Turning a One-Eighty!

2

Triangle Sum Theorem

MATERIALS

Patty paper
Centimeter ruler

Lesson Overview

Students explore and justify the relationships between angles and sides in a triangle. They establish the Triangle Sum Theorem and use the theorem as they explore the relationship between interior angle measures and the side lengths of triangles. They then practice applying the theorem.

Grade 6
Expressions, Equations, and Relationships

(8) The student applies mathematical process standards to use geometry to represent relationships and solve problems. The student is expected to:

(A) extend previous knowledge of triangles and their properties to include the sum of angles of a triangle, the relationship between the lengths of sides and measures of angles in a triangle, and determining when three lengths form a triangle.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G

Essential Ideas

- The Triangle Sum Theorem states that the sum of the measures of the interior angles of a triangle is 180°.
- The longest side of a triangle lies opposite the largest interior angle.
- The shortest side of a triangle lies opposite the smallest interior angle.

Lesson Structure and Pacing: 1 Day

Engage

Getting Started: Rip 'Em Up

Students draw a large triangle on a piece of paper, rip off the three interior angles of the triangle, and fit them together in a puzzle-like fashion to form a line. This investigation informally justifies the Triangle Sum Theorem, which will be stated at the beginning of Activity 2.1.

Develop

Activity 2.1: Analyzing Angles and Sides

Students use the Triangle Sum Theorem and analyze the relationship between the lengths of the sides of a triangle and the measures of their opposite angles. They determine the unknown angle measure in three triangles and measure the sides of the triangles. Students notice that the measure of an interior angle in a triangle is directly related to the length of the side of the triangle opposite that angle.

Demonstrate

Talk the Talk: So Many Angles!

Students demonstrate their knowledge of the triangle relationships learned in the lesson—the Triangle Sum Theorem. The diagrams are complex and require students to use sides and angles as elements of different triangles.

Getting Started: Rip 'Em Up

ENGAGE

Facilitation Notes

In this activity, students use patty paper to informally justify the Triangle Sum Theorem.

Provide students with patty paper for this activity.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Differentiation strategy

To assist all students,

- Have students shade or label the vertices to make connecting the angles more explicit.
- Have students cut out the triangles from the patty paper so that manipulating the angles is easier.

Questions to ask

- What are adjacent angles?
- Do adjacent angles share a common side?
- Do adjacent angles share a common vertex?
- What kind of triangle did you draw?
- Is your triangle an acute, obtuse, or right triangle?
- Is your triangle scalene, isosceles, or equilateral?
- When positioned adjacent to each other, do the three angles form a straight line?
- What is a straight angle?
- What is the measure of a straight angle?
- How many degrees are associated with a line?
- Is there another way to arrange your three angles? Do you get the same result?
- Is the sum of the three interior angles the same for everyone's triangle?

Summary

The sum of the measures of the three interior angles of a triangle is equal to 180°.

DEVELOP

Activity 2.1
Analyzing Angles and Sides

Facilitation Notes

In this activity, students use the Triangle Sum Theorem to determine the measure of a third interior angle of a triangle given the measure of two interior angles. They also explore the connection between the measure of an interior angle in a triangle and the length of the side of the triangle opposite that angle.

Ask a student to read the introduction and theorem. Discuss as a class.

Provide students with a ruler. Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

Differentiation strategies

To assist all students,

- Place this lesson in context with their prior knowledge of triangles. Review the Triangle Inequality Theorem that deals with the relationship among the lengths of the sides of a triangle. Explain that in this lesson, students will be dealing with the relationship among the angle measures and the relationship connecting angle measures and side measures.
- Explain how to label angles and sides of triangles. Have students label the vertex of each angle with a capital letter and the side opposite of an angle with the corresponding lowercase letter. When students list the angle and side measures, have them list them by label and number. This will assist students in recognizing and explaining the relationship between angles and the sides of a triangle.

Questions to ask

- Is this triangle an acute, obtuse, or right triangle?
- Is this triangle scalene, isosceles, or equilateral?
- How can you determine the unknown angle measure in a triangle?
- How can the Triangle Sum Theorem be helpful in this situation?
- What happens to the length of a side opposite an interior angle of a triangle as the measure of the angle increases?
- What happens to the length of a side opposite an interior angle of a triangle as the measure of the angle decreases?

- What happens to the measure of the angle opposite a side of a triangle as the length of the side increases?
- What happens to the measure of the angle opposite a side of a triangle as the length of the side decreases?
- What side is always the longest side in an obtuse triangle? Explain.
- If two interior angles of a triangle are the same measure, how do the length of the sides opposite these angles compare to each other?
- Which side is always the longest side in a right triangle? Explain.

Have students work with a partner or in a group to complete Questions 5 and 6. Share responses as a class.

Questions to ask

- What is the measure of the third angle of the triangle?
- How does the unknown interior angle measure compare to the measure of the other two interior angles?
- Which side lies opposite the largest interior angle of the triangle?
- Which side lies opposite the smallest interior angle of the triangle?
- How do you determine the longest side of the triangle?
- How do you determine the shortest side of the triangle?
- What situation could result in no one side being the longest side of a triangle?
- What situation could result in no one side being the shortest side of a triangle?
- Why does this relationship between the sides and angles make sense?

Misconception

With regard to Question 6 part (c), students may think that they can compare the sizes of all the angles in the figure without considering the angle measures within each triangle separately. If students consider only the angle measures, they might be unsure where to place g in the order of side lengths.

Summary

The Triangle Sum Theorem states that the sum of the measures of the interior angles of a triangle is 180°. The largest angle of a triangle lies opposite the longest side, and the smallest angle of the triangle lies opposite the shortest side.

DEMONSTRATE

Talk the Talk: So Many Angles!

Facilitation Notes

In this activity, students use the Triangle Sum Theorem and the relationship between lengths of sides and angle measures within a triangle to determine unknown angle measurements.

Have students work with a partner or in a group to complete the question. Share responses as a class.

Differentiation strategies

- To scaffold support,
  - Provide an enlarged diagram.
  - Suggest that students outline/color triangles different colors so they can see the different triangles in each diagram.
  - Review that the sum of the angles around a point is 180°.
- To extend the activity, have students identify two cases where two small triangles can be combined to create one larger triangle. Identify the angle measures for each of the larger triangles and calculate that the sum of the angle measures is 180°.

Questions to ask

- Which angle did you determine first? How did you determine the measure of this angle?
- Which angle did you determine next? How did you determine the measure of this angle?
- Was the Triangle Sum Theorem helpful? Where?

Summary

The Triangle Sum Theorem and the relationship between lengths of sides and angle measures within a triangle can be used to determine unknown angle measurements.

Turning a One-Eighty!

2

Triangle Sum Theorem

WARM UP

For each of the terms below, describe the angle measure and sketch an example.

1. Acute angle
2. Right angle
3. Obtuse angle
4. Straight angle

LEARNING GOALS

- Establish the Triangle Sum Theorem.
- Explore the relationship between the interior angle measures and the side lengths of a triangle.

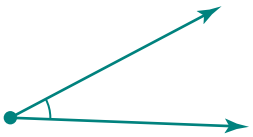
KEY TERM

- Triangle Sum Theorem

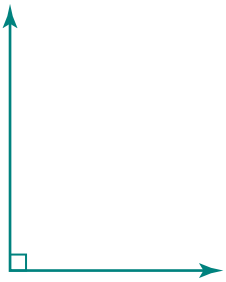
You already know a lot about triangles. What special relationships exist among the interior angles of a triangle and between interior and exterior angles of a triangle?

Warm Up Answers

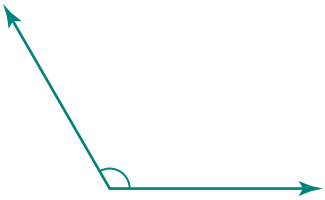
1. An angle with a measure greater than 0° and less than 90°.



2. An angle with a measure of exactly 90°.



3. An angle with a measure greater than 90° and less than 180°.



4. An angle with a measure of exactly 180°.

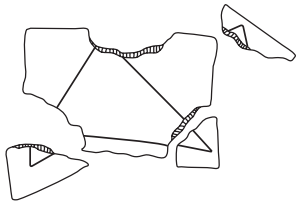
Answers

1. The three angles form a straight line, so the sum of the angles is 180°.
2. Answers may vary.

Getting Started

Rip 'Em Up

Draw any triangle on a piece of patty paper. Tear off the triangle's three angles. Arrange the angles so that they are adjacent angles.



1. What do you notice about these angles? Write a conjecture about the sum of the three angles in a triangle.

2. Compare your angles and your conjecture with your classmates'. What do you notice?

ELL Tip

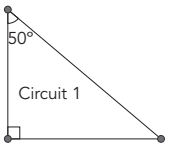
To help English Language Learners as they complete Question 1 of Activity 2.1, a word bank could be provided that shows the different classification of triangles for students to choose. Have the word bank printed on a small sheet of paper that can be easily placed on students' desks.

ACTIVITY
2.1 Analyzing Angles and Sides

In the previous activity, what you noticed about the relationship between the three angles in a triangle is called The *Triangle Sum Theorem*. The **Triangle Sum Theorem** states that the sum of the measures of the interior angles of a triangle is 180°.

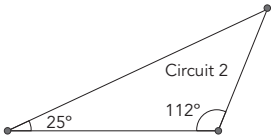
Trevor is organizing a bike race called the Tri-Cities Criterium. Criteriums consist of several laps around a closed circuit. Based on the city map provided to him, Trevor designs three different triangular circuits and presents scale drawings of them to the Tri-Cities Cycling Association for consideration.

1. Classify each circuit according to the type of triangle created.



2. Use the Triangle Sum Theorem to determine the measure of the third angle in each triangular circuit. Label the triangles with the unknown angle measures.

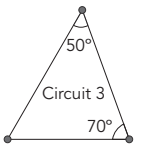
3. Measure the length of each side of each triangular circuit. Label the side lengths in the diagram.



The sharper the angles on a race course, the more difficult the course is for cyclists to navigate.

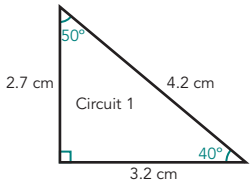
4. Perform the following tasks for each circuit.

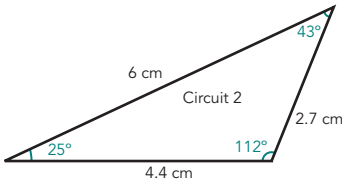
a. List the angle measures from least to greatest.

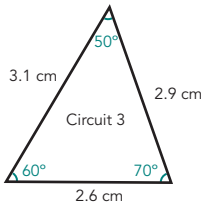


b. List the side lengths from shortest to longest.

Answers

1. Circuit 1 is a right triangle. Circuit 2 is an obtuse triangle. Circuit 3 is an acute triangle.
2. The measure of the third angle in Circuit 1 is 40°. The measure of the third angle in Circuit 2 is 43°. The measure of the third angle in Circuit 3 is 60°.
3. 





4a. Circuit 1: 40°, 50°, 90°
Circuit 2: 25°, 43°, 112°
Circuit 3: 50°, 60°, 70°

4b. Circuit 1:
2.7 cm, 3.2 cm, 4.2 cm
Circuit 2:
2.7 cm, 4.4 cm, 6 cm
Circuit 3:
2.6 cm, 2.9 cm, 3.1 cm

ELL Tip

Have students make a flash card for the definition of the Triangle Sum Theorem. On one side of the card have students write "Triangle Sum Theorem," and on the opposite side have students write the theorem. Encourage students to write the definition in their own words, using pictures or symbols.

Answers

4c. The shortest side is always across from the angle with the least measure of the triangle.

4d. The longest side is always across from the angle with the greatest measure of the triangle.

5a. The third angle has a measure of 62°.

5b. The longest side of the triangle lies opposite the 62° angle.

5c. The shortest side of the triangle lies opposite the 57° angle.

c. Describe what you notice about the location of the angle with the least measure and the location of the shortest side.

d. Describe what you notice about the location of the angle with the greatest measure and the location of the longest side.

5. Traci, the president of the Tri-Cities Cycling Association, presents a fourth circuit for consideration. The measures of two of the interior angles of the triangle are 57° and 61°. Determine the measure of the third angle, then describe the location of each side with respect to the measures of the opposite interior angles without drawing or measuring any part of the triangle.

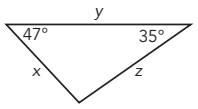
a. measure of the third angle

b. longest side of the triangle

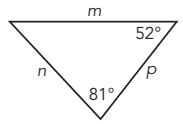
c. shortest side of the triangle

6. List the side lengths from shortest to longest for each diagram.

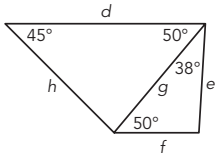
a.



b.



c.



Answers

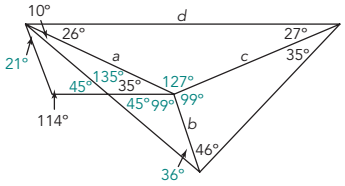
6a. *x, z, y*

6b. *p, n, m*

6c. *f, e, g, h, d*

Answers

1a.



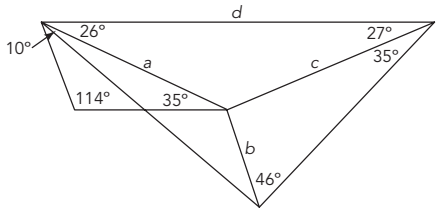
1b. *b, c, a, d*

NOTES

TALK the TALK

So Many Angles!

1. Consider the diagram shown.



a. Determine the measures of the eight unknown angle measures inside the figure.

b. List the labeled side lengths in order from least to greatest.