Angles and Shapes
Topic 3 Overview

How is *Angles and Shapes* organized?

This topic begins with students determining if three given line segments will construct a unique triangle or no triangle, thus developing the Triangle Inequality Theorem. Students then use hands-on tools to make and justify conjectures about the sum of the interior angles of a triangle and the relationship between triangle side and angle measures. Next, students review their understanding of area as additive, of composing and decomposing rectangles, and the formula for the area of a rectangle. From their knowledge of rectangles and area, students develop the formula for the areas of parallelograms, triangles, trapezoids, and use these formulas to determine the area of composite figures.

What is the entry point for students?

Students begin the topic building off their previous knowledge of triangles by exploring facts about triangles. They determine if any three given line segments will construct a triangle. They draw a triangle, rip it apart, and fit the angles together, noting that the angles form a line; therefore, the sum of the interior angles of a triangle is 180°. Students then use tools, including the Triangle Sum Theorem and rulers, to determine the relationship between the lengths of the sides of a triangle and the measures of the angles.

Students enter grade 6 with a conceptual understanding of area and fluency in computing the perimeter and area of rectangles. This topic goes beyond memorizing area formulas into developing area formulas for new shapes based on known area formulas. Students will decompose composite figures into familiar shapes that they know area formulas for, such as squares, rectangles, triangles, parallelograms and trapezoids. They determine the area of these familiar shapes in order to determine the total area of the composite figures.

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in this topic if they can:

- Determine if three given line segments will construct a triangle.
- Informally prove that the sum of the interior angles of a triangle is 180°.
- Determine the relationship between the lengths of the sides of a triangle and the measures of the angles.
- Recognize and know how to compose and decompose shapes into triangles and rectangles.

- Apply the techniques of composing and/or decomposing to determine the area of triangles, special quadrilaterals, and composite figures to solve mathematical and real-world problems.
- Discuss, develop, and justify formulas for triangles and parallelograms.

Why is *Angles and Shapes* important?

In this topic, students establish important triangle relationships through reasoning, logic, investigating, and testing conjectures. Students informally derive the area formulas of parallelograms, triangles, and trapezoids from their previous knowledge of the area of rectangles by decomposing and composing shapes. Area will continue to be used in problem-solving throughout this course. Students will calculate area of composite shapes, then connect that understanding to calculating the surface area of solids using their two-dimensional nets.

Throughout this topic, students are expected to follow lines of logic to reach conclusions, which is foundational for formal proof in high school. The geometric results established in the topic via informal arguments will be formally proven in high school, but their experiences in this topic provide them with opportunities to build intuition and justify results.

How do the activities in *Angles and Shapes* promote student expertise in the mathematical process standards?

All Carnegie Learning topics are written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work toward solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

Precision, the choice of appropriate tools, and the use of structure are highlighted in this topic. Students use objects along with mathematical and construction tools to determine relationships in triangles, derive area formulas for parallelograms, triangles, and trapezoids from the area formula of a rectangle, and calculate the area of composite figures. Students are encouraged to look for structure in shapes so they may compose and decompose shapes to make connections, develop, or solidify new area formulas.

Materials Needed

- Patty paper
- Protractor
- Ruler
- Compass
- Straightedge
- Scissors
- Glue sticks or tape
- Pieces of raw pasta (spaghetti or linguine)

Learning Together

ELPS: 1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G

Lesson	Lesson Name	TEKS	Days	Highlights
1	Consider Every Side: Constructing Triangles Given Sides	6.8A	2	Students use patty paper, pasta, and construction tools to explore the information required to create no triangles, unique triangles, or multiple triangles when given two or three possible side lengths. They learn that an infinite number of triangles can be made from only two side lengths. They also learn that unique triangles are formed when provided with three segments that are sufficiently long in relation to each other. Students should note that if all the measures of a triangle are the same as another triangle, even though they are in different orientations, the provided information creates a unique triangle. Students then summarize their knowledge of the conditions that form 0, 1, or multiple triangles.
2	Turning a One-Eighty!: Triangle Sum Theorem	6.8A	1	Students explore and justify the relationships between angles and sides in a triangle. They establish the Triangle Sum Theorem and use the theorem as they explore the relationship between interior angle measures and the side lengths of triangles. They then practice applying the theorem.
3	All About That Base… and Height: Area of Triangles and Quadrilaterals	6.8B 6.8C 6.8D	2	Students use previously known area formulas and the principle of area conservation to investigate the areas of parallelograms, triangles, and trapezoids. They use this knowledge to develop formulas for the areas of these shapes, practice calculating areas, and solving area-related problems. Students learn that the choice of base or height does not affect the area of the shape.
4	Slicing and Dicing: Composite Figures	6.8D 7.9C	2	In this lesson, students calculate the area of complex figures. They compare two methods: decomposing a figure into familiar shapes and composing a figure into a rectangle. Students then solve problems in context, including the area of countries, using map scales to approximate areas. They use given dimensions and problem solving to calculate the area of a triangle embedded in a square.

Suggested Topic Plan

*1 Day Pacing = 45 min. Session

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: 6.8A	**LESSON 1** continued	TEKS: 6.8A	TEKS: 6.8B, 6.8C, 6.8D	**LESSON 3** continued
LESSON 1	ACTIVITY 2	**LESSON 2**	**LESSON 3**	ACTIVITY 3
Consider Every Side	ACTIVITY 3	**Turning a One-Eighty!**	**All About That Base...and Height**	ACTIVITY 4
GETTING STARTED	TALK THE TALK	GETTING STARTED	GETTING STARTED	TALK THE TALK
ACTIVITY 1		ACTIVITY 1	ACTIVITY 1	
		ACTIVITY 2	ACTIVITY 2	
		TALK THE TALK		

Day 6	Day 7	Day 8
TEKS: 6.8D, 7.9C	**LESSON 4** continued	**END OF TOPIC ASSESSMENT**
LESSON 4	ACTIVITY 2	
Slicing and Dicing	TALK THE TALK	
GETTING STARTED		
ACTIVITY 1		

Assessments

There is one assessment aligned to this topic: End of Topic Assessment.