Which Warehouse?

2

Volume Composition and Decomposition

MATERIALS

None

Lesson Overview

A scenario about building a bench is provided. Students review estimating sums and differences of decimals and how to add and subtract decimals by adding or subtracting the digits in like place values. They then determine the volume of the bench, a composite solid, using decomposition into smaller rectangular prisms and composition into a larger rectangular prism. The two different strategies require either addition or subtraction of decimals. Students practice solving problems requiring addition and subtraction of decimal volumes.

Grade 6

Number and Operations

(3) The student applies mathematical process standards to represent addition, subtraction, multiplication, and division while solving problems and justifying solutions. The student is expected to:

(E) multiply and divide positive rational numbers fluently.

Expressions, Equations, and Relationships

(8) The student applies mathematical process standards to use geometry to represent relationships and solve problems. The student is expected to:

(D) determine solutions for problems involving the area of rectangles, parallelograms, trapezoids, and triangles and volume of right rectangular prisms where dimensions are positive rational numbers.

ELPS

1.A, 1.C, 1.D, 1.E, 1.F, 1.H, 2.C, 2.D, 2.G, 2.H, 2.I 3.A, 3.B, 3.C, 3.D, 3.E, 3.F, 4.A, 4.B, 4.C, 4.G, 4.K, 5.B, 5.C, 5.D, 5.E, 5.F, 5.G

Essential Ideas

- When adding or subtracting decimals, the decimal points must be lined up to ensure like place values are written in the same columns and combined appropriately.
- A rectangular prism is a prism that has rectangles as its bases.
- A composite solid is made up of more than one geometric solid.
- The formula for the volume of a cube is $V = lwh$, where l is the length, w is the width and h is the height, or $V = Bh$, where B is the area of the base and h is the height.
- The volume of composite solids is found by adding or subtracting volumes of common solids.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Measuring Water

Students determine how to measure exactly 4 gallons (924 cubic inches) of water using just two containers: one with a volume of 3 gallons (693 cubic inches) and one with a volume of 5 gallons (1155 cubic inches). This activity is designed to engage students in thinking about how volumes can be added and subtracted.

Develop

Activity 2.1: Adding and Subtracting Volumes

Students consider how to determine the volume of a composite solid with decimal side lengths using rectangular prisms. They review the algorithms for adding and subtracting decimals. Students then determine the volume of the original composite solid using addition of volumes and subtraction of volumes.

Day 2

Activity 2.2: Fluency with Decimal Operations

Students practice determining the volume of composite solids with decimal side lengths through composition and decomposition of rectangular prisms. They use multiple strategies for determining the volume of the same solid. Students solve volume problems that are more efficiently solved through either composition or decomposition, requiring the use of either addition or subtraction of decimals.

Demonstrate

Talk the Talk: The Volume Warehouse

Students are provided with floor plans and price per cubic foot for two different warehouse spaces. They must determine the total monthly costs of the two warehouses and decide which warehouse space to recommend. Floor plans and the price per cubic foot include decimal numbers.

ENGAGE

Getting Started: Measuring Water

Facilitation Notes

In this activity, students determine how to measure exactly 4 gallons (924 cubic inches) of water using just two containers: one with a volume of 3 gallons (693 cubic inches) and one with a volume of 5 gallons (1155 cubic inches). This activity is designed to engage students in thinking about how volumes can be added and subtracted.

Ask a student to read the scenario aloud and discuss as a class. Allow students to work in pairs or groups to problem-solve a solution. Circulate and ask students leading questions when needed. Have students share responses.

Questions to ask

- Which bucket did you fill first?
- Did you dump the contents of one bucket into the other bucket? Which one?
- If the 3-gallon bucket of water is emptied into the 5-gallon bucket, what additional information does this provide?
- If the 5-gallon bucket of water is emptied into the 3-gallon bucket, what additional information does this provide?
- What operation is modeled when one bucket of water is emptied into a second bucket of water?
- If the 5-gallon bucket of water is used to fill the 3-gallon bucket, how much water remains?
- How can a measurement of 2 gallons of water be helpful in this scenario?
- Why does it make sense to pour 2 gallons of water into an empty 3-gallon bucket?

Differentiation strategies

- To scaffold support, provide a template with a series of pairs of buckets to assist students in recording their steps.
- To extend the activity, provide an additional problem: You have a 4- gallon bucket and a 7-gallon bucket, and you want 5 gallons of water.

Summary

Subtraction can be used to problem-solve a real-world situation involving the volume of solids.

Activity 2.1
Adding and Subtracting Volumes

DEVELOP

Facilitation Notes

In this activity, students consider how to determine the volume of a composite solid with decimal side lengths using rectangular prisms. They review the algorithms for adding and subtracting decimals. Students then determine the volume of the original composite solid using addition of volumes and subtraction of volumes.

Ask a student to read the introduction aloud. Discuss the situation and answer Question 1 as a class. Ask different students to read each of the two Worked Examples aloud and discuss answers to Questions 2 through 5 as a class.

Differentiation strategies

- Be aware that some students may use a subtraction algorithm different than the one demonstrated here.
- To scaffold support for students, provide large grid graph paper or have them turn composition paper horizontally to help them keep columns lined up.

Questions to ask

- Can a decimal written to the nearest tenth be added to or subtracted from a decimal written to the nearest hundredth?
- Can a decimal written to the nearest hundredth be added to or subtracted from a decimal written to the nearest tenth?
- Can any two decimal values be added to or subtracted from each other?
- Can a number written in decimal form be added to or subtracted from a whole number?
- Can a whole number be subtracted from or added to a number written in decimal form?

In pairs or groups, allow students time to answer Questions 6 through 9 and share responses.

Differentiation strategy

To assist all students, have them build a model of the bench using cubic blocks. The model may help students visualize, solve, and explain the solution to this problem.

Questions to ask about Sofia's method

- What is the length of each cube that composes the bench?
- How can the bench be decomposed into rectangular prisms?
- What are the dimensions of each rectangular prism?

- Is there another way to decompose the bench?
- Will the different methods of decomposition affect the volume of the bench?
- What operation is used to solve this problem situation?

Questions to ask about Hunter's method

- What is the volume of the entire cube that forms the outline of the bench?
- What are the dimensions for the portion of the cube that makes up the seat of the bench? What is the volume of this portion of the cube?
- How does Hunter's total volume compare to Sophia's total volume?
- Whose method was easier to use? Why?
- Is there another alternate method for solving this problem situation?

Summary

The addition and subtraction of decimals are used in strategies to problem-solve a real-world situation involving the volume of right rectangular prisms.

Activity 2.2
Fluency with Decimal Operations

Facilitation Notes

In this activity, students practice determining the volume of composite solids with decimal side lengths through composition and decomposition of rectangular prisms. They use multiple strategies for determining the volume of the same solid. Students solve volume problems that are more efficiently solved through either composition or decomposition, requiring the use of either addition or subtraction of decimals.

Students work in pairs or groups to complete Questions 1 through 3. Circulate while students are working and note different solution methods for presentation purposes. Allow time as a class for students to share strategies used to solve each problem.

Questions to ask

- Are you using composition or decomposition to calculate the volume?
- Can the volume be determined using a different strategy?

- What values were used to determine the volume of this portion?
- How did you determine the placement of the decimal in the product?
- Which portions of the solid need to be subtracted?
- What operations did you use to calculate the volume?
- What do the shaded regions represent in this diagram?
- Did you use estimation at any point to check your calculations?

Differentiation strategies

Modify the assignment by decreasing the workload while also requiring all students to deal with both addition and subtraction of volume. Have students complete one of the following options:

- Questions 1 and 3b
- Questions 3a and 3b
- Question 2

Summary

The addition and subtraction of decimals algorithms in composition and decomposition strategies are used to solve for the volume of composite solids.

Talk the Talk: The Volume Warehouse

DEMONSTRATE

Facilitation Notes

In this activity, students are provided with floor plans and price per cubic foot for two different warehouse spaces. They must determine the total monthly costs of the two warehouses and decide which warehouse space to recommend. Floor plans and the price per cubic foot include decimal numbers.

Allow students to work in pairs or groups to solve this problem situation. Share different solution strategies as a class.

Questions to ask

- Are you using a composition or decomposition strategy for Warehouse A?
- Are you using a composition or decomposition strategy for Warehouse B?
- What values were used to determine the volume of this portion?
- How did you determine the placement of the decimal in the product?
- Which portions of the solid need to be subtracted or added?
- What operations did you use to calculate the volume?

- Did you use estimation?
- What are the key points in your report?
- Is the least expensive option always the best option?

Summary

A real world scenario uses composition and decomposition strategies, involving decimal dimensions, to solve for the costs associated with rental space. Findings are written in a report recommending one of the two possible locations.

2 Which Warehouse?

Volume Composition and Decomposition

WARM UP

Calculate each product.

1. 0.5×0.5
2. 0.1×0.9
3. 0.3×0.9
4. 0.8×0.7
5. 0.7×0.7
6. 0.4×0.4
7. 0.6×0.7
8. 0.6×0.8
9. 0.3×0.2
10. 0.2×0.8

LEARNING GOALS

- Fluently add, subtract, and multiply multi-digit decimals using the standard algorithms.
- Determine volumes of figures composed of rectangular prisms.

KEY TERMS

- composite solid
- trailing zeros

You have calculated areas by composing or decomposing complex shapes into familiar shapes. How can you used this same idea to determine the volume of composite solids?

Warm Up Answers

1. 0.25
2. 0.09
3. 0.27
4. 0.56
5. 0.49
6. 0.16
7. 0.42
8. 0.48
9. 0.06
10. 0.16

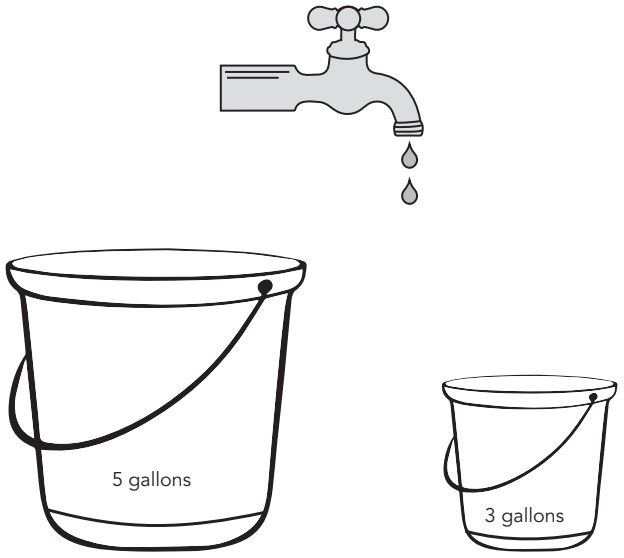
Answers

1. Fill up the first container with a volume of 5 gallons (1155 cubic in.) and pour that water into the second container with a volume of 3 gallons (693 cubic in.) to fill it up. This leaves 2 gallons (462 cubic inches) of water in the first container. Empty the second container. Pour the 2 gallons (462 cubic inches) of water from the first container into the second container. The second container is now 2 gallons (462 cubic inches) full and 1 gallon (231 cubic inches) empty. Fill up the first container and pour it into the second container until the second container is full. This leaves $5 - 1 = 4$ gallons ($1155 - 231 = 924$ cubic inches) of water in the first container.

Getting Started

Measuring Water

You have two empty containers, each with a different volume, as shown. You also have a source of water.



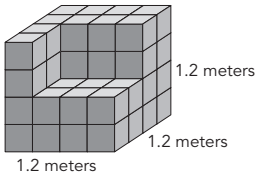
(1155 in.^{3}) (693 in.^{3})

1. Using just these containers, how can you measure out a volume of exactly 4 gallons (924 in.^{3})?

ACTIVITY
2.1 Adding and Subtracting Volumes

As part of Let's Build Together, an organization that builds recreation centers for communities in need, your class is building a concrete bench for use in a community garden.

Your class has been provided with a drawing of your assignment. You need to determine how much concrete is needed to construct the bench.



The bench is a *composite solid*. A **composite solid** is made up of more than one geometric solid.

1. How might you determine the amount of concrete needed to construct your group's bench? What information do you need to know?

Sofia and Hunter propose different strategies for determining the volume of the bench. Sofia's strategy requires the addition of volumes and Hunter's strategy requires the subtraction of volumes. Because of the decimal side lengths of this bench, let's start by reviewing how to add and subtract with decimals.

Answers

1. Answers will vary.

Answers

2a. It helps him line up place values.

2b. 12.850

3. To add decimals, line up the decimal points so that the digits with the same place value are in the same column, adding trailing zeros if it is helpful. Then add the numbers in each column as usual, regrouping as necessary. Place a decimal point in the answer below the decimal points of the addends.

Let's consider adding decimals.

WORKED EXAMPLE

$3.421 + 9.5 + 12.85 = ?$

Estimating first helps you check your answers. You know what answer to expect.

Before calculating the sum, estimate the answer so you know the approximate sum.

$$3 + 10 + 13 = 26$$

To calculate the exact sum, line up the decimals so that like place values are in the same column. You can use the decimal point as a reference point to help you align numbers in the correct place-value column.

$$\begin{array}{r} 3.421 \\ 9.5 \\ +\ 12.85 \\ \hline 25.771 \end{array}$$

The estimate of 26 and the sum of 25.771 are reasonably close, so the sum appears to be correct.

Lijo added *trailing zeros* to his decimal numbers. **Trailing zeros** are a sequence of 0s in a decimal representation of a number, after which no non-zero digits follow. Trailing zeros do not affect the value of a number.

2. Lijo says that he can write 9.5 as 9.500 to help calculate the sum 3.421 + 9.5 + 12.85.

a. How does this help Lijo calculate the sum?

b. How might Lijo rewrite 12.85 in this sum?

3. Summarize how to add decimals.

ELL Tip

Help students understand the difference between estimating and rounding. Estimating gives us a rough idea of what an answer should be, so the answer does not have to be precise. Unlike estimating, rounding involves adjusting a number to a given place value.

You can use a similar algorithm for subtracting decimals. Let's consider two different subtraction problems.

WORKED EXAMPLE

	18.205 − 3.91	22.4 − 8.936
First, estimate the answer so you know the approximate difference.	18 − 4 = 14	22 − 9 = 13
Then, line up the decimals so that like place values are in the same column and subtract.	$\begin{array}{r} {\scriptstyle 7\ 11\ 10} \\ 1\,\not{8}.\not{2}\,\not{0}5 \\ -\,3.9\,1\,0 \\ \hline 1\,4.2\,9\,5 \end{array}$	$\begin{array}{r} {\scriptstyle 1\ 11\ 13\ 9\ 10} \\ \not{2}\,\not{2}.\not{4}\,\not{0}\,\not{0} \\ -\,8.9\,3\,6 \\ \hline 1\,3.4\,6\,4 \end{array}$
Compare the answer to your estimate to check your work.	The estimate of 14 and the difference of 14.295 are reasonably close, so the difference appears to be correct.	The estimate of 13 and the difference of 13.464 are reasonably close, so the difference appears to be correct.

4. Analyze both subtraction problems.

a. What do the subtraction problems have in common?

b. What is different about the subtraction problems in the Worked Example?

5. Summarize how to subtract decimals.

ELL Tip

The word "algorithm" appears in this sentence. Explain to students that an algorithm is an organized step-by-step process for solving a problem. The term comes from the name of the Persian mathematician Al-Khwárizmí and his mathematical treatise written in the 800s. Telling students this will help them connect the term to a story, particularly one that is not of English origin, which will in turn help them to remember the word and its meaning.

Answers

4a. In both Worked Examples, the decimal points were lined up so that the digits with the same place value were in the same column. Trailing zeros were added as place holders to the numbers in both Worked Examples. Both Worked Examples required regrouping to subtract.

4b. In In the first subtraction problem in the Worked Example, the trailing zero was added to the subtrahend. In the second subtraction problem in the Worked Example, the trailing zeros were added to the minuend.

5. To subtract decimals, line up the decimal points so that the digits with the same place value are in the same column. Add trailing zeros to the minuend or to the subtrahend as needed. Then subtract the numbers in each column, regrouping as necessary. Place a decimal point in the answer below the decimal points of the minuend and subtrahend.

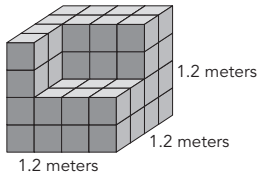
Answers

6. Answers will vary. The width of each of the cubes in the bench is 0.3 meter.

   One possibility is to create 3 rectangular prisms: (1) on the left with dimensions 0.3 meter by 1.2 meters by 1.2 meters, (2) on the bottom front with dimensions 0.6 meter by 0.9 meter by 0.6 meter, and (3) in the back with dimensions 0.6 meter by 0.9 meter by 1.2 meters. Then find the volume of each rectangular prism and add the three volumes. The total volume is 1.404 cubic meters.

7. The volume of the 1.2-meter cube is 1.728 cubic meters. The portion of the cube that forms the seat of the bench is 0.9 meters by 0.6 meters by 0.6 meters and has a volume of 0.324 cubic meters. Subtract the volume of the missing portion from the volume of the whole cube: 1.728 cubic meters − 0.324 cubic meters = 1.404 cubic meters

Let's go back to determining the amount of concrete needed for your group's bench.



6. Sofia proposes that the class decompose the bench into rectangular prisms, calculating the volume of each prism, and then adding up the volumes. Use Sofia's strategy to determine the volume of the bench.

7. Hunter proposes that the class first calculate the total volume of a 1.2 meter cube. Then, they can subtract out the portion of the cube that forms the seat of the bench. Determine the volume of the bench using Hunter's strategy.

8. **Compare the volume calculated using Sofia's strategy with the volume calculated using Hunter's strategy.**

NOTES

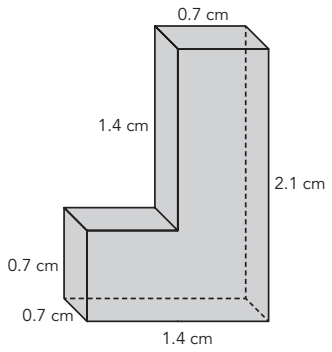
9. **How are Sofia's and Hunter's strategies for determining the volume of composite solids like the strategies used to determine the area of composite figures?**

ACTIVITY 2.2

Fluency with Decimal Operations

You have seen that you can add, subtract, and of course multiply positive rational numbers, like decimals, to determine volumes. Let's apply what you know to solve problems.

1. **Determine the volume of the figure.**



Answers

8. The volumes are the same.
9. When determining area of composite figures or volume of composite solids, you can decompose into non-overlapping figures or solids and add the areas or volumes. You can also compose into a larger figure or solid, determine the area or volume of the larger figure or solid as well as the area or volume of any unnecessary portions, and then subtract the areas or volumes.

Answers

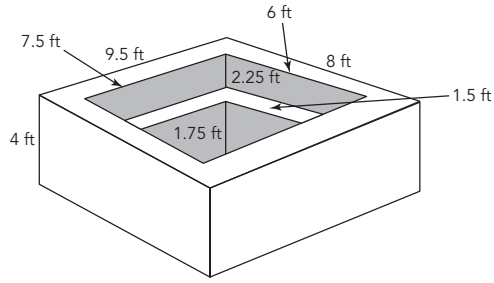
1. 1.372 cubic centimeters

Answers

2a. 113.625 cubic feet

2b. 179.125 cubic feet

2. Regina is building a hot tub next to her swimming pool. The interior dimensions are 6 feet by 7.5 feet. It includes solid bench seating on all four sides. The bench has a width of 1.5 feet. The bench is positioned 1.75 feet from the ground and 2.25 feet from the top as shown.

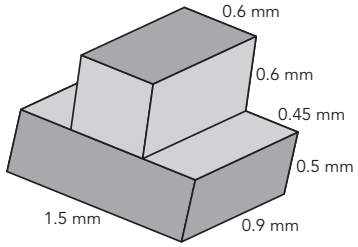


a. When the hot tub is filled, the water level will be 0.25 feet from the top. How much water will it take to fill the hot tub?

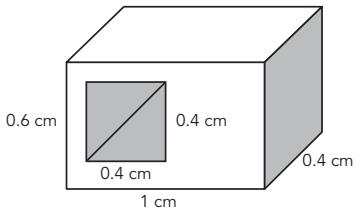
b. How many cubic feet of concrete is needed to build the hot tub?

3. **Calculate the volume of each figure. Show your work.**

a.



b.



Answers

3a. 0.999 cubic millimeters

3b. 0.176 cubic centimeters

Answers

1. Volume of A = 50,355.76 cubic feet; Volume of B = 80,554.496 cubic feet

   Cost of A, if full: $12,588.94;

   Cost of B, if full: $12,083.17

   Recommendations will vary.

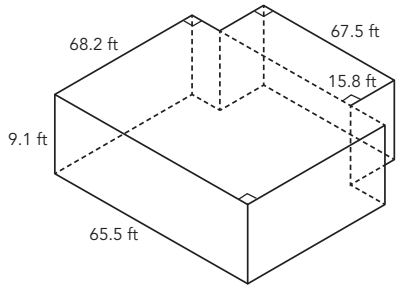
NOTES

TALK the TALK

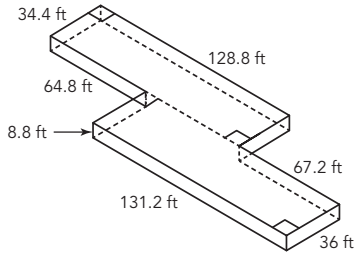
The Volume Warehouse

A business is shopping for warehouse space. Two of their choices are shown.

Warehouse A



Warehouse B



The total cost each month for space in Warehouse A is $0.25 times the number of cubic feet used. The total cost each month for space in Warehouse B is $0.15 times the number of cubic feet used.

1. **Which warehouse space would you recommend? What information would you need to make this decision? Write your findings in a report to your Director of Finance.**

ELL Tip

This activity ends with a request to write up the findings in a report. ELL students may have difficulty concentrating on both the mathematics and the writing assignment. The two tasks should be evaluated independently. To help the students explain their process in a writing assignment, pair up the ELL students with a native speaker. The two students can work together to solve the mathematics problem and to write up the solution in report format. Have some of the students read their reports out loud. By doing so they receive practice not only in writing, but also in speaking.