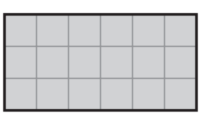
Yours IS to Reason Why!

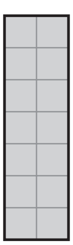
4

Fraction by Fraction Division

WARM UP

Write the multiplication-division fact family for each rectangular array.

1. 

2. 

LEARNING GOALS

- Model the division of fractions using bar models and number lines.
- Compute and interpret quotients of fractions and interpret remainders in real-world problems.
- Divide with mixed numbers.

KEY TERMS

- rational number
- reciprocal
- multiplicative inverse
- complex fraction

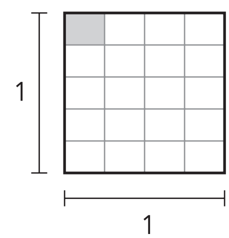
You know how to multiply and divide with whole numbers and positive rational numbers. How can you apply what you know to understand how to divide two fractions or mixed numbers?

Getting Started

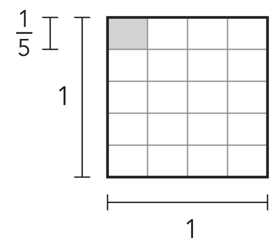
All in the Fact Family

You can write a multiplication-division fact family using fractions. Consider the unit square model divided into 5 equal rows and 4 equal columns.

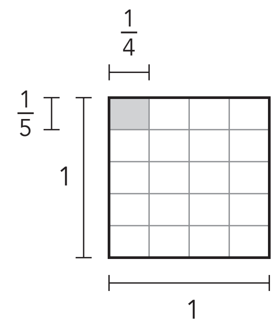
WORKED EXAMPLE



The shaded area represents the fraction $\frac{1}{20}$ because 1 rectangle is shaded of the 20 total rectangles.



The height of the shaded rectangle is $\frac{1}{5}$ of the height of the model.



The width of the shaded rectangle is $\frac{1}{4}$ of the width of the model.

So, the shaded area of the rectangle represents the product $\frac{1}{5} \times \frac{1}{4} = \frac{1}{20}$.

Fractions are part of a larger set of numbers called *rational numbers*. A **rational number** is a number you can write in the form $\frac{a}{b}$, where a and b are both whole numbers greater than zero.

1. **Write a multiplication-division fact family for the model.**

2. **Describe how the model shows the division of fractions.**

3. **Draw a diagram to model the multiplication of two different fractions. Write a multiplication-division fact family with fractions for your diagram. Show your work.**

ACTIVITY 4.1

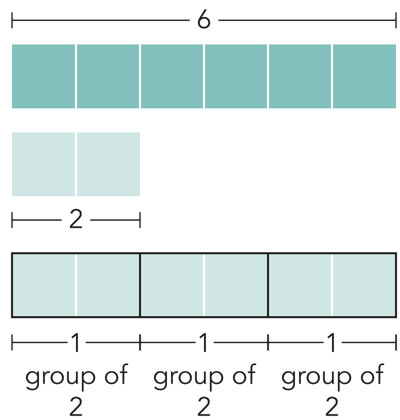
Whole Number ÷ Fraction

Division is the process of breaking a number up into an equal number of parts.

WORKED EXAMPLE

The model shows 6 ÷ 2. The expression asks, "How many groups of two are in 6?"

There are 3 groups of 2 in 6, so 6 ÷ 2 = 3.

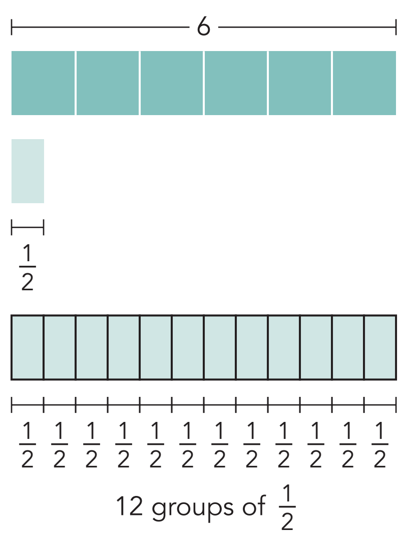


When you divide with fractions, you are asking the same question.

WORKED EXAMPLE

The model shows $6 \div \frac{1}{2}$. The expression asks, "How many halves, or groups of $\frac{1}{2}$, are in 6?"

There are twelve $\frac{1}{2}$ parts in 6, so $6 \div \frac{1}{2} = 12$.



"The *dividend* is the number divided into equal groups, the *divisor* is the number that divides the dividend, and the *quotient* is the result of the division."

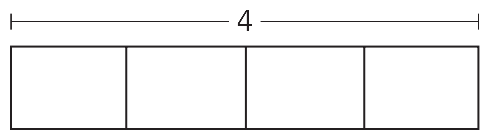
1. Label the dividend, divisor, and quotient in each model.

Consider each situation.

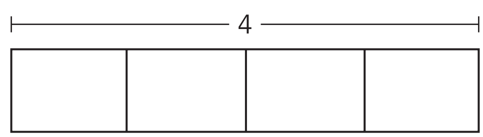
Suppose you are creating snack-sized portions from 4 cups of trail mix.

2. Determine the number of portions you can make if you use different measuring scoops. Draw a diagram and write the appropriate number sentence.

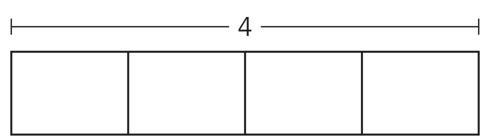
a. $\frac{1}{2}$ **cup measuring scoop**



b. $\frac{1}{4}$ **cup measuring scoop**



c. $\frac{1}{3}$ **cup measuring scoop**



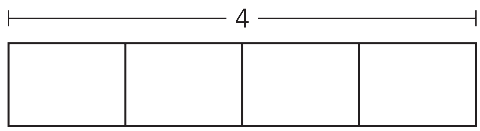
d. How did the number of portions change as the size of the measuring scoop changed?

e. Analyze the three number sentences you wrote. Consider the relationship between the dividend, divisor, and quotient in each model. What patterns do you notice? Explain your reasoning.

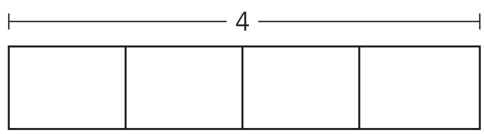
Suppose you are organizing games for a field day event that lasts 4 hours.

3. Determine how many games you can organize for the event if each game lasts the given amount of time. Draw a diagram and write the appropriate number sentence.

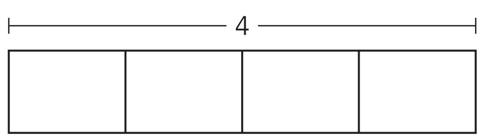
a. $\frac{2}{3}$ **hour**



b. $\frac{2}{5}$ **hour**



c. $\frac{4}{5}$ **hour**



d. Consider the relationship between the dividend, divisor, and quotient in each number sentence. How do these number sentences compare to the ones you wrote in Question 2? What patterns do you notice? Explain your reasoning.

Consider the relationships between the different division statements that you just modeled.

4. How is the quotient of $4 \div \frac{1}{3}$ related to the quotient of $4 \div \frac{2}{3}$? Explain your reasoning.

5. From a previous example, you know that $6 \div \frac{1}{2} = 12$. Use reasoning to determine each quotient.

a. $6 \div \frac{1}{4}$ b. $6 \div \frac{1}{8}$ c. $6 \div \frac{1}{16}$

6. Jamilla is throwing a small party. She has 4 pizzas and decides that everyone at her party should receive a serving size that is $\frac{3}{5}$ of a pizza. Jamilla says she has $6\frac{2}{3}$ servings, but Devon says she has $6\frac{2}{5}$ servings. Draw a diagram of the situation, and solve for the quotient to determine who is correct. Explain why the other person is not correct.

7. Jonathon has 4 pizzas and decides that everyone at his party should receive a serving size that is $\frac{3}{4}$ of a pizza. Draw a diagram and determine the number of servings he will have for his party.

ACTIVITY
4.2 Fraction ÷ Fraction

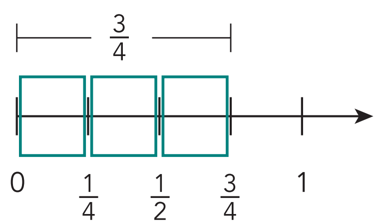
In this activity, you will use a number line model to determine a quotient when the dividend and divisor are both fractions.

You want to determine how many $\frac{1}{4}$-inch pieces are in $\frac{3}{4}$ inch.

WORKED EXAMPLE

The model shows $\frac{3}{4} \div \frac{1}{4}$.

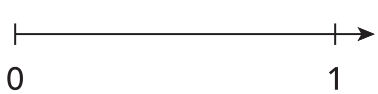
The division expression asks, "How many $\frac{1}{4}$s are in $\frac{3}{4}$?"



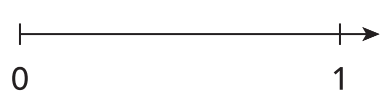
1. What is the quotient $\frac{3}{4} \div \frac{1}{4}$ represented in the model shown in the Worked Example? Interpret the quotient in terms of the situation.

2. Write a question to describe what each division expression is asking. Then, draw a model to determine the quotient. Write a sentence to describe your answer.

a. $\frac{1}{2} \div \frac{1}{8}$



b. $\frac{3}{4} \div \frac{3}{8}$



Create a model to represent each situation and then answer the question.

3. **Mason has $\frac{2}{3}$ of a foot of ribbon. He needs to divide the ribbon into $\frac{1}{6}$-foot pieces. How many pieces can he cut from the ribbon?**

4. **Niquelle places signposts along a trail that is $4\frac{1}{2}$ miles long. The signposts divide the trail into sections that are $\frac{3}{4}$ mile long. How many sections does Niquelle create along the trail?**

5. **Aster has a board that is $3\frac{1}{3}$ feet long. She needs to cut the board into $\frac{5}{12}$-foot pieces. Into how many pieces can she cut the board?**

ACTIVITY 4.3

From Model to Procedure

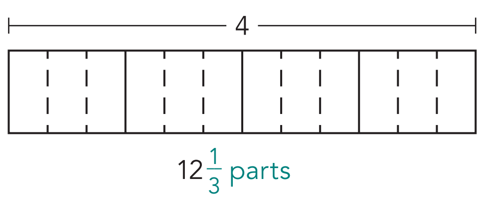
You have drawn models to answer, "How many groups of a certain size are in a number?"

In this activity, you will analyze a few models that you drew to develop an efficient way to perform division of fractions without a model.

Let's revisit the strategy to draw a model for $4 \div \frac{1}{3} = 12$ and $4 \div \frac{2}{3} = 6$.

WORKED EXAMPLE

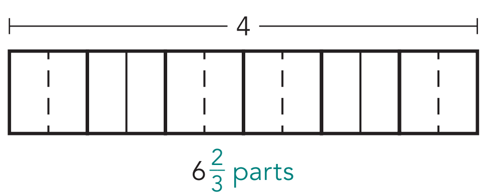
To create the model $4 \div \frac{1}{3}$, you partitioned 4 wholes to create twelve $\frac{1}{3}$ parts, so $4 \div \frac{1}{3} = 12$.



Notice there is one step to this process.

STEP 1 The denominator of the divisor is 3. Partition each whole into $\frac{1}{3}$ parts. This is the same as $4 \cdot 3 = 12$.

To create the model $4 \div \frac{2}{3}$, you partitioned 4 units to create twelve $\frac{1}{3}$ parts and then grouped 2 $\frac{1}{3}$ parts together, so $4 \div \frac{2}{3} = 6$.



Notice there are two steps to this process.

STEP 1 The denominator of the divisor is 3. Partition each whole into $\frac{1}{3}$ parts. This is the same as $4 \cdot 3 = 12$.

STEP 2 The numerator of the divisor is 2. Group two of the $\frac{1}{3}$ parts together. This is the same as $12 \div 2 = 6$.

The combination of steps is the same as $4 \cdot 3 \div 2 = 6$.

You can rewrite a fraction as division or division as a fraction: $\frac{a}{b} = a \div b$. So, you can rewrite the final number sentence, $4 \cdot 3 \div 2 = 6$, as $4 \cdot \frac{3}{2} = 6$.

1. **Compare the resulting equivalent number sentences from the Worked Examples. What do you notice?**

- $4 \div \frac{1}{3} = 12$ and $4 \cdot 3 = 12$
- $4 \div \frac{2}{3} = 6$ and $4 \cdot \frac{3}{2} = 6$

A **reciprocal**, or *multiplicative inverse*, is one of a pair of numbers whose product is 1. The **multiplicative inverse** of the number $\frac{a}{b}$ is the number $\frac{b}{a}$, where a and b are nonzero numbers.
$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$

You just developed a rule you can use to rewrite any division sentence as multiplication. You can rewrite any division sentence as multiplication by taking the reciprocal of the divisor.

2. **Rewrite each division sentence as a multiplication sentence and then calculate the quotient.**

a. $5 \div \frac{1}{4}$

b. $\frac{1}{8} \div 2$

c. $\frac{4}{5} \div \frac{1}{3}$

d. $\frac{3}{10} \div \frac{1}{3}$

3. **Explain why Alexa is incorrect and provide the correct reciprocal.**

Alexa

The reciprocal of $3\frac{8}{5}$ is $3\frac{5}{8}$.

Solve each problem. Show your work and be sure to label your answer.

4. If you have $5\frac{2}{3}$ pounds of trail mix, how many bags can you make so that each bag contains $1\frac{5}{6}$ pounds?

5. A cook in a restaurant made $47\frac{1}{2}$ cups of mashed potatoes. If there are $1\frac{1}{4}$ cups of mashed potatoes in a serving, how many servings did he make?

6. The hiking trail through Glacier Gorge in Rocky Mountain National Park is $9\frac{3}{5}$ miles round trip. If you hike $1\frac{3}{5}$ miles an hour, how many hours will the round trip take?

ACTIVITY 4.4

Different Strategies to Divide Numbers

You have learned that one way to divide two fractions is to rewrite the division problem as multiplication by the reciprocal of the divisor.

In this activity, you will consider two other strategies for dividing fractions.

WORKED EXAMPLE

In the same way that you can "multiply across," or multiply the numerators and multiply the denominators, to determine the product of two fractions, you can also "divide across" to determine the quotient of two fractions.

Determine the quotient $\frac{7}{8} \div \frac{1}{2}$.

Divide the numerators. Then divide the denominators.

$$\frac{7}{8} \div \frac{1}{2} = \frac{7 \div 1}{8 \div 2}$$
$$= \frac{7}{4}$$

1. Divide across to determine each quotient.

a. $\frac{3}{8} \div \frac{1}{4}$

b. $\frac{4}{9} \div \frac{2}{3}$

c. $\frac{2}{5} \div \frac{1}{5}$

d. $\frac{1}{4} \div \frac{3}{4}$

2. Think about the structure of the fractional parts of each dividend and divisor. What was special about the numbers that allowed you to divide across?

Kareem is trying to use the dividing across strategy to determine the quotient for the expression $\frac{6}{7} \div \frac{2}{3}$ but gets stuck.

$$\frac{6}{7} \div \frac{2}{3} = \frac{6 \div 2}{7 \div 3} = \frac{3}{7 \div 3}$$

Analyze Stella's reasoning to help Kareem.

Stella

This is still a fraction, and I know that 3 divided by 3 is 1. So, the answer is $\frac{1}{7}$.

$$\frac{\overset{1}{\cancel{3}}}{7 \div \cancel{3}_1} = \frac{1}{7}$$

3. Explain why Stella's reasoning is not correct.

Analyze Catherine's reasoning to help Kareem.

Catherine

We know how to rewrite a division sentence as a fraction.

$$\frac{3}{7 \div 3} = \frac{3}{\frac{7}{3}}$$

We also want the denominator to be 1, so we will need to use multiplicative inverses.

$$\frac{3}{7 \div 3} = \frac{3}{\frac{7}{3}} \cdot \frac{\frac{3}{7}}{\frac{3}{7}} = \frac{3 \cdot \frac{3}{7}}{1}$$

$$= \frac{9}{7}$$

So the answer is $\frac{9}{7}$.

A **complex fraction** is a fraction that has a fraction in either the numerator, the denominator, or both the numerator and the denominator.

4. Explain how Catherine maintained equivalent fractions when she multiplied by $\frac{\frac{3}{7}}{\frac{3}{7}}$.

You can use multiplicative inverses to rewrite any division sentence as multiplication.

WORKED EXAMPLE

$$\frac{5}{8} \div \frac{3}{4} = \frac{\frac{5}{8}}{\frac{3}{4}}$$

Rewrite the division expression as a complex fraction.

$$= \frac{\frac{5}{8}}{\frac{3}{4}} \cdot \frac{\frac{4}{3}}{\frac{4}{3}}$$

Multiply the numerator and denominator by the multiplicative inverse of $\frac{3}{4}$.

$$= \frac{\frac{5}{8} \cdot \frac{4}{3}}{\frac{3}{4} \cdot \frac{4}{3}} = \frac{\frac{5}{8} \cdot \frac{4}{3}}{1}$$

Perform multiplication and rewrite the denominator as 1.

$$= \frac{5}{\cancel{8}_{2}} \cdot \frac{\cancel{4}^{1}}{3} = \frac{5}{6}$$

How can you, a savvy mathematician, look at the structure of a division problem and choose the most efficient strategy?

5. Use any method to determine each quotient.

a. $\frac{9}{10} \div \frac{1}{3}$

b. $\frac{5}{6} \div \frac{1}{3}$

c. $\frac{4}{9} \div \frac{1}{2}$

d. $\frac{3}{8} \div \frac{1}{2}$

e. $\frac{7}{9} \div \frac{3}{7}$

f. $\frac{5}{2} \div \frac{2}{3}$

NOTES

TALK the TALK

Going (Almost) Numberless

Look back at the dividends, divisors, and quotients in this lesson. What generalizations can you make about the division of fractions?

1. **Complete each statement with *greater than*, *less than*, or *the same as*.**

   a. **If you divide a quantity greater than 1 by a value between 0 and 1, the quotient will be ______________ the original quantity.**

   b. **If you divide a quantity between 0 and 1 by a value greater than 1, the quotient will be ______________ the original quantity.**

   c. **If you divide a quantity between 0 and 1 by a value between 0 and 1, the quotient will be ______________ the original quantity.**

2. **Complete each statement with *always*, *sometimes*, or *never*.**

   a. **If you divide a mixed number by another mixed number, the quotient will ______________ be greater than 1.**

   b. **If you mulitply a fraction between 0 and 1 by another fraction between 0 and 1, the product will ______________ be less than 1.**

   c. **If you divide a whole number by a fraction between 0 and 1, the quotient will ______________ be less than 1.**

   d. **If you multiply a fraction between 0 and 1 by a mixed number, the product will ______________ be greater than 1.**

NOTES

3. Consider the quotients of $\frac{5}{6} \div \frac{1}{2}$ and $\frac{5}{6} \div 2$. Describe how these quotients are different.