Consider Every Side

1

Constructing Triangles Given Sides

MATERIALS

Patty paper
Protractor
Ruler
Compass
Straightedge
Scissors
Pieces of raw pasta (spaghetti or linguine)

Lesson Overview

Students use patty paper, pasta, and construction tools to explore the information required to create no triangles, unique triangles, or multiple triangles when given two or three possible side lengths. They learn that an infinite number of triangles can be made from only two side lengths. They also learn that unique triangles are formed when provided with three segments that are sufficiently long in relation to each other. Students should note that if all the measures of a triangle are the same as another triangle, even though they are in different orientations, the provided information creates a unique triangle. Students then summarize their knowledge of the conditions that form 0, 1, or multiple triangles.

Grade 6

Expressions, Equations, and Relationships

(8) The student applies mathematical process standards to use geometry to represent relationships and solve problems. The student is expected to:

(A) extend previous knowledge of triangles and their properties to include the sum of angles of a triangle, the relationship between the lengths of sides and measures of angles in a triangle, and determining when three lengths form a triangle.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G

Essential Ideas

- Constructing a triangle given the length of two sides does not result in the construction of a unique triangle.
- Constructing a triangle given the length of three segments, such that the sum of two segment lengths is greater than the third length, results in the construction of a unique triangle.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Tri-, Tri-, and Tri- Again

Students review properties of triangles by determining if statements about triangles are always or sometimes true. They should recall types of triangles, including *right*, *equilateral*, *scalene*, and *isosceles*.

Develop

Activity 1.1: Pasta Triangles

Students use a piece of raw pasta such as spaghetti or linguine, break it into three pieces, and attempt to form a triangle. They measure the length of each piece and note if the pieces formed a triangle. A chart is provided for the purposes of collecting the data from all students and comparing the results. Students then make conjectures about the conditions for three sides that form a triangle.

Day 2

Activity 1.2: A Triangle Given Three Segments

Students use patty paper and construction tools to create triangles given three segments. They investigate the conditions that would make it impossible for three line segments to form a triangle.

Activity 1.3: Triangle Inequality Theorem

Students use the idea that in Euclidean geometry the shortest distance between two points is a straight line to arrive at the *Triangle Inequality Theorem.* The Triangle Inequality Theorem states that the sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

Demonstrate

Talk the Talk: None, One, or Many?

Students use the Triangle Inequality Theorem to determine if a triangle is possible for the given segment lengths.

Getting Started: Tri-, Tri-, and Tri- Again

ENGAGE

Facilitation Notes

This activity begins with two statements about triangles. Students identify which statements are always true or sometimes true and provide examples.

Students should have access to construction tools and measuring tools; however, sketches are acceptable if students can support them mathematically.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

Questions to ask

- What is an acute triangle?
- What is an obtuse triangle?
- What is a right triangle?
- What is an isosceles triangle?
- What is an equilateral triangle?
- What is a scalene triangle?
- Can a triangle have more than one right angle?

Summary

Triangles can be classified by the length of their sides or by the measures of their interior angles.

Activity 1.1

Pasta Triangles

Facilitation Notes

In this activity, students are given three side lengths. They determine the circumstances in which a triangle could be constructed. Raw pasta is used to model the length of each side of the triangle. This activity lays the groundwork for the Triangle Inequality Theorem.

Students should have access to raw pasta and measuring tools.

Ask a student to read the introduction and Question 1 aloud. Complete Questions 1 through 3 as a class.

As students work, look for

- Evidence that students used lengths for isosceles and equilateral triangles.
- Errors in the data, especially the assumption that if the lengths of two pieces equal the length of the third piece, it forms a triangle. This can happen depending on the way the students connect the pieces.
- Organization of data, such as listing the side lengths from smallest to largest for each triangle.

Have students work with a partner or in a group to complete Question 4. Share responses as a class.

Questions to ask

- Do the three broken pieces of pasta appear to be different lengths?
- Are any of the pieces of pasta out of the three the same length?
- What do you suppose is one reason the three pieces of pasta will not form a triangle?
- What do you suppose is one reason the three pieces of pasta will form a triangle?
- Do you think everyone's pieces of pasta formed a triangle? Why or why not?
- How many of your classmates had the same length pieces?
- If the three pieces of pasta were the same length, did they form a triangle?
- Considering only the lengths of pasta that formed a triangle, if you calculate the sum of the lengths of two of the pieces of pasta, how does the sum compare to the length of the third piece?

Summary

When the sum of the lengths of two sides of a triangle is greater than the length of the third side, the three side lengths form a triangle.

Activity 1.2
A Triangle Given Three Segments

Facilitation Notes

In this activity, students are given two sets of three line segments. They use patty paper to create a triangle when possible. In this situation, only one triangle is possible.

Students should have access to patty paper.

Ask a student to read the introduction aloud. Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- How many different triangles can be determined using the three given sides in Set 1?
- How many different triangles can be determined using the three given sides in Set 2?
- What does it mean to say the result is a unique triangle?
- Does it matter what order you connected the sides? Why not?
- What is another set of three side lengths that will not form a triangle?

Differentiation strategy

Assign different groups one of the two sets of sides rather than having the entire class working with both sets.

Summary

Given the length of three line segments, either one triangle or no triangle can be determined.

Activity 1.3
Triangle Inequality Theorem

Facilitation Notes

In this activity, students use the idea that in Euclidean geometry, the shortest distance between two points is a straight line, to arrive at the Triangle Inequality Theorem. The Triangle Inequality Theorem states that the sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

Ask a student to read the introduction and Question 1 aloud. Discuss the information and diagrams as a class.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

Questions to ask

- If $\overline{AC} + \overline{CB} = \overline{AB}$, why does that mean $\overline{AC}$ lies on $\overline{AB}$?
- Would line segments $\overline{AC}$, $\overline{CB}$, and $\overline{AB}$ form a triangle? Why or why not?
- What would a diagram look like to model Question 2?
- What would a diagram look like to model Question 3?
- Why do you really only need to test the lengths of the two shorter segments compared to the longest segment?
- If the sum of the lengths of the two shorter line segments is less than the length of the third side, do the three sides form a triangle?
- If the sum of the lengths of the two shorter line segments is equal to the length of the third side, do the three sides form a triangle?
- If the sum of the lengths of the two shorter line segments is greater than the length of the third side, do the three sides form a triangle?
- Why do you think the theorem is called the Triangle Inequality Theorem?

Misconception

Students may misinterpret the Triangle Inequality Theorem and erroneously think that as long as the sum of any one set of lengths is greater than the third length, the three lengths will form a triangle, while the theorem implies that the inequality be true for any combination (or all combinations) of the three lengths. Provide a counterexample to contradict this thinking. Using the segment lengths 2 cm, 7 cm, and 4 cm, $2 + 7 > 4$; however, these 3 segments will not form a triangle. Suggest that students really only need to test that the sum of the two smallest lengths is greater than the longest length in order for a triangle to be formed.

Summary

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Talk the Talk: None, One, or Many?

DEMONSTRATE

Facilitation Notes

In this activity, students are given three side lengths and use the Triangle Inequality Theorem to determine if they form a triangle.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

Questions to ask

- When do three side lengths determine a unique triangle?
- When do three side lengths determine no triangles?
- When do two side lengths determine a triangle?
- When do two side lengths determine no triangles?

Summary

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

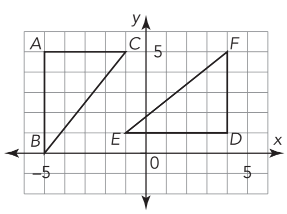
NOTES

Consider Every Side 1

Constructing Triangles Given Sides

WARM UP

Use the coordinate plane to determine each distance. Show your work.



1. What is the distance from point *F* to point *D*?
2. What is the distance from point *A* to point *B*?
3. What is the distance from point *C* to point *A*?
4. What is the distance from point *E* to point *D*?

LEARNING GOALS

- Use patty paper to investigate triangles.
- Construct triangles from three angle measures or side lengths, identifying when the conditions determine a unique triangle, more than one triangle, or no triangle.

KEY TERM

- Triangle Inequality Theorem

You know how to draw a triangle. Can you construct a specific triangle if you are given only two or three possible side lengths? Is there more than one possible triangle that you can construct?

Warm Up Answers

1. 4 units
2. 5 units
3. 4 units
4. 5 units

Answers

1. Sometimes.
   In an equilateral or equiangular triangle, the angles have the same measure, 60°. In other triangles, the angles are not the same.
2. Always.
   The definition of a triangle is a closed figure, or a polygon, with three angles.
3. Sometimes.
   In an isosceles triangle, two sides have the same measure. In an equilateral triangle, all sides have the same measure. In a scalene triangle, none of the sides have the same measure.
4. Sometimes.
   In a right triangle, one angle is a right angle that measures 90°.

Getting Started

Tri-, Tri-, and Tri- Again

Classify each statement as *always* or *sometimes* true about triangles.

- For each *always* true statement, explain your reasoning.
- For each *sometimes* true statement, provide an example and a counterexample.

1. The angles of a triangle have the same measure.

2. A triangle has three angles.

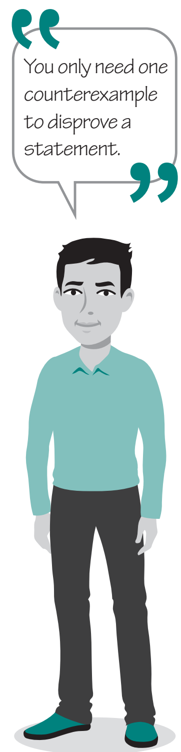
3. Two sides of a triangle have the same measure.

4. One angle of a triangle measures 90 degrees.

ACTIVITY 1.1 Pasta Triangles

Let's investigate the conditions necessary for forming a triangle with different side lengths.

1. Sarah says that when you know 2 segment lengths, you can form many different triangles. She claims she can also use any 3 segment lengths to form a triangle. Sam does not agree. He thinks some combinations will not work. Who is correct? Use a counterexample to disprove the incorrect statement.



Sam then claims that he can just look at the three lengths and know immediately if they will work. Sarah is unsure. She decides to explore this for herself.

Help Sarah by working through the following investigation.

To begin, you will need a piece of strand pasta. Break the pasta at two random points so the strand is divided into three pieces.

- Try to form a triangle from your three pieces of pasta.
- Measure each of your three pieces of pasta in centimeters.
- Repeat the experiment with a new piece of pasta.

2. Record your measurements and the measurements of your group members in the table provided.

Answers

1. Sample answer.
   Sam is correct. The lengths 4 cm, 5 cm, and 12 cm do not determine a triangle.
2. Answers will vary.

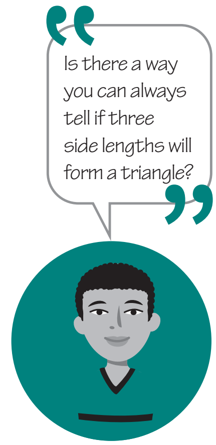
Answers

3. Answers will vary.

3. **Now, add to your table. Collect and record your classmates' measurements.**

Piece 1 (cm)	Piece 2 (cm)	Piece 3 (cm)	Forms a Triangle? (yes or no)

4. **Examine the lengths of the pasta pieces that did form a triangle. Compare them with the lengths of the pasta pieces that did not form a triangle. Make a conjecture about the conditions under which it is possible to form a triangle.**



Answers

4. Conjectures will vary.

ACTIVITY 1.2 A Triangle Given Three Segments

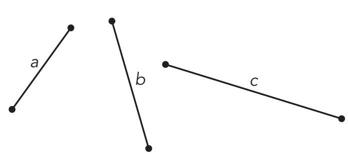
Let's continue to investigate Sarah's question and your conjecture with patty paper.

You and your partner should use different sets of segments for this investigation.

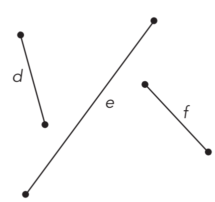
1. **Trace each of the three segments onto its own sheet of patty paper.**

   a. **Overlay the sheets to determine if you can create a triangle. If you can, record the triangle on its own sheet of patty paper.**

Set 1:



Set 2:



Answers

1. See next page.

Answers

1. For Set 1, students should only be able to create 1 triangle. For Set 2, students should not be able to create any triangles.
2. Answers may vary. For any two segments you choose, the sum of their lengths must be greater than the length of the third segment.

b. Now create as many different triangles as you can, using the given segments as sides of a possible triangle. Use a different sheet of patty paper to record each unique triangle.

c. What do you notice? How many different triangles were you able to create?

Triangles are *congruent* when all of their corresponding angle measures and corresponding side lengths are the same.

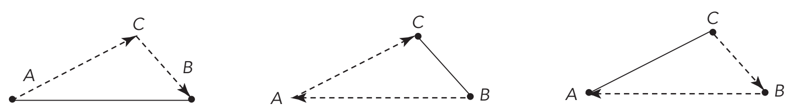
2. Use the patty paper examples from Set 1 and Set 2 to make a conjecture about when three segments can be used to create a triangle. Test your conjecture by creating additional triangles.

ACTIVITY 1.3

Triangle Inequality Theorem

In Euclidean geometry—the geometry of straight lines on flat planes—the shortest distance between two points is a straight line.

- any distance *AC* + *CB* will be greater than the distance *AB*
- any distance *BA* + *AC* will be greater than or equal to the distance *BC*
- any distance *CB* + *BA* will be greater than the distance *AC*



1. How could you use this fact to test whether three line segments can form a triangle? Explain your reasoning.

The measure of $\overline{AB}$ can be expressed in two different ways. *AB* is read as "the distance from point *A* to point *B*." $m\overline{AB}$ is read as "the measure of line segment *AB*."

2. Provide examples of line segments that cannot possibly form a triangle.

Answers

1. For any given line segment, if the sum of the lengths of the other two line segments is greater than the length of the third side, the line segments form a triangle.
2. Answers will vary. Any three segment lengths that have a pair whose sum is less than or equal to the length of the third segment cannot form a triangle.

Answers

3. If $\overline{AC} + \overline{CB} = \overline{AB}$, this would mean that $\overline{AC}$ lies on $\overline{AB}$. These three segments would not form a triangle. The sum of two side lengths must be greater than, not equal to, the length of the third side.

4a. No, the measurements would not form a triangle because the sum of the lengths of the two shorter segments is less than the length of the third segment.

4b. No, the measurements would not form a triangle because the sum of the lengths of the two shorter segments is less than the length of the third segment.

3. What would it mean for the distance *AC* + *CB* to be equal to the distance *AB*? Would these three segments form a triangle?

4. Based on your observations, determine if it is possible to form a triangle using segments with the given measurements. Explain your reasoning.

a. 2 cm, 5.1 cm, 2.4 cm

b. 9.2 cm, 7 cm, 1.9 cm

A theorem is a mathematical rule that can be formally proven.

The rule that you have been using is known as the *Triangle Inequality Theorem*. The **Triangle Inequality Theorem** states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

NOTES

TALK the TALK

None, One, or Many?

Determine if the given information could be used to form a unique triangle, many different triangles, or no triangles. Explain your reasoning.

1. 3 in., 2.9 in., 5 in.

2. 112 mm, 300 mm

3. 5 yd, 10 yd, 21 yd

4. 8 ft, 9 ft, 11 ft

5. 13.8 km, 6.3 km, 7.5 km

Answers

1. This information could be used to create a unique triangle because 3 + 2.9 = 5.9, and 5.9 is greater than 5.
2. This information could be used to form many different triangles because only two segments are given.
3. This information could not be used to create a triangle because 5 + 10 = 15, and 15 is not greater than 21.
4. This information could be used to create a unique triangle because 8 + 9 = 17, and 17 is greater than 11.
5. This information could not be used to create a triangle because 6.3 + 7.5 = 13.8, and 13.8 is not greater than 13.8.