1

Depth, Width, and Length

Deepening Understanding of Volume

WARM UP

Determine the least common multiple of the numbers in each pair.

1. 2, 10
2. 3, 8
3. 6, 14
4. 10, 15

LEARNING GOALS

- Determine the volume of right rectangular prisms with fractional edge lengths using unit cubes with unit fractional dimensions.
- Connect the volume formulas $V = lwh$ and $V = Bh$ with a unit-cube model of volume for rectangular prisms.
- Apply the formulas $V = lwh$ and $V = Bh$ to determine volumes in real-world problems.
- Fluently add, subtract, and multiply multi-digit decimals using the standard algorithms.

KEY TERMS

- point
- line segment
- polygon
- geometric solid
- polyhedron
- face
- edge
- vertex
- right rectangular prism
- cube
- pyramid
- volume

You know about three-dimensional figures such as cubes and other rectangular prisms. You also know how to operate with positive rational numbers. How can you use what you know to calculate measurements of any rectangular prism, even one with fractional edge lengths?

Getting Started

Common Figures

Cut out the cards found at the end of the lesson. Sort the figures into two or more groups. Name each category and be prepared to share your reasoning.

ACTIVITY 1.1

Name that Figure

It is important to speak a common language when studying mathematics.

A word you may have used in the past may actually have a more precise definition when dealing with mathematics. For example, the word *point* has many meanings outside of math. However, the mathematical definition of **point** is a location in space. A mathematical point has no size or shape, but it is often represented by using a dot and is named by a capital letter. A **line segment** is a portion of a line that includes two points and all the points between those two points. Knowing these definitions will help you learn the meanings of other geometric words.

Recall, a **polygon** is a closed figure formed by three or more line segments.

A **geometric solid** is a bounded three-dimensional geometric figure.

A **polyhedron** is a three-dimensional solid figure that is made up of polygons. A **face** is one of the polygons that makes up a polyhedron. An **edge** is the intersection of two faces of a three-dimensional figure. The point where multiple edges meet is known as a **vertex** of a three-dimensional figure.

Let's revisit the different figures you sorted.

1. **Sort the figures into one of these three categories and explain your reasoning.**

Polygon	**Polyhedron**	**Neither**

Three polyhedra are shown.

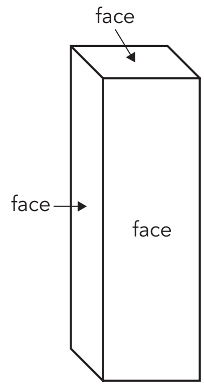


Figure A

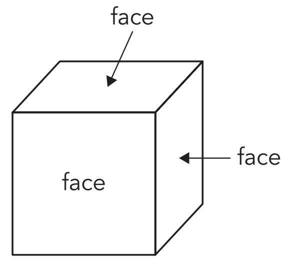


Figure B

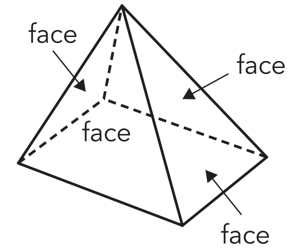


Figure C

A unit cube is a cube whose sides are all 1 unit long.

Figure A is a *right rectangular prism*. A **right rectangular prism** is a polyhedron with three pairs of congruent and parallel rectangular faces.

Figure B is an example of a *cube*, which is a special kind of right rectangular prism. A **cube** is a polyhedron that has congruent squares as faces.

Figure C is an example of a *rectangular pyramid*. A **pyramid** is a polyhedron with one base and the same number of triangular faces as there are sides of the base.

2. Describe the different faces of each polyhedron.

3. Study the right rectangular prism. Identify the three pairs of congruent parallel faces.

4. Study the cube.

a. Describe the locations of the cube faces you can see and the locations of the faces you cannot see.

b. What do you know about the length, width, and height of the cube?

c. Describe how the cube is also an example of a right rectangular prism.

NOTES

5. **Compare the numbers of faces, edges, and vertices of the cube and the other right rectangular prism. Write what you notice.**

6. **Study the rectangular pyramid. How do the faces of the rectangular pyramid differ from the faces of the rectangular prisms?**

7. **List examples in the real-world objects that are shaped like right rectangular prisms or pyramids.**

ACTIVITY 1.2 Volume of Rectangular Prisms

Volume is the amount of space occupied by an object. The volume of an object is measured in cubic units.

The volume of a cube is calculated by multiplying the length times the width times the height.

$$\text{Volume of a cube} = l \times w \times h$$

1. **Calculate the volume of each cube with the given side length.**

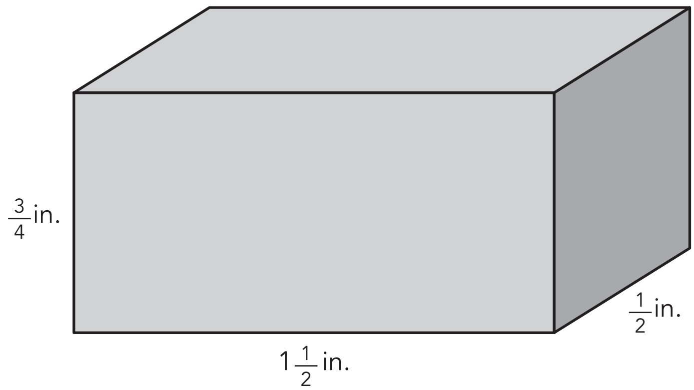
   a. $\frac{9}{10}$ **centimeter**

   b. $1\frac{1}{3}$ **centimeters**

2. **Suppose a cube has a volume of 27 cubic meters. What are the dimensions of the cube?**

To determine the volume of a rectangular prism, you can also pack the prism with cubes. You may have done this in elementary school.

Consider the rectangular prism shown. What do you notice about the side lengths? Can you determine its volume by packing it with cubes?



WORKED EXAMPLE

To determine the volume of the right rectangular prism with dimensions $1\frac{1}{2} \times \frac{1}{2} \times \frac{3}{4}$, you can fill the prism with cubes. However, the unit cubes that you may have used in elementary school will not work here. Instead, smaller unit cubes with fractional side lengths are required.

Assign a unit fraction to the dimensions of each cube. Use the least common multiple (LCM) of the fraction denominators to determine the unit fraction.	$\text{LCM}(2, 4) = 4$ So, each cube will measure $\frac{1}{4}$ in. $\times \frac{1}{4}$ in. $\times \frac{1}{4}$ in. The volume of each unit cube is $\frac{1}{64}$ cubic inches.
Determine the number of cubes needed to pack the prism in each dimension.	length: $1\frac{1}{2} \div \frac{1}{4} = 6$ width: $\frac{1}{2} \div \frac{1}{4} = 2$ height: $\frac{3}{4} \div \frac{1}{4} = 3$
Determine the number of cubes that make up the right rectangular prism.	$6 \times 2 \times 3 = 36$
Multiply the number of cubes by the volume of each cube to determine the volume of the right rectangular prism.	$36 \times \frac{1}{64} = \frac{36}{64}$ $= \frac{9}{16}$

The volume of the right rectangular prism is $\frac{9}{16}$ cubic inches.

NOTES

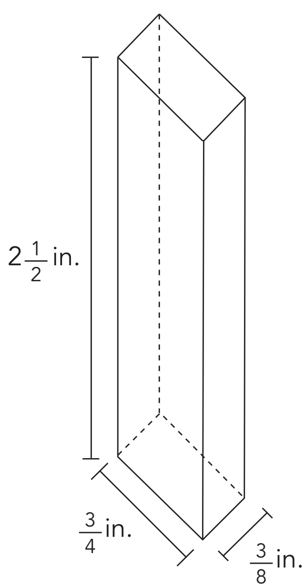
3. Interpret the Worked Example.

   a. How was the number of cubes needed to pack the prism in each dimension determined?

   b. Instead of cubes with a width of $\frac{1}{4}$ inch, suppose you used cubes each with a width of $\frac{1}{8}$ inch. How does this change the volume of the rectangular prism?

4. Use the method from the worked example to determine the volume of each rectangular prism.

   a.



   b. $1\frac{3}{4}$ in. by $2\frac{1}{3}$ in. by $\frac{1}{2}$ in.

ACTIVITY 1.3

Volume Formulas

You have calculated the volume of a rectangular prism using the formula $V = lwh$, where V is the volume, l is the length, w is the width, and h is the height. You also know that the area of a rectangle can be calculated using the formula $A = l \cdot w$.

Consider the two formulas:

$$V = l \cdot w \cdot h$$
$$A = l \cdot w$$

If B is used to represent the area of the base of a rectangular prism, then you can rewrite the formula for area: $B = l \cdot w$.

Now consider the two formulas:

$$V = l \cdot w \cdot h$$
$$B = l \cdot w$$

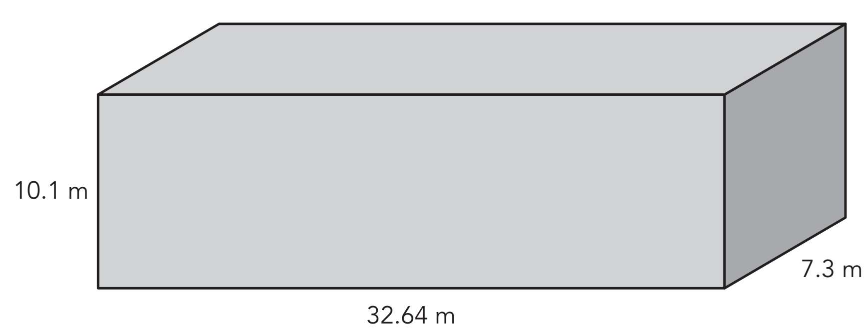
You can use the formula $V = Bh$ to calculate the volume of any prism. However, the formula for calculating the value of B will change depending on the shape of the base.

Using both of these formulas, you can rewrite the formula for the volume of a rectangular prism as $V = B \cdot h$, where V represents the volume, B represents the area of the base, and h represents the height.

In order to calculate the volume of various geometric solids you will need to perform multiplication. In this activity, you will calculate the volume of rectangular prisms with decimal side lengths.

It is good practice to estimate before you actually calculate. If you have an estimate, you can use it to decide whether your answer is correct.

Consider the right rectangular prism shown.



To calculate the volume of the prism, first calculate the area of the base, *B*, by multiplying 32.64 meters by 7.3 meters.

Kenny said, "I use estimation to help place the decimal point correctly in the product."

WORKED EXAMPLE

The area of the base is 32.64 meters × 7.3 meters.

He estimates his two numbers.

32.64 is close to 30
7.3 is close to 7
$30 \times 7 = 210$

So he knows his product is close to 210, but larger since he rounded down. Next, he calculates the product of 32.64 × 7.3.

$$
\begin{array}{r}
32.64 \\
\times \quad 7.3 \\
\hline
9792 \\
228480 \\
\hline
238.272
\end{array}
$$

Kenny knows the product will be close to but greater than 210, so he must place the decimal point after the 8. The area of the base of the rectangular prism is 238.272 square meters.

1. Calculate the volume of the right rectangular prism.

2. Each number sentence represents the base, *B*, times height, *h*, of different rectangular prisms. Complete each number sentence by inserting a decimal point to show the correct volume.

   a. 53.6 sq. ft × 0.83 ft = 44488 cu. ft

   b. 7.9 sq. cm × 0.6 cm = 474 cu. cm

   c. 0.94 sq. m × 24.9 m = 23406 cu. m

3. Casey thought that using a pattern would help her understand how to calculate the product in a decimal multiplication problem.

   a. Complete the table.

Problem	Product	Problem	Product	Problem	Product
32 × 100		3.2 × 100		0.32 × 100	
32 × 10		3.2 × 10		0.32 × 10	
32 × 1		3.2 × 1		0.32 × 1	
32 × 0.1		3.2 × 0.1		0.32 × 0.1	
32 × 0.01		3.2 × 0.01		0.32 × 0.01	
32 × 0.001		3.2 × 0.001		0.32 × 0.001	

   b. Describe any patterns that you notice.

4. A rectangular prism with $B = 26$ square centimeters and $h = 31$ centimeters has a volume of 806 cubic centimeters. Use this information to determine the volume of the other rectangular prisms.

a. 2.6 sq. cm × 31 cm

b. 2.6 sq. cm × 3.1 cm

c. 0.26 sq. cm × 3.1 cm

d. 2.6 sq. cm × 0.31 cm

e. 0.26 sq. cm × 31 cm

f. 2.6 sq. cm × 0.031 cm

g. 0.026 sq. cm × 0.31 cm

h. 0.26 sq. cm × 0.31 cm

5. Look at the patterns in Question 4.

a. How can some of the rectangular prisms have the same volume?

b. How can you tell without multiplying which rectangular prisms will have the same volume?

NOTES

TALK the TALK

Fractionally Full

1. Determine the volume of a right rectangular prism with dimensions $1\frac{1}{4}$ feet × 1 foot × $\frac{1}{2}$ foot using the unit fraction method you learned in this lesson.

2. Haley makes earrings and packages them into cube boxes that measure $\frac{1}{6}$-foot wide. How many $\frac{1}{6}$-foot cubic boxes can she fit into a shipping box that is $1\frac{1}{6}$ feet by $\frac{1}{3}$ foot by $\frac{1}{3}$ foot?

3. The school athletic director has a storage closet that is $4\frac{1}{2}$ feet long, $2\frac{2}{3}$ feet deep, and 6 feet tall.

   a. She wants to put carpet in the closet. How much carpeting will she need?

   b. The athletic director wants to store cube boxes that are $\frac{1}{2}$ foot wide. How many boxes will the storage closet hold?

4. Estimate the volume of each right rectangular prism. Then calculate its volume.

   a.

   b.

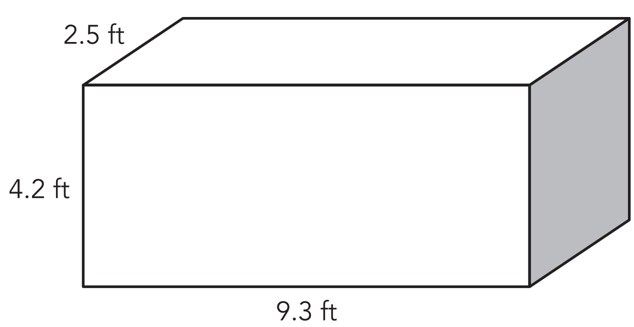


Figure 1

Figure 2

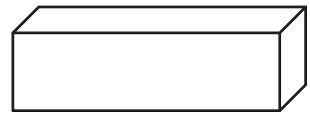


Figure 3

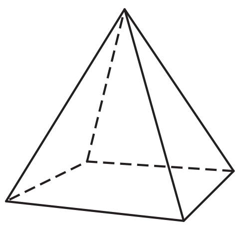


Figure 4

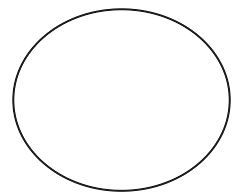


Figure 5

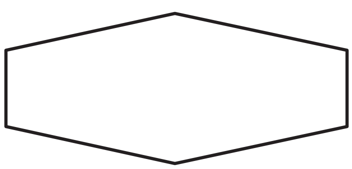


Figure 6

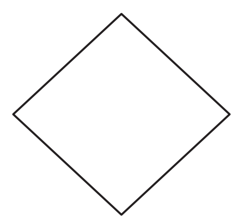


Figure 7

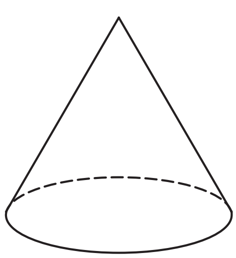


Figure 8