Searching for Common Ground

2

Identifying Common Factors and Common Multiples

MATERIALS

Scissors

Lesson Overview

Students construct rectangles with given areas and relate their dimensions to factors and common factors. They create prime factorizations to determine the greatest common factor (GCF) and least common multiple (LCM) of two numbers. Students examine the rows and columns of an area model to identify multiples and the LCM. They describe the relationship between the product, GCF, and LCM.

Grade 6

Expressions, Equations, and Relationships

(7) The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:

(A) generate equivalent numerical expressions using order of operations, including whole number exponents and prime factorization.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G

Essential Ideas

- Prime factorization can be used to determine common factors and common multiples of two numbers.
- The greatest common factor (GCF) of two numbers is the largest factor shared by the two numbers.
- The least common multiple (LCM) of two numbers is the smallest non-zero multiple shared by the two numbers.
- The Commutative and Distributive Properties are properties that can be used to generate equivalent expressions.
- If two numbers a and b are relatively prime, then the GCF(a, b) = 1 and the LCM(a, b) = ab.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: How Many Rectangles Can You Build?

Students create rectangular arrays and compositions of rectangles to explore factors and common factors.

Develop

Activity 2.1: Prime Factors

Students create factor trees to write prime factorizations; they use exponents to express repeated factors.

Activity 2.2: Common Factors

Students determine the GCF of two numbers using prime factors organized in a factor table. They use the GCF to rewrite a numeric expression using the Distributive Property.

Day 2

Activity 2.3: Common Multiples

Students use a rectangular array to generate lists of common multiples of two numbers and identify the least common multiple (LCM) of the numbers.

Activity 2.4: Using Prime Factors to Determine the LCM

Students determine the LCM of two numbers using prime factors organized in a factor table. They analyze the relationship between the product, GCF, and LCM of two numbers.

Demonstrate

Talk the Talk: Bringing it Back Around

Students summarize their understanding of common factors and common multiples. They rewrite a sum using the Distributive Property and write general statements about GCFs and LCMs.

Getting Started: How Many Rectangles Can You Build?

ENGAGE

Facilitation Notes

In this activity, students create rectangular arrays to explore factors. They combine different rectangles to investigate common factors.

Read and discuss the introduction and directions as a class. Have students work with a partner or in groups to complete Questions 1 through 3. Share responses as a class.

Misconception

When completing Question 1, students may think that a rectangle with dimensions 3×4 is different from a rectangle with dimensions 4×3. Rotate a rectangular cutout to demonstrate that both sets of dimensions represent the same rectangle.

As students work, look for

- Statements connecting factors and dimensions of the rectangle.
- Students who count unit squares instead of using factor pairs to determine area. Students should use factor pairs to form their rectangles.

Questions to ask

- If two rectangles are unique, what does that mean?
- Is a 3×4 rectangle the same as a 4×3 rectangle? Explain.
- What are factors?
- How do you know if you have thought of all possible factors?
- Is it possible for every number to have factors? Explain.

Have students work with a partner or in groups to complete Questions 4 through 9. Share responses as a class.

Questions to ask

- How can you tell what smaller rectangles can be combined to make a larger rectangle?
- How do you know what value to use for length and what value to use for width?
- What does the sum in each of your dimensions of the combined rectangle represent?
- Explain how the table demonstrates the use of the Distributive Property.
- Explain why you can't create a combined rectangle with a length of 6?
- How are common factors and the number of rectangles you composed related?

Summary

Rectangular area models are used to represent multiplication.

DEVELOP

ACTIVITY 2.1

Prime Factors

Facilitation Notes

In this activity, students create factor trees to write prime factorizations; they use exponents to express repeated factors.

Have a student read the introduction aloud. Discuss the Worked Example as a class. Ask students to work with a partner or in groups to complete

Questions 1 through 3. Share responses as a class.

As students work, look for

Confusion between listing all factors and writing the prime factorization of two numbers.

Differentiation strategy

To scaffold support for making a factor tree, show students how to use the calculator and the concept of remainders to check if a number is a factor of another number.

Questions to ask

- What is meant by *prime factorization*?
- In the Worked Example, why does the number 2 have only one line branching from it?
- What is another factor pair you can use to start your factor tree?
- How can you check that your prime factorization is correct?

Have a student read the text and definitions about exponents aloud. Discuss as a class. Ask students to work with a partner or in groups to complete Questions 4 and 5. Share responses as a class.

Questions to ask

- What is the benefit of using exponential notation?

Summary

You can use a factor tree to write the prime factorization of a number. You can use exponential notation to express a factor that is listed more than once.

ACTIVITY 2.2
Common Factors

Facilitation Notes

In this activity, students determine the GCF of two numbers using prime factors organized in a factor table. They use the GCF to rewrite a numeric expression using the Distributive Property.

Have a student read the introduction aloud. Discuss the Worked Example as a class. Ask students to work with a partner or in groups to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- Show how to use a factor tree to get the prime factors of 56 and 42.
- Why do you think it makes sense to list the factors in numerical order?
- How is 14 identified as a common factor when it is not listed in the table?
- Do you have to use the GCF to rewrite an expression using the Distributive Property? Why or why not?

Have students work with a partner or in groups to complete Questions 3 and 4. Share responses as a class.

Misconception

Students may think that two numbers that are relatively prime must each be prime numbers. Use the numbers 8 and 9 to clarify that relatively prime numbers are prime in relation to one another, meaning they have no common factors.

Questions to ask

- Explain how you used the table to organize the prime factors.
- How did you know that 6 was a common factor?
- What is the connection between the GCF and the Distributive Property?

Summary

You can use prime factorization to determine the greatest common factor (GCF). You can rewrite algebraic expressions using the Distributive Property and any common factor, including the GCF.

Activity 2.3
Common Multiples

Facilitation Notes

In this activity, students use a rectangular array to generate lists of common multiples of two numbers.
They identify the least common multiple (LCM) of the numbers.

Have a student read the introduction aloud. Discuss the model as a class. Ask students to work with a partner or in groups to complete Questions 1 through 3. Share responses as a class.

Differentiation strategy

To scaffold support, suggest that students use colored pencils to make sense of the multiples generated from the rows and columns.

Questions to ask

- How do you know how large to make the area model?
- Why doesn't it make a difference which value is used for the number of rows and which value is used for the number of columns?
- Are the multiples of a given number larger or smaller than it?
- Explain how you can use the area model to list the multiples.

Have students work with a partner or in groups to complete Questions 4 through 6. Share responses as a class.

Questions to ask

- Did you construct a rectangular array to determine the least common multiple? Why or why not?
- What is an example of another two numbers where the least common multiple is not the product of the two numbers?
- Explain how you generated your example.
- What is the difference between Questions 4 and 5?
- What is an example of another two numbers where the least common multiple is the product of the two numbers?
- Explain how you generated your example.

Summary

You can use rectangular area models to determine the least common multiple (LCM) of two numbers.

Activity 2.4
Using Prime Factors to Determine the LCM

Facilitation Notes

In this activity, students determine the LCM of two numbers using prime factors organized in a factor table. They analyze the relationship between the product, GCF, and LCM of two numbers.

Have a student read the introduction aloud. Discuss the Worked Example as a class. Ask students to work with a partner or in groups to complete Questions 1 and 2. Share responses as a class.

Differentiation strategy

To scaffold support for Question 1, have students use one colored pencil to shade in one pair of the common cells in their factor table that represents the GCF. Then have them use a second colored pencil to shade in the cells that relate to the LCM. The shaded cells result in the product of the two values.

Misconception

For all students, using the terms *greatest* and *least* may sound counter-intuitive compared to their answers. Stress that they should concentrate on key terms. In GCF, *factor* is the key term, and a common factor cannot be larger than both values. In LCM, *multiple* is the key term, and a multiple may be larger than both values.

Questions to ask

- Why does this factor table look familiar?
- Explain how to use the table to determine the GCF.
- What other method can you use to determine the LCM? Why isn't this method efficient?
- Why isn't the LCM of 56 and 42 equal to the product of the two numbers?
- What are the steps to determine the LCM?
- What are the steps to determine the GCF?
- Is the LCM a factor of the product?
- Is the GCF a factor of the product?
- Explain your thinking to determine the relationship.
- Does the relationship you identified also work for two numbers that are relatively prime? Provide an example.

Summary

You can compose and decompose numbers using common factors and common multiples.

DEMONSTRATE

Talk the Talk: Bringing It Back Around

Facilitation Notes

In this activity, students summarize their understanding of common factors and common multiples. They rewrite a sum using the Distributive Property and write general statements about GCFs and LCMs.

Have a student read the introduction aloud. Ask students to work with a partner or in groups to complete Questions 1 through 5. Share responses as a class.

As students work, look for

- Three different responses to determine the GCF: list all factors, use prime factorization and a factor table, and divide the product by the LCM.
- Three different responses to determine the LCM: use an area model to list all multiples, use a factor table, and divide the product by the GCF.

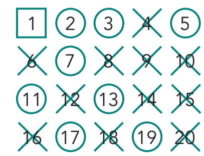
Questions to ask

- What property does the expression $a(b + c)$ represent?
- What method did you use to determine all possible values of a?
- What methods can you use to determine the GCF?
- Provide an example of two numbers that have a greatest common factor of 1.
- What methods can you use to determine the LCM?
- Provide an example of two numbers whose least common multiple is their product.

Summary

Numbers can be composed and decomposed using common factors and common multiples.

Warm Up Answers



Searching for Common Ground

2

Identifying Common Factors and Common Multiples

WARM UP

In the array of numbers shown, circle the prime numbers, cross out the composite numbers, and use a box to identify any number that is neither prime nor composite.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

LEARNING GOALS

- Identify the factors and multiples of numbers and the common factors and multiples of two whole numbers.
- Use powers and exponents to write the prime factorization of a number.
- Write and evaluate numeric expressions using the Distributive Property to model composing and decomposing the areas of rectangles.
- Rewrite the sum of two whole numbers with a common factor as a product using the Distributive Property.

KEY TERMS

- common factor
- base
- power
- exponent
- greatest common factor (GCF)
- relatively prime
- multiple
- Commutative Property
- least common multiple (LCM)

You have decomposed rectangles to determine areas and products of numbers. How can you use shapes to see relationships between pairs of numbers?

Answers

1. 12: 1 × 12, 2 × 6, 3 × 4
   16: 1 × 16, 2 × 8, 4 × 4
2. Each side length represents a factor and the dimensions of each rectangle represent factor pairs.
3. 12: 1, 2, 3, 4, 6, 12
   16: 1, 2, 4, 8, 16

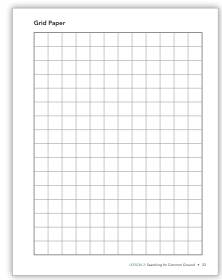
Getting Started

How Many Rectangles Can You Build?

Understanding the area of rectangles is helpful when learning about factors. A rectangular area model is one way to represent multiplication. You and your partner will create area models for the numbers 12 and 16.

Number assigned to me __________

Number assigned to my partner ___________



Use the grid paper located on page 13 to create and cut out as many unique rectangles as possible with the area of your assigned number.

1. **Label each rectangle with its dimensions. List the dimensions of all of the rectangles that you created for your assigned number.**

2. **How did you represent factors and factor pairs in your rectangles?**

3. **List all of the factors of 12 and 16.**

Together with your partner, combine one of your rectangles and one of your partner's rectangles to make a bigger rectangle. Use this method to create additional rectangles.

2 • TOPIC 1: Factors and Multiples

4. **Complete the table with the information for each combined rectangle that you and your partner created.**

Dimensions of Rectangle with an Area of 12	Dimensions of Rectangle with an Area of 16	Dimensions of the Combined Rectangle	Area of the Combined Rectangle as a Sum of the Smaller Rectangles	Total Area of Combined Rectangle
$l \times w_1$	$l \times w_2$	$l(w_1 + w_2)$	$A_1 + A_2$	

5. **How are the dimensions of the combined rectangle related to its total area?**

6. **For each combined rectangle you and your partner created, write a numeric expression that relates the dimensions of the combined rectangle to the sum of the areas of the smaller rectangles.**

Consider any factors shared between your number and your partner's number.

Common factors are the factors shared between the numbers.

7. **How are the common factors represented in the combined rectangles that you and your partner created?**

8. **How did you represent common factors in the numeric expressions that you and your partner wrote?**

9. **List the common factors of the two numbers.**

Answers

4. See table below.
5. The area is the length times the width. The width is the dimension that the two rectangles have in common, and the length is the sum of both of the lengths of the smaller rectangles.
6. Sample answer.

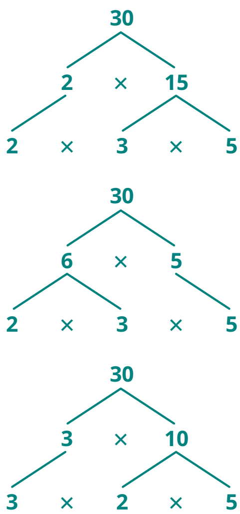
   $4(4 + 3) = 4 \times 4 + 4 \times 3$
7. You represent the common factors as the side length. The common factors are the sides that line up from one smaller rectangle to the other.
8. The common factor is the number outside the parentheses when you distribute.
9. 1, 2, 4

4.

Dimensions of Rectangle with an Area of 12	Dimensions of Rectangle with an Area of 16	Dimensions of the Combined Rectangle	Area of the Combined Rectangle as a Sum of the Smaller Rectangles	Total Area of Combined Rectangle
1×12	1×16	$1(12 + 16)$	$12 + 16$	28
2×6	2×8	$2(6 + 8)$	$12 + 16$	28
4×3	4×4	$4(3 + 4)$	$12 + 16$	28

Answers

1. $30 = 2 \times 3 \times 5$
2. There are three different factor trees for 30.



ACTIVITY 2.1

Prime Factors

You just determined the factors for a given number, as well as the common factors that two numbers share.

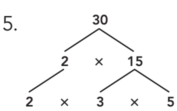
In this activity, you will learn to determine the prime factors of a given number.

A factor tree is a way to organize the prime factors of a number. Choose any factor pair to get started.

WORKED EXAMPLE

Use a factor tree to write the prime factorization for 30.

- Begin with the number 30.
- Pick any whole number factor pair of 30, other than 1 and 30.
- Draw a branch from 30 to each factor, 2 and 15.
- Since both of the factors are not prime, you are not finished.
- Use branches to write a factor pair for 15.
- Because 2, 3, and 5 are all prime, this factor tree is complete.



1. Use the factor tree to write the prime factorization of 30.

The factor tree in the Worked Example is not the only factor tree that you can create for 30.

2. How many different factor trees are there for 30?

3. Construct a factor tree and write the prime factorization for each number.

4 • TOPIC 1: Factors and Multiples

a. 24 b. 81 c. 96

You may have noticed that each prime factorization in Question 3 had repeat factors. You can represent repeated multiplication as a *power*. A **power** has two elements: the base and the exponent.

The **base** of a power is the factor multiplied by itself repeatedly, and the **exponent** of the power is the number of times you use the base as a factor.

$$2 \times 2 \times 2 \times 2 = 2^4$$

base → 2^4 ← exponent

power

4. Identify the base and exponent in each power. Then, write each power in words.

a. 7^5 b. 4^8

5. Write the prime factorization for each number in Question 3 using powers.

a. 24 b. 81 c. 96

Answers

3a. See below

3b. See below

3c. See below

4a. base: 7, exponent: 5; seven to the fifth power

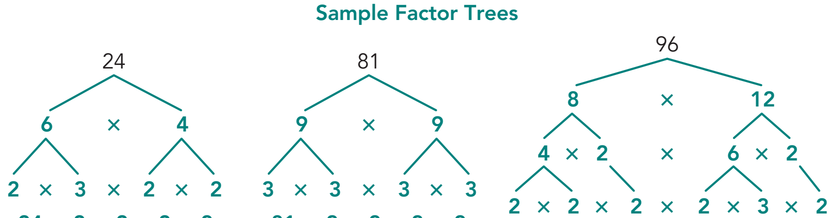
4b. base: 4, exponent: 8; four to the eighth power

5a. $24 = 2^3 \cdot 3$

5b. $81 = 3^4$

5c. $96 = 2^5 \cdot 3$

Sample Factor Trees



24 = 2 × 2 × 2 × 3

81 = 3 × 3 × 3 × 3

96 = 2 × 2 × 2 × 2 × 2 × 3

Answers

1. $56 \div 14 = 4$
   $42 \div 14 = 3$
   If the result in both cases is an integer, the divisor is a common factor.
2. There is a space between the 2 and the 7 because 3 is not a factor of 56.

ACTIVITY 2.2

Common Factors

Suppose you want to determine the common factors of 56 and 42, but you do not have grid paper or scissors to create rectangles. Is there another way?

WORKED EXAMPLE

One way to determine common factors is to use prime factorization. Start by writing each number as a product of its prime factors.

$56 = 2 \cdot 2 \cdot 2 \cdot 7$
$42 = 2 \cdot 3 \cdot 7$

Organize the prime factors into a table. Only list shared factors in the same column.

Number	Prime Factors				
56	2	2	2		7
42	2			3	7

The common factors of the two numbers are the numbers that are in both rows and the product of the numbers that are in both rows.

The common factors of 56 and 42 are 2, 7, and 14.

1. **How do you know that 14 is a common factor of 56 and 42?**

2. **Why is there a space between 2 and 7 in the top row of the table?**

3. Consider the numbers 54 and 84.

a. Create a table of prime factors.

b. Identify all of the common factors of 54 and 84.

c. Of the common factors, which factor is the largest?

The **greatest common factor (GCF)** is the largest factor two or more numbers have in common.

4. Rewrite each sum using the GCF and the Distributive Property.

Two numbers that do not have any common factors other than 1 are **relatively prime**.

a. 56 + 42

b. 54 + 84

Answers

3a.

54	2		3	3	3	
84	2	2	3			7

3b. The common factors are 2, 3, and 6.

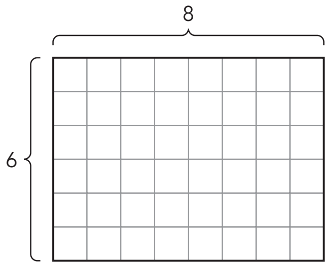
3c. The factor 6 is the largest.

4a. 14(4 + 3)

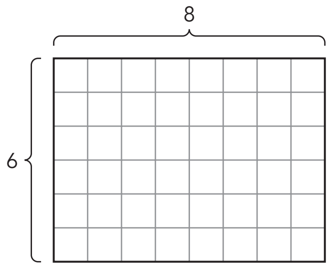
4b. 6(9 + 14)

Answers

1. See the model.



2. See the model.



ELL Tip

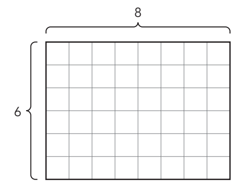
Discuss how the meaning of the everyday term *commute* relates to the Commutative Property. *Commute* means *to travel*; according to the Commutative Property, terms can travel or move to a different order.

ACTIVITY **2.3** Common Multiples

You can use rectangular arrays to determine multiples and common multiples.

Consider the area model for $6 \cdot 8 = 48$.

A **multiple** is the product of a given whole number and another whole number.



One way to think about the area model is to analyze the collection of columns. The addition of each new column creates a multiple of 6.

- The first column is a 6×1 rectangle representing the first multiple of 6, or 6.
- The first and second columns together are a 6×2 rectangle representing the second multiple of 6, or 12.
- The whole rectangle represents 6×8, or 48.

The **Commutative Property**, when applied for multiplication, states that for any numbers *a* and *b*, the product $a \cdot b$ is equal to the product $b \cdot a$.

1. List the first eight multiples of 6 by labeling each column of the area model.

Next, think about the area model as a collection of 6 rows. The first row alone creates an 8×1 rectangle, which represents the first multiple of 8, or 8. Including all rows, the 8×6 rectangle represents the sixth multiple of 8, or 48.

2. List the first six multiples of 8 by labeling each row of the area model.

ELL Tip

Connect the terms *multiple* and *multiply*. The multiples of a number are created by taking a number and multiplying it by 1, 2, 3, etc.

While 48 is a multiple shared by both 6 and 8, it is not the **least common multiple (LCM)**. The LCM is the smallest multiple (other than zero) that two or more numbers have in common.

Analyze the multiples of 6 and 8 that you labeled on the area model.

3. Identify the least common multiple of 6 and 8.

LCM(6, 8) = _____

As demonstrated by the rectangular array, for any two whole numbers *a* and *b*, a common multiple is $a \cdot b$. However, this number may not be the *least* common multiple of *a* and *b*.

4. Determine the least common multiple of 6 and 9.

a. List the first 9 multiples of 6.

b. List the first 6 multiples of 9.

c. What is the least common multiple of 6 and 9?

5. Determine the least common multiple of 7 and 8.

LCM(7, 8) = _____

6. Using prime factorization, how can you determine whether the least common multiple of two numbers is the product of the two numbers or is less than the product of the two numbers?

Answers

3. 24

4a. 6, 12, 18, 24, 30, 36, 42, 48, 54

4b. 9, 18, 27, 36, 45, 54

4c. 18

5. 56

6. If the two numbers have a GCF other than 1, the LCM is less than the product of the numbers; otherwise it is the product.

Answers

1a.

12	2	2	3	
10	2			5

Product: 120
GCF: 2
LCM: $2 \cdot 2 \cdot 3 \cdot 5 = 60$

ACTIVITY 2.4

Using Prime Factors to Determine the LCM

Suppose you want to determine the LCM of 56 and 42 without drawing an area model. Is there another way?

WORKED EXAMPLE

Organize the prime factors into a table. Only list shared factors in the same column.

Number	Prime Factors				
56	2	2	2		7
42	2			3	7

Determine the prime factors that the numbers share.

Shared prime factors: 2, 7

Determine the prime factors that the numbers *do not* share.

Non-shared prime factors: 2, 2, 3

The least common multiple of the two numbers is the product of their shared prime factors and non-shared prime factors.

$2 \cdot 2 \cdot 2 \cdot 3 \cdot 7 = 168$

LCM(56, 42) = 168

1. For each pair of numbers, determine their product. Then, use a factor table to determine their least common multiple and their greatest common factor.

a. 12 and 10

b. 9 and 15

c. 9 and 10

d. 5 and 9

2. Write a sentence to describe the relationship between the product, GCF, and LCM.

Answers

1b.

9	3	3	
15	3		5

Product: 135
GCF: 3
LCM: $3 \cdot 3 \cdot 5 = 45$

1c.

9		3	3	
10	2			5

Product: 90
GCF: 1
LCM: $2 \cdot 3 \cdot 3 \cdot 5 = 90$

1d.

5			5
9	3	3	

Product: 45
GCF: 1
LCM: $3 \cdot 3 \cdot 5 = 45$

2. The LCM multiplied by the GCF is equal to the product.

   The product divided by the GCF is equal to the LCM.

   The product divided by the LCM is equal to the GCF.

Answers

1a. 2(18 + 12); 3(12 + 8); 4(9 + 6); 6(6 + 4); 12(3 + 2); 1(36 + 24)

1b. You can find all the common factors of 36 and 24. The *a* terms are all of those common factors.

2. Yes, because you can always make a factor tree or table.

3. The numbers are relatively prime.

4. Yes, you can divide the product of the numbers by the GCF to get the LCM.

5. The numbers are relatively prime.

NOTES

TALK the TALK

Bringing It Back Around

You have composed and decomposed numbers using factors and multiples.

Use the relationship between factors and multiples to answer each question.

1. **Consider the sum 36 + 24.**

   a. **Express the sum 36 + 24 as many ways as possible as the product *a*(*b* + *c*).**

   b. **How can you use factors to determine whether you have listed all possible products *a*(*b* + *c*) that are equivalent to 36 + 24?**

2. **Can you always determine the greatest common factor of any two numbers? Explain your reasoning.**

3. **If the greatest common factor of two numbers is 1, what can you say about the numbers?**

4. **Can you always determine the least common multiple of any two numbers? Explain your reasoning.**

5. **If the least common multiple of two numbers is the product of those numbers, what can you say about the two numbers?**

Grid Paper

Why is this page blank?

So you can cut out your rectangles on the other side.