Factors and Multiples Summary

KEY TERMS

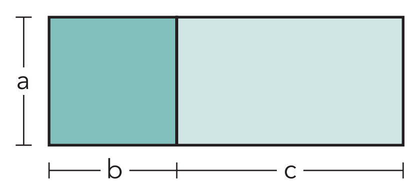
- numeric expression
- equation
- Distributive Property
- base
- power
- exponent
- common factor
- relatively prime
- greatest common factor (GCF)
- multiple
- Commutative Property
- least common multiple (LCM)

LESSON 1 Taking Apart Numbers and Shapes

A **numeric expression** is a mathematical phrase that contains numbers and operations. An **equation** is a mathematical sentence that uses an equals sign to show that two quantities are the same as one another.

The equation $5 \times 27 = 135$ shows that the expression 5×27 is equal to the expression 135.

There are many ways to rewrite equivalent expressions using properties of operations. The **Distributive Property**, when applied for multiplication, states that for any numbers *a*, *b*, and *c*, $a(b + c) = ab + ac$.



For example, you can use the Distributive Property to rewrite the expression $4(2 + 15)$.

$$4\,(2 + 15) = 4 \cdot 2 + 4 \cdot 15$$

You can read and describe the expression $4(2 + 15)$ in different ways.

For example, you can say:
- four times the quantity of two plus fifteen,
- four times the sum of two and fifteen, or
- the product of four and the sum of two and fifteen.

You can describe the expression $4(2 + 15)$ as a product of two factors. The quantity $(2 + 15)$ is both a single factor and a sum of two terms.

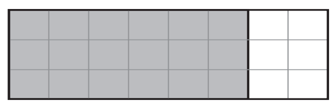
You can also use grouping symbols to show that you need to multiply each set of factors before you add them, $(4 \cdot 2) + (4 \cdot 15)$.

LESSON 2

Searching for Common Ground

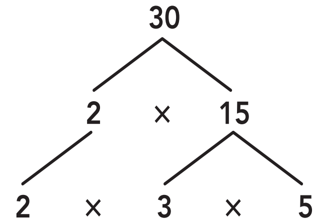
Understanding the area of rectangles is helpful when learning about factors. A rectangular area model is one way to represent multiplication. You can determine the factors of a number by creating rectangles with a given area. You can combine rectangles with a shared side length, or *common factor*, to create larger rectangles. **Common factors** are the factors shared between the numbers.

In this example, the common factor, or shared sided length, of the two smaller rectagles is three.



Dimensions of Rectangle with an Area of 18	Dimensions of Rectangle with an Area of 6	Dimensions of the Combined Rectangle	Area of the Combined Rectangle as a Sum of the Smaller Rectangles	Total Area of Combined Rectangle
3×6	3×2	$3(6 + 2)$	$18 + 6$	24

One way to determine the prime factors of a number is by creating a factor tree. You can use a factor tree to organize the prime factors of a given number.



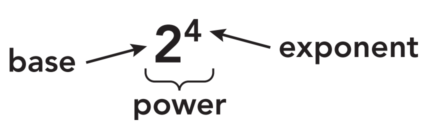
The example shows the prime factorization for 30.

- Pick any whole number factor pair of 30, other than 1 and 30.
- Draw a branch from 30 to each factor, 2 and 15.
- Since both of the factors are not prime, you are not finished.
- Use branches to write a factor pair for 15.
- Because 2, 3, and 5 are all prime, this factor tree is complete.

The prime factors of 30 are 2, 3, and 5.

In some cases, the prime factorization has repeat factors. You can represent repeated multiplication as a power. A **power** has two elements: the base and the exponent. The **base** of a power is the factor multiplied by itself repeatedly, and the **exponent** of the power is the number of times you use the base as a factor.

$$2 \times 2 \times 2 \times 2 = 2^4$$



Previously, we used area models to determine common factors between numbers. Another way to determine common factors is to use prime factorization.

For example, you can use prime factorization to determine common factors of 56 and 42. Start by writing each number as a product of its prime factors.

$56 = 2 \cdot 2 \cdot 2 \cdot 7$
$42 = 2 \cdot 3 \cdot 7$

Organize the prime factors into a table, where only shared factors are listed in the same column.

Number	Prime Factors				
56	2	2	2		7
42	2			3	7

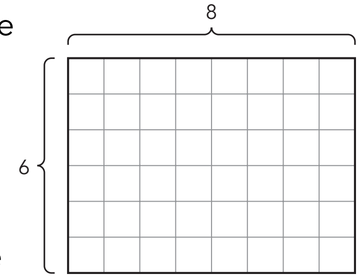
The common factors of the two numbers are the numbers that are in both rows and the product of the numbers that are in both rows.

The common factors of 56 and 42 are 2, 7, and 14.

The **greatest common factor (GCF)** is the largest factor two or more numbers have in common. In the example of 56 and 42, the greatest common factor is 14. Two numbers that do not have any common factors other than 1 are **relatively prime**.

The **Commutative Property**, when applied for multiplication, states that for any numbers *a* and *b*, the product $a \cdot b$ is equal to the product $b \cdot a$.

You can use rectangles to determine multiples and common multiples. A **multiple** is the product of a given whole number and another whole number. One way to think about the area model is to analyze the collection of rows and columns in a rectangle.



In the example shown, the addition of each new row creates a multiple of 8. The addition of each new column creates a multiple of 6.

While 48 is a multiple shared by both 6 and 8, it is not the **least common multiple (LCM)**. The LCM is the smallest multiple (other than zero) that two or more numbers have in common.

You can use tables like the one above with 56 and 42 to determine the LCM. You can determine the prime factors that the numbers share and the prime factors that the numbers do not share. The least common multiple of the two numbers is the product of their shared prime factors and non-shared prime factors.

$2 \cdot 2 \cdot 2 \cdot 3 \cdot 7 = 168$

LCM(56, 42) = 168

You can compose and decompose numbers using factors and multiples.

LESSON 3

Composing and Decomposing Numbers

You can solve real-world problems that involve common factors or common multiples by thinking about the question you are trying to answer. Remember that common factors help you think about how to divide, or share things equally, and common multiples help you think about how things with different cycles can occur at the same time.

For example, a local bus arrives at the stop near Aaron's house every 15 minutes. An express bus arrives at the same stop every 9 minutes. Aaron sees both a local and an express bus arrive at the stop at 10 A.M. What is the next time that he would expect to see both buses arrive at the stop?

The problem is asking about when the two different cycles of the buses will occur again at the same time, so you can use the least common multiple of 15 and 9 to answer the question.

The multiples of 15 are 15, 30, 45, 60, 75, . . .
The multiples of 9 are 9, 18, 27, 36, 45, 54, . . .

The least common multiple of 15 and 9 is 45, therefore the two buses should arrive at the stop at the same time every 45 minutes. The next time Aaron would expect to see both buses at the stop is 10:45 A.M.

In another example, Ramona is filling window box planters that will be sold to benefit a local charity. She has 56 pansies, 42 tulips, and 28 marigolds. What is the greatest number of planters she can fill if she wants to use all of the flowers and have the same number of each type of flower in each planter? How many of each flower type will be in a planter?

The problem is asking you to share each type of flower among an equal number of groups, so you can use the greatest common factor of 56, 42, and 28 to answer the question.

You can use prime factorization to determine the prime factors for each type of flower.

Pansies: $56 = 2 \times 2 \times 2 \times 7$

Tulips: $42 = 2 \times 3 \times 7$

Marigolds: $28 = 2 \times 2 \times 7$

The greatest common factor of 56, 42, and 28 is 14. Therefore, Ramona can fill 14 planters. Each planter will contain 4 pansies, 3 tulips, and 2 marigolds.