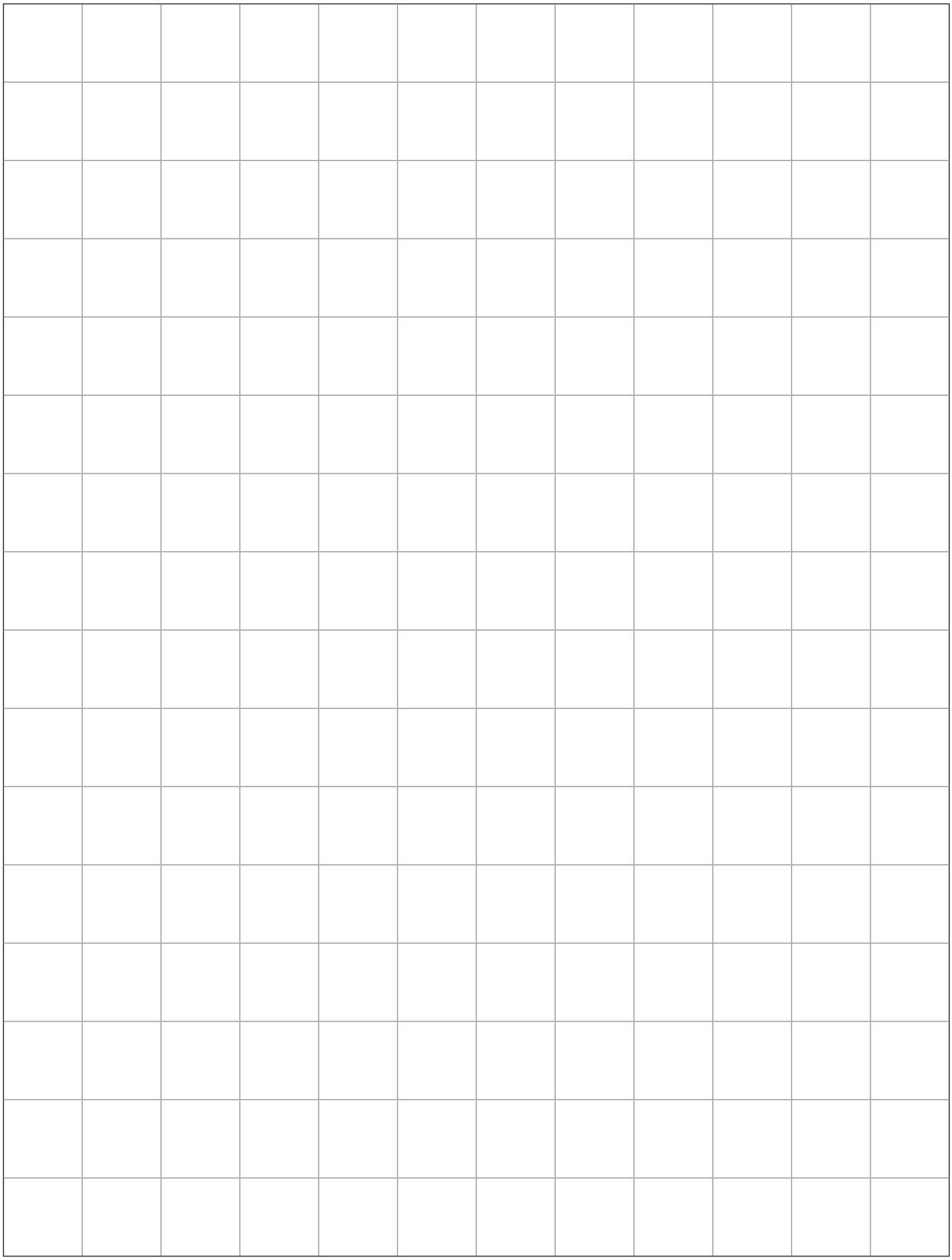
2. **Can you always determine the greatest common factor of any two numbers? Explain your reasoning.**

3. **If the greatest common factor of two numbers is 1, what can you say about the numbers?**

4. **Can you always determine the least common multiple of any two numbers? Explain your reasoning.**

5. **If the least common multiple of two numbers is the product of those numbers, what can you say about the two numbers?**

Grid Paper



Why is this page blank?

So you can cut out your rectangles on the other side.