Did You Get the Part?

3

Multiplying Fractions

MATERIALS

None

Lesson Overview

Students review the area model for multiplication and apply it to multiplying mixed numbers. They analyze two methods for multiplying mixed numbers and then use these methods to answer questions in the context of a real-world scenario.

Grade 6
Number and Operations

(3) The student applies mathematical process standards to represent addition, subtraction, multiplication, and division while solving problems and justifying solutions. The student is expected to:

(B) determine, with and without computation, whether a quantity is increased or decreased when multiplied by a fraction, including values greater than or less than one.
(E) multiply and divide positive rational numbers fluently.

ELPS

1.A, 1.C, 1.E, 1.F, 1.H, 2.C, 2.I, 3.D, 3.E, 3.G, 3.H, 4.F

Essential Ideas

- Area models can be used to illustrate the multiplication of two fractions, which is essentially the same as taking a part of a part.
- An area model representing the multiplication of two mixed numbers can be tiled with fractional unit squares to express the product as an improper fraction.
- The product of two fractions represented by an area model is the same as the product of the fractions calculated using the standard algorithm.

Lesson Structure and Pacing: 1 Day

Engage

Getting Started: A Part of a Part

Students use area models to represent the product of fractions less than 1 and connect the models to the standard algorithm for multiplying fractions.

Develop

Activity 3.1: Using Area Models to Multiply Mixed Numbers

Students decompose an area model to determine the product of two mixed numbers. They then tile the same area model with fractional unit squares and connect the product represented by the model to the product of two mixed numbers written as an improper fraction.

Activity 3.2: Multiplying Mixed Numbers

Students calculate the product of mixed numbers in contextual and mathematical problems.

Demonstrate

Talk The Talk: Going in a General Direction

Students complete statements generalizing about the size of the product when multiplying by a fraction less than one. They describe the algorithm for multiplying fractions or mixed numbers in their own words.

Facilitation Notes

In this activity, students use area models to represent the product of fractions less than one. They connect the models to the standard algorithm for multiplying fractions.

Ask a student to read the introductory paragraph aloud. Then have students complete Question 1 and 2 with a partner. Share responses as a class.

Questions to ask

- What does the shading in the model represent?
- What does the region with overlapping shading represent?
- Why are 4 rectangles shaded the same? What fraction does that represent in the problem?
- Why are 2 rectangles shaded the same? How do they represent the fraction $\frac{1}{4}$?
- Use the order of shading to explain how the model can be used to express $\frac{1}{2}$ of $\frac{1}{4}$?
- Use the order of shading to explain how the model can be used to express $\frac{1}{4}$ of $\frac{1}{2}$?
- What property tells you that the order doesn't matter in multiplication?

Differentiation strategies

- To scaffold support with Question 1, review how to write equivalent fractions.
- To extend the activity, challenge students to extend the model to demonstrate $\frac{1}{4} \times \frac{1}{2} \times \frac{1}{8}$. If the area model represented the product of two fractions, one representing the base and the other representing the height of the model, challenge students to think about if their model would have to change to represent the product of three fractions. Students may connect this multiplication problem to calculating the volume of a rectangular prism. Or, students may determine that they can use one area model to represent the product of two of the fractions in the problem. Then use that product and multiply it with the remaining fraction in the problem using a second area model. Ask students to mathematically justify why that works.

Misconception

Students may think the 4 shaded rectangles in the diagram represent $\frac{1}{4}$. The 4 shaded rectangles represent $\frac{4}{8}$ or $\frac{1}{2}$. Likewise, the 2 shaded rectangles represent $\frac{2}{8}$ or $\frac{1}{4}$.

Direct students to complete Question 3 and 4 individually. Have students compare their responses with a partner or in a small group. Discuss responses as a class to make the connection between the area model and the algorithm for multiplying two fractions less than 1.

Questions to ask

- How is this multiplication problem different from the previous one?
- How does the model demonstrate the product $\frac{1}{2}$?
- What extra step could you include in your algorithm to eliminate rewriting $\frac{6}{12}$ as $\frac{1}{2}$?

Differentiation strategy

To scaffold support, review how to divide out common factors before multiplying across.

Summary

You can use area models to illustrate the multiplication of two fractions, which is essentially the same as taking a part of a part.

DEVELOP

Activity 3.1

Using Area Models to Multiply Mixed Numbers

Facilitation Notes

In this activity, students decompose an area model to determine the product of two mixed numbers. They then tile the same area model with fractional unit squares and connect the product represented by the model to the product of two mixed numbers written as an improper fraction.

To introduce the activity and assess prior knowledge, ask students to define and provide an example of a mixed number to a partner. Randomly select a few students to share their responses.

Then, ask a student to read the introduction aloud. Ask students to complete Question 1 and 2 with a partner. Share responses as a class.

Questions to ask

- What is ceramic tile?
- How many different-sizes of tiles are required to cover the table?
- How many tiles have an area of $\frac{1}{2}$ sq. ft?
- What are the dimensions of the tiles with an area of $\frac{1}{2}$ sq. ft?
- What is the area of the smallest tile? How do you know?
- What are the dimensions of the smallest tile?
- How did you calculate the portion of tiles needed to cover the tabletop?
- Why are the portion of tiles and the area of the table both $8\frac{3}{4}$ square feet?

As students work, look for

Use of the area model or operations with fractions to determine the portion of tiles needed to cover the tabletop.

Differentiation strategy

To scaffold support, review the terms *mixed number*, *proper fraction*, and *improper fraction*. Remind students that the fractions multiplied in the Getting Started, those less than 1, are proper fractions. Also review how to convert a mixed number to an improper fraction and vice versa.

Ask students to work with a partner to complete Questions 3 through 6. Share responses as a class.

As students work, look for

Modification of the original model into smaller square tiles or creation of a new model by beginning with the small square tiles.

Questions to ask

- What is the benefit of using smaller tiles?
- Why did you use tiles with an area of $\frac{1}{4}$ sq. ft?
- How did you know how many tiles to place across the length and width of the tabletop?
- Did you express the size of the tile in terms of its area or its dimensions?
- How can you determine the dimensions of one tile from the model?
- Verify the dimensions of one tile by using the area formula.

Misconception

Students may think it takes 10 tiles with an area of $\frac{1}{4}$ sq. ft to span a length of $2\frac{1}{2}$ ft. It takes five tiles with an area of $\frac{1}{4}$ sq. in. to span the length of $2\frac{1}{2}$ ft because the tiles are $\frac{1}{2}$ ft $\times$ $\frac{1}{2}$ ft in length.

Summary

An area model can be used to represent the product of two mixed numbers.

Activity 3.2
Multiplying Mixed Numbers

Facilitation Notes

In this activity, students calculate the product of mixed numbers in contextual and mathematical problems.

Ask a student to read the introductory paragraphs aloud. Direct students to individually read to understand Question 1 along with Megan's and CJ's strategies. Then ask students to work with a partner to discuss how Megan and CJ differ in strategy but produce the same correct solution. Select a few students to share their reasoning.

As students work, look for

Correct mathematical language. Students may say that CJ *cancelled out* the 3s. Have them replace that language with *divided out* which denotes an actual mathematical operation. They may also say that they *reduced* the fraction $\frac{21}{3}$ to 7. Because *reduce* implies the value was changed, suggest the phrases *rewrite* or *express in lowest terms*.

Questions to ask

- Why was 3 rewritten as a fraction?
- What are the steps to multiply a mixed number by a whole number?
- Whose method do you prefer? Why?

Have students complete Question 2 individually. Then, have them compare their methods and solutions with a partner. Select students who used different strategies to share their work to facilitate a class discussion. Look to select students that used Megan's strategy, CJ's strategy, an area model or something different.

Questions to ask

- How can you use estimation to check the reasonableness of your responses?
- How did you determine which strategy to use?

Misconception

When multiplying mixed numbers, students may multiply the whole number parts and the fractional parts, for example, $4\frac{1}{2} \times 2\frac{1}{4} = 8\frac{1}{8}$ rather than $10\frac{1}{8}$. Revisit the problem and area model from Activity 1 to demonstrate why this method does not work; all of the $\frac{1}{2}$ sq. ft tiles that are part of the product were overlooked.

Have students complete Question 3 with a partner or group. Share responses as a class.

Questions to ask

- When you multiply two mixed numbers, can the result be a whole number? Provide an example.
- Is the product larger or smaller than both factors? Why do you think that occurs?

Differentiation strategy

To extend this activity, ask additional questions, such as:

- What is the serving size in cups per person?
- What portion of the trail mix is nuts and seeds?

Summary

To multiply two mixed numbers, first write them as improper fractions, and then use the standard algorithm. The product can be left as an improper fraction or rewritten as a mixed number.

Talk the Talk: Going in a General Direction

DEMONSTRATE

Facilitation Notes

In this activity, students complete statements generalizing about the size of the product when multiplying by a fraction less than one. They describe the algorithm for multiplying fractions or mixed numbers in their own words.

Ask a student to read the introduction aloud. Have students complete Questions 1 and 2 with a partner or group. Share responses as a class.

Questions to ask

- Provide an example to support your *always* response.
- Provide two examples to support your *sometimes* response, one for each type of product.

- How does your algorithm address mixed numbers?
- How does your algorithm address writing the product in lowest terms?

As students work, look for

Mathematical terminology, such as *numerator, denominator, factors, products, mixed numbers, improper fractions*, and *lowest terms*.

Misconception

Students sometimes overgeneralize and think that when you multiply any two numbers, you always get a larger number. Ask them to edit their statement to specify for what types of numbers that statement holds true.

Summary

The size of the product of two fractions depends on the size of the factors.

Did You Get the Part?

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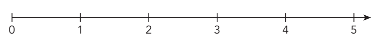
Multiplying Fractions

WARM UP

Plot each value on the number line shown.



1. 2

2. $\frac{1}{4}$

3. $4\frac{1}{3}$

4. $\frac{7}{2}$

LEARNING GOALS

- Connect an area model to the standard algorithm for multiplying two fractions.
- Multiply two fractions using the standard algorithm.
- Calculate the products of fractions in real-world and mathematical problems.

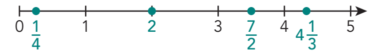
KEY TERM

- algorithm

You have used area models to represent products of whole numbers. How can you use area models to represent products of fractions?

Warm Up Answers

1–4.



Answers

1. $\frac{1}{4}$ is represented by dividing the length of the unit square into 4 equal parts and shading 1 part. $\frac{1}{2}$ is represented by dividing the width of the unit square into two equal parts and shading 1 part.
2. $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$; There are 8 equal parts in the model with 1 part shaded by overlapping regions.

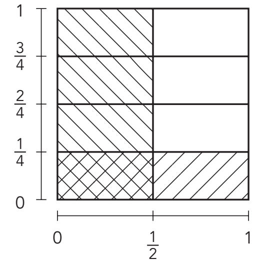
Getting Started

A Part of a Part

Previously, you used an area model to represent products, determine factors, and list multiples of given numbers. In the same way that area models represent whole number multiplication, area models can represent fraction multiplication.

Consider the expression $\frac{1}{4} \times \frac{1}{2}$ represented in the area model shown.

1. How are the factors $\frac{1}{4} \times \frac{1}{2}$ represented in the model?

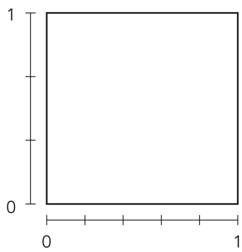


2. What is the product of $\frac{1}{4} \times \frac{1}{2}$? Describe how the product is represented in the model.

2 • TOPIC 2: Positive Rational Numbers

Consider the expression $\frac{2}{3} \times \frac{3}{5}$.

3. Model the expression and determine the product.

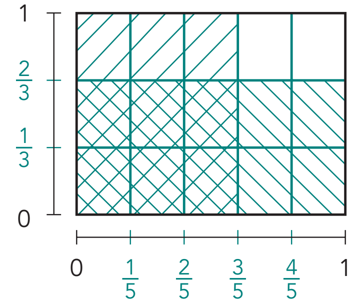


4. Show how the *algorithm* for multiplying two fractions less than 1 gives the same product as the model.

An **algorithm** is a process or description of steps you can follow to complete a mathematical calculation.

Answers

3.



$\frac{6}{15} = \frac{2}{5}$

4. $\frac{2}{3} \times \frac{3}{5} = \frac{6}{15} = \frac{2}{5}$

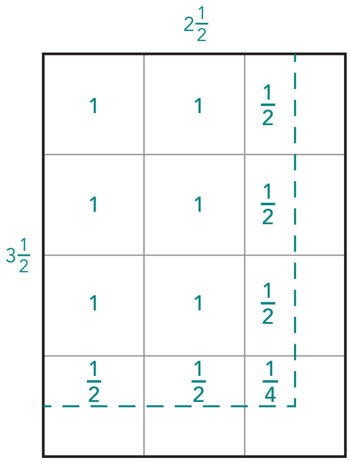
To multiply two fractions, you multiply across the numerators and denominators.

The numerator represents the part of the whole, shown in the model as the number of overlapping individual rectangles.

The denominator represents the total number of equal parts in the whole, shown in the model as the total number of individual rectangles.

Answers

1.



2. She will use $8\frac{3}{4}$ tiles.
   She uses 6 full tiles, five $\frac{1}{2}$ tiles, and one $\frac{1}{4}$ tile.
   $(6 \times 1) + \left(5 \times \frac{1}{2}\right) + \frac{1}{4}$
   $= 6 + 2\frac{1}{2} + \frac{1}{4}$
   $= 8\frac{3}{4}$

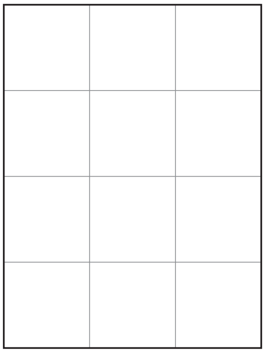
ACTIVITY **3.1**

Using Area Models to Multiply Mixed Numbers

Bree is tiling the top of a table that measures $3\frac{1}{2}$ feet by $2\frac{1}{2}$ feet. She has 12 ceramic tiles that each measure 1 foot by 1 foot.

Consider the 4 × 3 area model that represents the 12 tiles Bree will use.

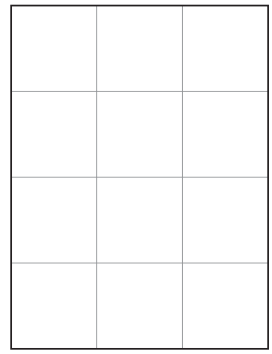
1. Create a model to represent the dimensions of the table.



2. What portion of the tiles will Bree use to cover the tabletop?

Suppose Bree doesn't want to cut the 1 ft × 1 ft tiles. Instead, she wants to buy smaller sized square tiles that she can use to cover the entire tabletop.

3. Continue to divide the model into equal-sized tiles. Describe your model.



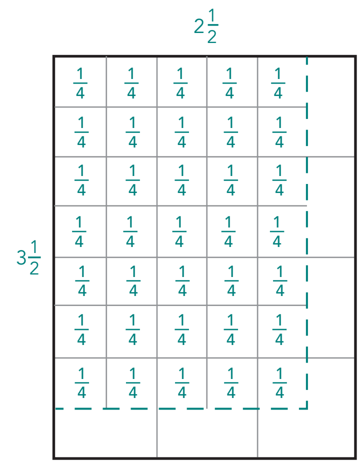
4. What size tiles should she buy?

5. How many tiles of that size does she need?

6. Show that the area represented in the model is the same as the product of the side lengths.

Answers

3. See model. The model shows $\frac{35}{4}$ or thirty-five $\frac{1}{4}$-square unit tiles.



4. Bree should buy $\frac{1}{2}$ ft × $\frac{1}{2}$ ft tiles, or 6 in. × 6 in. tiles.

5. She needs to purchase 35 tiles.

6. $3\frac{1}{2} \times 2\frac{1}{2} = \frac{7}{2} \times \frac{5}{2} = \frac{35}{4}$

Relate the terms *proper fractions* and *improper fractions* by defining the prefix *im-* as *not*. In a *proper fraction*, the numerator is less than the denominator. In an *improper fraction*, the numerator is greater (not less) than the denominator. Provide non-mathematical examples of *possible* and *impossible*.

ELL Tip

Help students make sense of mixed numbers by defining the adjective mixed. *Mix* means *to combine two or more items. Mixed numbers* are a combination of whole numbers and fractions. Provide non-mathematical examples such as music *mix* or trail *mix* (in the next activity).

Answers

1. CJ divided the first fraction by 3 (by dividing the numerator by 3) and multiplied the second fraction by 3 (by dividing the denominator by 3). This keeps the product the same, but when you multiply across, the product is already 7 instead of $\frac{21}{3}$.

ACTIVITY **3.2**

Multiplying Mixed Numbers

You can use the standard algorithm to multiply a whole number and a mixed number, a mixed number and a fraction less than one, or two mixed numbers when solving real-world and mathematical problems.

The teachers at Riverside Middle School decide to make trail mix for an upcoming field trip. Ms. Hadley shares her Hawaiian Trail Mix Extravaganza recipe with the other teachers. The recipe shown is for 1 batch.

RECIPE

Hawaiian Trail Mix Extravaganza

- $3\frac{3}{8}$ cups of macadamia nuts
- $2\frac{1}{3}$ cups of almonds
- $2\frac{1}{4}$ cups of pumpkin seeds
- $1\frac{1}{3}$ cups of sunflower seeds
- $3\frac{3}{8}$ cups of dried cherries
- $2\frac{5}{6}$ cups of honey
- $4\frac{5}{8}$ cups of popped pop corn
- $4\frac{1}{2}$ cups of raisins
- $\frac{3}{4}$ cups of corn syrup
- $2\frac{3}{4}$ cups of granola

Feeds 12 People

1. **Megan and CJ determined the number of cups of almonds it will take to make 3 batches. How is CJ's strategy different than Megan's?**

Megan

$3 \times 2\frac{1}{3}$

$\frac{3}{1} \times \frac{7}{3} = \frac{21}{3}$

$= 7$

7 cups

CJ

$3 \times 2\frac{1}{3}$

$\frac{\cancel{3}}{1} \times \frac{7}{\cancel{3}} = 7$

7 cups

ELL Tip

Trail mix is a type of snack, typically a combination of granola, fruits, and nuts. It got its name because it is easy to carry and nibble on while taking a hike along a trail.

2. Determine the number of cups of each ingredient it will take to make $4\frac{1}{2}$ batches. Show your work.

   a. corn syrup

   b. sunflower seeds

   c. pumpkin seeds

3. Calculate each product. Write your answer as a mixed number. Show your work.

   a. $2\frac{1}{2} \times 3\frac{2}{5}$

   b. $2\frac{2}{3} \times 4\frac{1}{4}$

   c. $1\frac{3}{4} \times \frac{2}{5}$

   d. $1\frac{1}{2} \times \frac{5}{6}$

   e. $3\frac{3}{4} \times 2$

   f. $2\frac{5}{8} \times 3$

Answers

2a. $4\frac{1}{2} \times \frac{3}{4}$

$\frac{9}{2} \times \frac{3}{4} = \frac{27}{8}$

$= 3\frac{3}{8}$

$3\frac{3}{8}$ cups

2b. $4\frac{1}{2} \times 1\frac{1}{3}$

$\frac{9}{2} \times \frac{4}{3} = \frac{36}{6}$

$= 6$

6 cups

2c. $4\frac{1}{2} \times 2\frac{1}{4}$

$\frac{9}{2} \times \frac{9}{4} = \frac{81}{8}$

$= 10\frac{1}{8}$

$10\frac{1}{8}$ cups

3a. $\frac{\cancel{5}^1}{2} \times \frac{17}{\cancel{5}_1} = \frac{17}{2}$

$= 8\frac{1}{2}$

3b. $\frac{\cancel{8}^2}{3} \times \frac{17}{\cancel{4}_1} = \frac{34}{3}$

$= 11\frac{1}{3}$

3c. $\frac{7}{\cancel{4}_2} \times \frac{\cancel{2}^1}{5} = \frac{7}{10}$

3d. $\frac{\cancel{3}^1}{2} \times \frac{5}{\cancel{6}_2} = \frac{5}{4}$

$= 1\frac{1}{4}$

3e. $\frac{15}{\cancel{4}_2} \times \frac{\cancel{2}^1}{1} = \frac{15}{2}$

$= 7\frac{1}{2}$

3f. $\frac{21}{8} \times \frac{3}{1} = \frac{63}{8}$

$= 7\frac{7}{8}$

Answers

1a. always

1b. sometimes

2. To multiply any two fractions or mixed numbers, first rewrite mixed numbers as improper fractions. Then, multiply across the numerators and denominators of the fractions being multiplied. You can then choose to rewrite the product in lowest terms and/or rewrite an improper fraction as a mixed number.

NOTES

TALK the TALK

Going in a General Direction

Look back at the factors and products in this lesson. What generalizations can you make about the multiplication of fractions?

1. Determine whether each statement is *always, sometimes,* or *never* true. Provide examples.

a. If a fraction between 0 and 1 is multiplied by another fraction between 0 and 1, the product will be less than 1.

b. If a fraction between 0 and 1 is multiplied by a mixed number, the product will be greater than 1.

2. Describe the algorithm for multiplying any two fractions or mixed numbers.