Positive Rational Numbers Summary

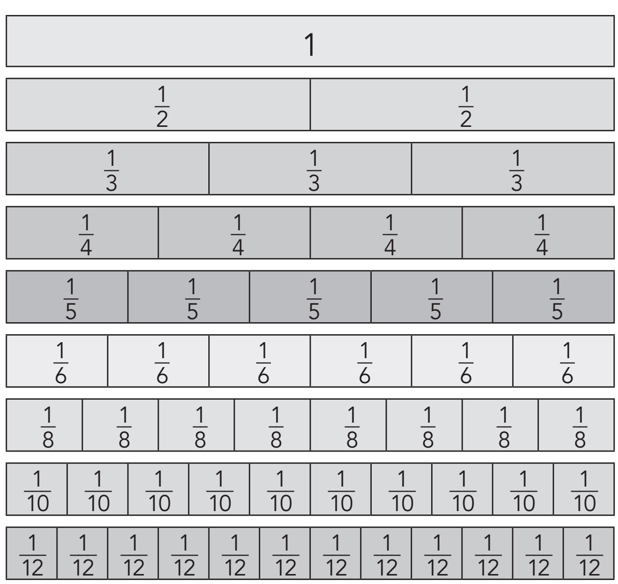
KEY TERMS

- unit fraction
- equivalent fraction
- benchmark fractions
- algorithm
- rational number
- reciprocal
- multiplicative inverse
- complex fraction

LESSON 1 Rocket Strips

A **unit fraction** is a fraction that has a numerator of 1 and a denominator that is a positive integer. Fractions that represent the same part-to-whole relationship are **equivalent fractions.**

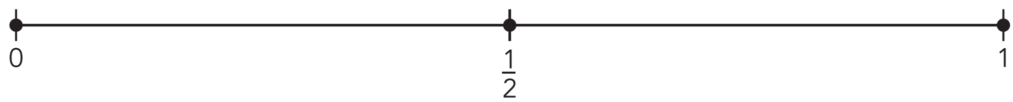
To list a set of fractions in ascending order means to list the set from least to greatest. To list a set of fractions in descending order means to list the set from greatest to least.



LESSON 2 Searching for Common Ground

Benchmark fractions are common fractions you can use to estimate the value of fractions.

Three common benchmark fractions are 0, $\frac{1}{2}$, and 1.



A fraction is close to 0 when the numerator is very small compared to the denominator.

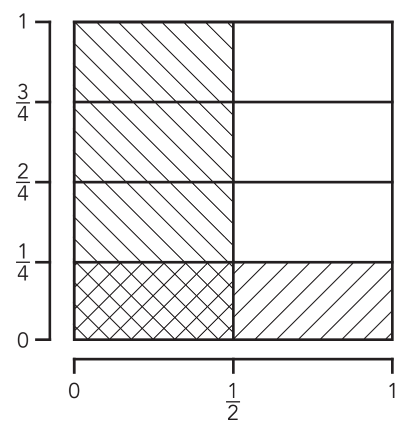
A fraction is close to $\frac{1}{2}$ when the numerator is about half the size of the denominator.

A fraction is close to 1 when the numerator is very close in size to the denominator.

LESSON 3 Did You Get the Part?

Area models can be used to represent products, to determine factors, and to list multiples of given numbers. In the same way that area models represent whole number multiplication, area models can represent fraction multiplication.

The expression $\frac{1}{4} \times \frac{1}{2}$ is represented in the area model shown.



To multiply two fractions, you multiply across the numerators and denominators. The numerator represents the part of the whole, shown in the model as the number of overlapping individual rectangles. The denominator represents the total number of equal parts in the whole, shown in the model as the total number or individual rectangles.

An **algorithm** is a process or description of steps you can follow to complete a mathematical calculation. The algorithm for multiplying fractions is to multiply across the numerators and denominators.

For example, determine the product of $1\frac{3}{4} \times 2\frac{1}{2}$.

Rewrite the factors as improper fractions.

$$1\frac{3}{4} \times 2\frac{1}{2} = \frac{7}{4} \times \frac{5}{2}$$

Multiply across numerators and denominators.

$$\frac{7}{4} \times \frac{5}{2} = \frac{35}{8}$$

Rewrite the improper fraction as a mixed number.

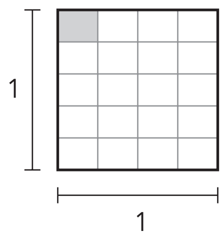
$$\frac{35}{8} = 4\frac{3}{8}$$

You can use the standard algorithm to multiply a whole number and a mixed number, a mixed number and a fraction less than one, or two mixed numbers when solving real-world and mathematical problems.

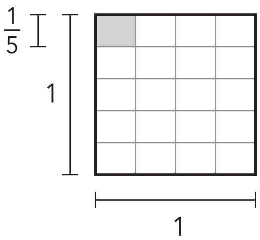
LESSON 4

Yours IS to Reason Why!

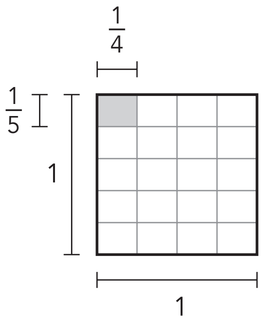
You can write a multiplication-division fact family using fractions. Consider the unit square model divided into 5 equal rows and 4 equal columns.



The shaded area represents the fraction $\frac{1}{20}$, because 1 rectangle is shaded of the 20 total rectangles.



The height of the shaded rectangle is $\frac{1}{5}$ of the height of the model.



The width of the shaded rectangle is $\frac{1}{4}$ of the width of the model.

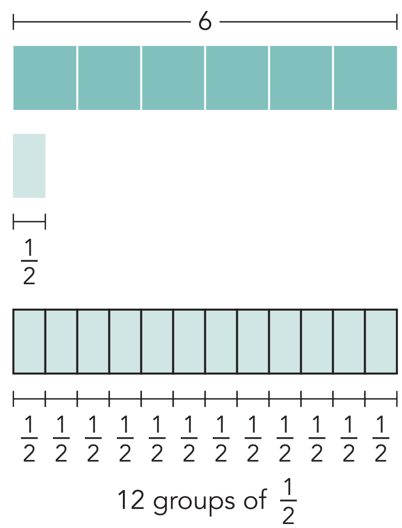
So, the shaded area of the rectangle represents the product $\frac{1}{5} \times \frac{1}{4} = \frac{1}{20}$.

Fractions are part of a larger set of numbers called rational numbers. A **rational number** is a number that can be written in the form $\frac{a}{b}$, where a and b are both whole numbers greater than 0.

Division is the process of breaking a number up into an equal number of parts. The expression $6 \div 2$ asks, "How many twos are in 6?" When you divide with fractions, you are asking the same question.

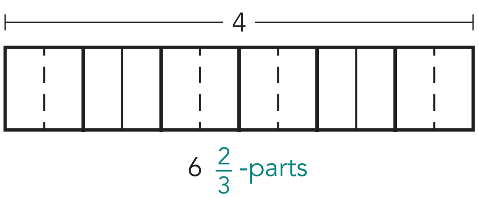
The model shows $6 \div \frac{1}{2}$. The expression asks, "How many halves, or groups of $\frac{1}{2}$, are in 6?"

There are twelve $\frac{1}{2}$ parts in 6, so $6 \div \frac{1}{2} = 12$.



You can use bar models or strip diagrams to solve division problems when you are dividing a whole number by a fraction that is not a unit fraction.

To create the model $4 \div \frac{2}{3}$, you partitioned 4 units to create twelve $\frac{1}{3}$-parts and then grouped 2 $\frac{1}{3}$-parts together, so $4 \div \frac{2}{3} = 6$.



Notice there are two steps to this process.

STEP 1 The denominator of the divisor is 3. Partition each whole into $\frac{1}{3}$-parts. This is the same as $4 \cdot 3 = 12$.

STEP 2 The numerator of the divisor is 2. Group two of the $\frac{1}{3}$-parts together. This is the same as $12 \div 2 = 6$.

The combination of steps is the same as $4 \cdot 3 \div 2 = 6$.

You can rewrite any division sentence as multiplication by taking the reciprocal of the divisor. A **reciprocal,** or **multiplicative inverse,** is one of a pair of numbers whose product is 1. The multiplicative inverse of the number $\frac{a}{b}$ is the number $\frac{b}{a}$, where *a* and *b* are nonzero numbers.

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$

For example, $\frac{5}{8} \div \frac{15}{16} = \frac{5}{8} \cdot \frac{16}{15} = \frac{2}{3}$

A **complex fraction** is a fraction that has a fraction in either the numerator, the denominator, or both the numerator and denominator. You can use multiplicative inverses to rewrite any division sentence as multiplication.

$$\frac{5}{8} \div \frac{3}{4} = \frac{\frac{5}{8}}{\frac{3}{4}}$$

Rewrite the division expression as a complex fraction.

$$= \frac{\frac{5}{8}}{\frac{3}{4}} \cdot \frac{\frac{4}{3}}{\frac{4}{3}}$$

Multiply the numerator and denominator by the multiplicative inverse of $\frac{3}{4}$.

$$= \frac{\frac{5}{8} \cdot \frac{4}{3}}{\frac{3}{4} \cdot \frac{4}{3}} = \frac{\frac{5}{8} \cdot \frac{4}{3}}{1}$$

Perform multiplication and rewrite the denominator as 1.

$$= \frac{5}{\cancel{8}_{2}} \cdot \frac{\cancel{4}^{1}}{3} = \frac{5}{6}$$