# Breaking the Fourth Wall

3

Surface Area of Rectangular Prisms and Pyramids

## MATERIALS

Scissors
Tape

## Lesson Overview

Students apply mathematical and spatial reasoning to determine the surface areas of prisms and pyramids using nets, drawings, and measurements. Students solve a variety of surface area problems and distinguish between volume and surface area measurements.

## Grade 6
## Number and Operations

**(3) The student applies mathematical process standards to represent addition, subtraction, multiplication, and division while solving problems and justifying solutions. The student is expected to:**

(E) multiply and divide positive rational numbers fluently.

## Grade 7
## Expressions, Equations, and Relationships

**(9) The student applies mathematical process standards to solve geometric problems. The student is expected to:**

(D) solve problems involving the lateral and total surface areas of a rectangular prism, rectangular pyramid, triangular prism, and triangular pyramid by determining the area of the shape's net.

## ELPS

1.A, 1.C, 1.D, 1.E, 1.G, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.D, 4.G, 4.K, 5.E

## Essential Ideas

- A *net* is a two-dimensional representation of a three-dimensional geometric figure.
- The surface area of a three-dimensional figure can be calculated by determining the areas of each face of the figure.

# Lesson Structure and Pacing: 2 Days

## Day 1

### Engage

**Getting Started: Breaking Down a Cube**

Students decide whether a net can be folded into a cube. This activity engages students in visuospatial thinking about the two-dimensional faces of cubes, given what they have learned about rectangular prisms.

### Develop

**Activity 3.1: Nets of Rectangular Prisms**

Students use the cube they built in the Getting Started to find the area of one face, as well as the total surface area. Students calculate the total surface area from given nets, and they draw nets from given rectangular prisms to calculate the total surface area. Students use decimal and fraction multiplication to determine the surface areas of these figures.

## Day 2

**Activity 3.2: Prisms and Pyramids**

Students complete a table to identify and compare the numbers of faces, edges, and vertices of prisms and pyramids. They identify various prisms and pyramids by their nets.

**Activity 3.3: Nets of Other Solids**

Students cut out and build three-dimensional models of a triangular prism and a triangular pyramid from their nets and determine their surface areas. Students calculate the total surface area from given nets, and they draw nets from given solid figures to calculate the total surface area.

**Activity 3.4: Surface Area Problems**

Students solve real-world problems involving the surface areas of various candle molds.

### Demonstrate

**Talk the Talk: Nothing but Net**

Students solve various surface area problems. They draw a net to calculate the total surface area of a prism, and they label a given net to calculate the total surface area of a pyramid.

# Getting Started: Breaking Down a Cube

ENGAGE

## Facilitation Notes

In this activity, students decide whether a net can be folded into a cube. This activity engages students in visuospatial thinking about the two-dimensional faces of cubes, given what they have learned about rectangular prisms.

Ask a student to read the definition aloud and discuss as a class. Have students complete Question 1 to assemble the net of a cube. Allow students to work with a partner or in a group to answer Questions 2 through 4. Share responses as a class.

**Questions to ask**

- Do you have to cut out the net to determine whether it folds into a cube?
- What characteristic is important to look for?
- Which net is obviously not the net of a cube? Why?
- How many faces are on the net of a cube?
- How many sides are on a cube?
- How many edges are on a cube?
- Is a cube a right rectangular prism?

**Differentiation strategies**

- To scaffold support for students, have them concentrate on one row of Question 2.
- To extend the activity, challenge students to draw a different net for a cube.

## Summary

A net is a two-dimensional representation of a three-dimensional geometric figure. A net can be assembled to create a model of a geometric solid, such as a cube. The number of faces, edges, and vertices are considered when identifying alternate nets for cubes.

# Activity 3.1

## Nets of Rectangular Prisms

DEVELOP

## Facilitation Notes

In this activity, students use the cube they built in the Getting Started to find the area of one face, as well as the total surface area. Students calculate the total surface area from given nets, and they draw nets from given rectangular prisms to calculate the total

surface area. Students use decimal and fraction multiplication to determine the surface areas of these figures.

Ask a student to read the properties of a net and the definition of surface area of a polyhedron aloud and discuss as a class. Allow students to work with a partner or in a group to answer Questions 1 through 7. Circulate and ask students leading questions when needed. Share responses as a class.

**Differentiation strategies**

To scaffold support for students,

- Have a pre-assembled prism available to help them visualize how the net forms the prism.
- Have students color opposite faces of the prism the same color; this will help them visualize that opposite faces are congruent and parallel.
- Provide a template to help them organize the multi-step process.

To extend the activity, have students determine the surface area of prisms with triangular or trapezoidal bases.

**Questions to ask**

- Is area associated with square units or cubic units?
- Is volume associated with square units or cubic units?
- What is the difference between area and surface area?
- How many different rectangles are used to determine the surface area of a cube?
- What is the difference between a cube and a unit cube?
- Is your estimate close to the actual calculation?
- How many different rectangles were used to determine the surface area of the right rectangular prism?
- Are both of these figures nets of right rectangular prisms?
- Is there an alternate method for drawing this net?
- How many dimensions are needed on the net to calculate the total surface area of the solid figure?
- Which dimensions are needed on the net to calculate the total surface area of the solid figure?

## Summary

Nets can be drawn to represent three-dimensional figures, such as right rectangular prisms. The surface area of a polyhedron is the total area of all of its two-dimensional faces.

# Activity 3.2
## Prisms and Pyramids

### Facilitation Notes

In this activity, students complete a table to identify and compare the numbers of faces, edges, and vertices of prisms and pyramids. Students identify various prisms and pyramids by their nets.

Ask a student to read the information and definition of a pyramid and slant height aloud. Allow students to work with a partner or in a group to answer Questions 1 through 3. Share responses as a class.

**Questions to ask**

- What is the difference between a prism and a pyramid?
- What is the difference between height and slant height?
- How many sides or faces are not visible in this figure?
- How many edges are not visible in this figure?
- What is the difference between a triangular pyramid and a square pyramid?

**Differentiation strategies**

To extend the activity,

- Have students investigate the relationship among vertices, edges, and faces of polyhedron, and then check their findings by researching Euler's formula: $V - E + F = 2$.
- Have students investigate how the total surface area of a pyramid is related to the total surface area of a rectangle prism with the same base and height.

### Summary

Nets can be drawn to represent prisms and pyramids with rectangular and non-rectangular bases. The characteristics of pyramids and prisms are identified.

# Activity 3.3
## Nets of Other Solids

### Facilitation Notes

In this activity, students cut out and build three-dimensional models of a triangular prism and a triangular pyramid from their nets and determine their surface areas. They calculate total surface area when given a net. They draw nets to represent solid figures and calculate the total surface area.

Allow students to work with a partner or in a group to cut out and measure the nets of the triangular pyramid and the triangular prism. For the triangular pyramid, ensure that students measure each side length. In order to calculate the total surface area, students also need to measure the height of each triangle. Have them answer Questions 1 through 3, and share responses as a class.

**Questions to ask**

- What is the name of the solid figure represented by this net?
- What are the missing dimensions? How do you know?
- How many rectangles are used to calculate the surface area of the solid figure represented by this net?
- How many triangles are used to calculate the surface area of the solid figure represented by this net?
- Did you first estimate to determine the surface area?
- Which face is considered the base in this net?
- What polygon describes the base of this solid figure?

**Differentiation strategy**

To extend the activity, have students draw the nets to scale for Question 3.

### Summary

Nets can be drawn to represent three-dimensional figures, including triangular pyramids and triangular prisms. The surface area of a polyhedron is the total area of all of its two-dimensional faces.

## Activity 3.4
### Surface Area Problems

### Facilitation Notes

In this activity, students solve real-world problems involving the surface areas of various candle molds.

Students identify four solids and calculate the surface area of each in a real-world scenario. Allow students to work with a partner or in a group to answer Questions 1 and 2. Share responses as a class.

**Differentiation strategies**

- To scaffold support for students, provide enlarged copies of the nets to assemble.
- To extend the activity, have students try to determine the surface area of a cylinder.

**Questions to ask**

- What is the name of the solid figure represented by this net?
- What are the missing dimensions? How do you know?
- Do the dashed lines represent folds or heights?
- How many rectangles are used to calculate the surface area of the solid figure represented by this net?
- How many triangles are used to calculate the surface area of the solid figure represented by this net?
- Did you first estimate to determine the surface area?
- Which face is considered the base in this net?
- What polygon describes the base of this solid?

### Summary

A net is a two-dimensional drawing of a three-dimensional geometric figure. A net can be used to determine the total surface area of a 3-D figure, which is the sum of the areas of each of its faces.

## Talk the Talk: Nothing but Net

DEMONSTRATE

### Facilitation Notes

In this activity, students solve various surface area problems. They draw and label nets to calculate the total surface area of solid figures and explain how to determine the total surface area of a pyramid.

Students identify four solids and calculate the surface area of each in a real-world scenario. Allow students to work with a partner or in a group to answer Questions 1 and 2. Share responses as a class.

**Questions to ask**

- Is there a different way to draw the net of the rectangular prism?
- Do you have all of the measures needed to calculate the surface area of this prism?
- Does it matter where you label the given dimensions on the net?
- How many rectangles are used to calculate the total surface area?
- What are the dimensions of each rectangle used to calculate the total surface area?

- What is the name of the solid figure represented by this net?
- Which face is considered the base in this net?
- How many triangles are used to calculate the surface area of the pyramid?

## Summary

Drawing and labeling a net of a three-dimensional figure can be used to model and solve problems about surface area.

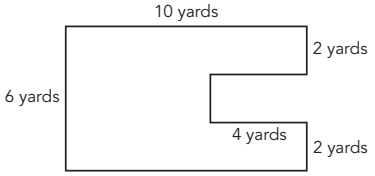
# 3 Breaking the Fourth Wall

## Surface Area of Rectangular Prisms and Pyramids

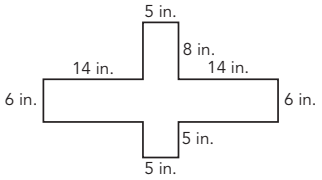
### WARM UP

Calculate the area of each composite figure.

1.




2.




### LEARNING GOALS

- Represent solid figures using two-dimensional nets made up of rectangles and triangles.
- Use nets of solid figures to determine the surface areas of the figures.
- Solve real-world and mathematical problems involving surface area.
- Fluently multiply and divide multi-digit decimals using standard algorithms.

### KEY TERMS

- net
- surface area
- pyramid
- slant height

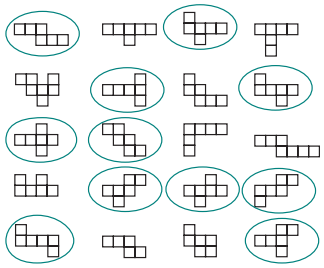
You know how to determine how many cubic units fill a rectangular prism. How can you calculate the number of square units it takes to cover the outside of a prism?



### Warm Up Answers

1. 52 square yards
2. 263 square inches

## Answers

1. Check students' models.
2. 
3. Answers will vary.
4. Answers will vary.

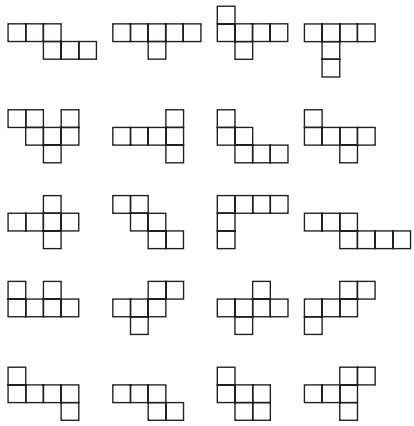
## Getting Started

### Breaking Down a Cube

A **net** is a two-dimensional representation of a three-dimensional geometric figure. A net is cut out, folded, and taped to create a model of a geometric solid.

1. **Cut, fold, and tape the cube net found at the end of the lesson.**

2. **Are there other nets that form a cube? Circle the 11 cutouts that can form a cube.**



3. **How did you determine which are nets of cubes?**

4. **What do all of the nets for a cube have in common? Consider the number of faces, edges, and vertices in your explanation.**

ACTIVITY
3.1

## Nets of Rectangular Prisms

A net has all these properties:

- The net is cut out as a single piece.
- All of the faces of the geometric solid are represented in the net.
- The faces of the geometric solid are drawn such that they share common edges.

The **surface area** of a polyhedron is the total area of all its two-dimensional faces.

Consider the cube you created.

1. **How is the area of a face of a cube measured? Analyze the two responses and explain why Leticia is incorrect in her reasoning.**

**Leticia**

This is a 3D figure, which means that its measurements are cubic units.

**Isaiah**

Surface area is still measuring area, which is always measured in square units.




## Answers

1. Leticia is not thinking about the faces of the three-dimensional figure. She may be thinking about the volume of a three-dimensional figure.

## ELL Tip

At this point, students may confuse surface area and volume, particularly early ELL students. Have students relate these concepts to familiar situations by asking them to categorize them as situations involving surface area and situations involving volume. Non-verbal assent, such as a thumbs-up or thumbs-down, can be used to demonstrate their agreement with statements like, "The amount of wrapping paper it takes to wrap a present is an example of surface area," or, "The amount of orange juice that can fit in a jug is an example of volume." Make sure that some of the examples are categorized correctly and some are categorized incorrectly. Ask students to generate their own examples.

## Answers

2. Answers will vary.
3. 24 square inches
4. 6 square units

5a. Sample estimate.
108 square cm

5b. $2(5.7 \times 2.9) + 2(5.7 \times 4.3) + 2(4.3 \times 2.9) = 107.02$ square cm

Sample explanation.
I can think about the rectangular prism as a box and find the area of each face.

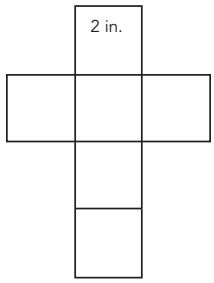
Bottom and top:
$2 \times 5.7 \times 2.9 = 33.06$

Left and right sides:
$2 \times 4.3 \times 2.9 = 24.94$

Front and back sides:
$2 \times 5.7 \times 4.3 = 49.02$

Surface area: $33.06 + 24.94 + 49.02 =$ 107.02 sq cm

**2. Describe a strategy that you can use to determine the surface area of a cube.**

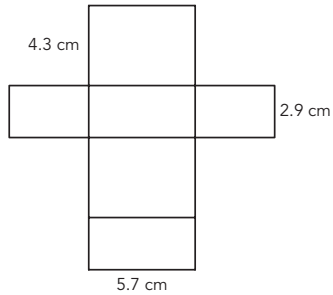



**3. Consider the cube net shown. Calculate the surface area.**

**4. What is the surface area of a unit cube?**

**5. Let's consider a different rectangular prism.**

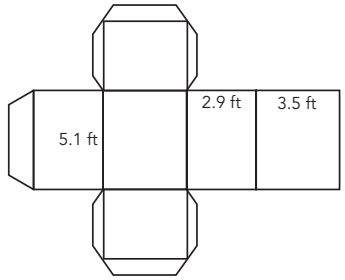
**a. Use the net to estimate the surface area of the right rectangular prism.**



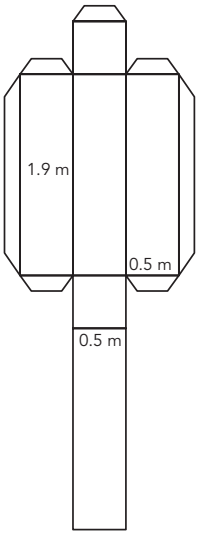

**b. Calculate the surface area of the right rectangular prism. Explain your calculation.**

6. **Calculate the surface area of the solid figure represented by each net.**

a.

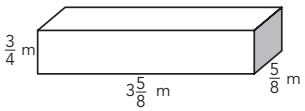



b.

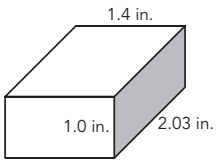



7. **Draw a net to represent each solid figure. Label each net with measurements, and then calculate the surface area of the solid figure.**

a.




b.






## Answers

6a. 85.58 square feet

6b. 4.3 square meters

7a. Check students' nets.
$10\frac{29}{32}$ square meters

7b. Check students' nets.
12.544 square inches

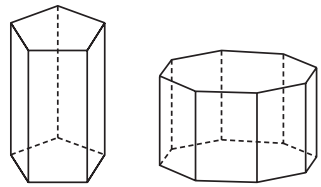
## Answers

1. See table below.

ACTIVITY 3.2

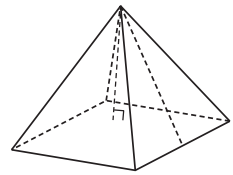
# Prisms and Pyramids

The base of a prism does not have to be rectangular. The base of a prism can be a triangle, pentagon, hexagon, and so on.



A **pyramid** is a polyhedron with one base and the same number of triangular faces as there are sides of the base. The vertex of a pyramid is the point at which all the triangular faces intersect.

A **slant height** of a pyramid is the distance measured along a triangular face from the vertex of the pyramid to the midpoint of an edge of the base.



**1. Analyze the figures shown. Then, complete the table using the figures.**

Figure A

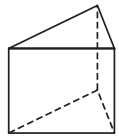


Figure B

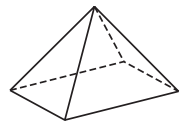


Figure C

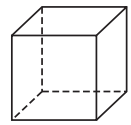
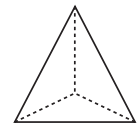


Figure D



| Figure | Is it a Prism or Pyramid? | Shape of Base | Number of Faces | Number of Vertices | Number of Edges |
|---|---|---|---|---|---|
| A | | | | | |
| B | | | | | |
| C | | | | | |
| D | | | | | |



1.

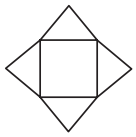
| Figure | Is it a Prism or Pyramid? | Shape of Base | Number of Faces | Number of Vertices | Number of Edges |
|---|---|---|---|---|---|
| A | Prism | Triangle | 5 | 6 | 9 |
| B | Pyramid | Rectangle | 5 | 5 | 8 |
| C | Prism | Rectangle | 6 | 8 | 12 |
| D | Pyramid | Triangle | 4 | 4 | 6 |

NOTES

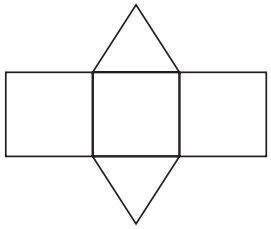
2. **Write the names of Figures A, B, C, and D from your completed table.**

3. **Label each net with the name of the solid figure it forms.**

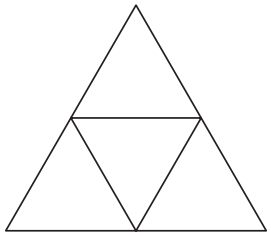
a.



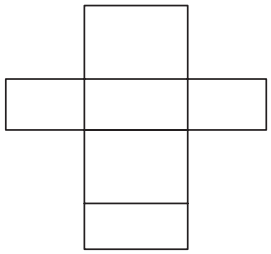
b.



c.



d.





## Answers

2. Figure A: Triangular Prism;
   Figure B: Rectangular Pyramid;
   Figure C: Rectangular Prism;
   Figure D: Triangular Pyramid

3a. Square pyramid

3b. Triangular prism

3c. Triangular pyramid

3d. Rectangular prism

## Answers

1a. Check students' measurements, as they will vary based on printing dimensions and scale.

1b. The surface area of the triangular prism is approximately 61.56 square centimeters, and the surface area of the triangular pyramid is approximately 81.9 square centimeters.

Note: These values are based on the nets in the student edition, when printed at 100% scale on $8\frac{1}{2}$ by 11 in. paper.

1c. Check students' models.

1d. Check that students correctly identify the triangular prism and the triangular pyramid.

2a. 638 square inches

2b. 2.4 square meters

ACTIVITY 3.3

# Nets of Other Solids

**1. Locate the nets for the triangular prism and triangular pyramid at the end of the lesson.**

**a. Measure the edge lengths of each net with a centimeter ruler. Label the lengths.**

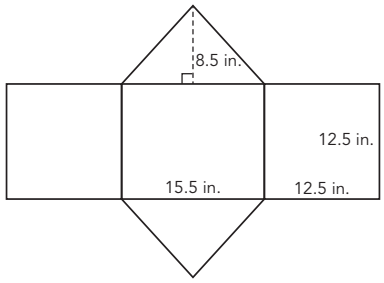
**b. Calculate the surface area of each solid figure.**

**c. Cut out, fold, and tape each net.**

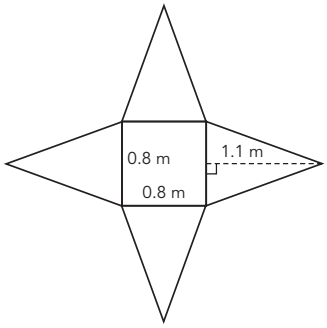
**d. Name each solid figure.**


**2. Calculate the surface area of the solid figure represented by each net.**

**a.**

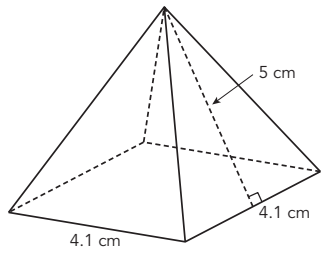



**b.**

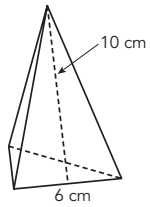


3. **Draw a net to represent each solid figure. Label each net with measurements, and then calculate the surface area of the solid figure.**

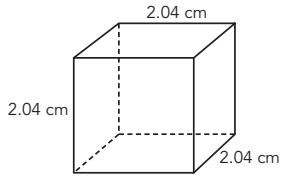
**a.**




**b. The slant heights are all equal. The height of the base is 5.2 cm.**



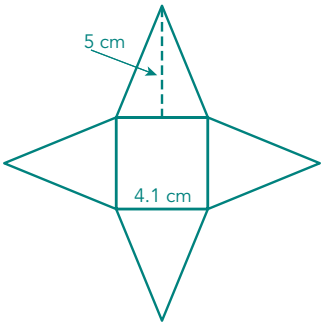

**c.**





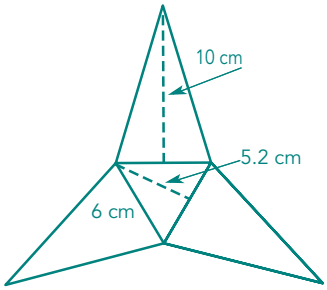

## Answers

3a.



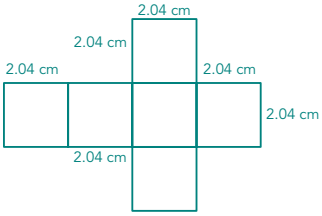

57.81 square centimeters

3b.




105.6 square centimeters

3c.

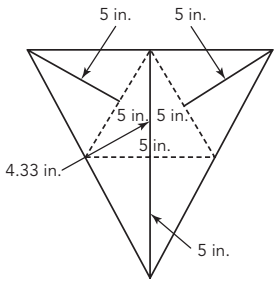



24.9696 square centimeters

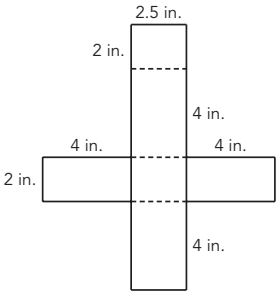
ACTIVITY
# 3.4 Surface Area Problems

Scents-R-Us produces candles in a variety of shapes. To produce each candle, the company first creates a mold, then pours hot wax into the mold. When the hot wax cools and solidifies, the mold is removed.

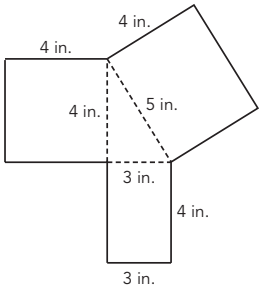
Candle Mold A



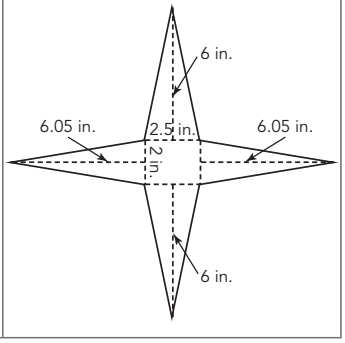

Candle Mold B




Candle Mold C




Candle Mold D



1. **Classify the shape of each candle, based on the candle mold.**

   a. **Candle Mold A**

   b. **Candle Mold B**

   c. **Candle Mold C**

   d. **Candle Mold D**

2. **Use each candle mold to answer each question.**

   a. **Calculate the surface area of each candle.**

   b. **How could Scents-R-Us use the surface area of the candles to determine how to price each candle?**



## Answers

1a. Triangular pyramid

1b. Rectangular prism

1c. Triangular prism

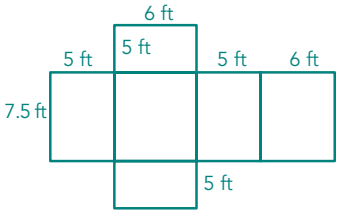
1d. Rectangular pyramid

2a. Candle A: 48.325 square inches; Candle B: 46 square inches; Candle C: 54 square inches; Candle D: 32.1 square inches.

2b. Answers will vary.

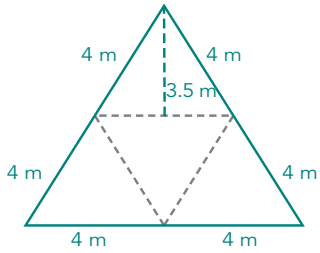
## Answers

1a. Sample drawing.




1b. 225 square feet

2a.




2b. 28 square meters

3. Answers will vary.

NOTES

## TALK the TALK

### Nothing but Net

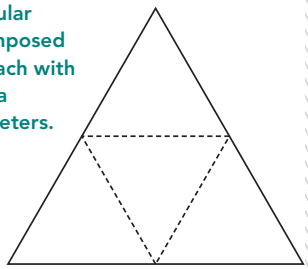
1. A rectangular prism has a height of 6 feet, a length of 7.5 feet, and a width of 5 feet.

   a. Draw a net of the rectangular prism and label its measurements.

   b. Calculate the surface area of the prism.

2. Consider the net of the triangular pyramid shown. The net is composed of four equilateral triangles, each with a side length of 4 meters and a height of approximately 3.5 meters.

   a. Label the pyramid with its measurements.



   b. Calculate the surface area of the pyramid.

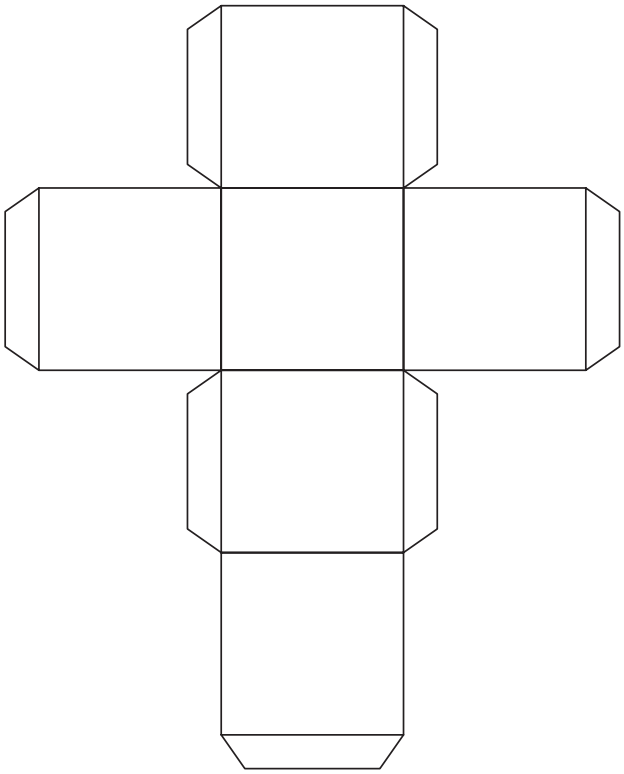
3. Explain in your own words how to determine the surface area of a pyramid.



### ELL Tip

The last portion of this section asks students to explain. Have the students engage in a partner interview, where one person begins the explanation and the other asks clarifying questions. When they are satisfied with the explanation, the two students should switch roles. This exercise not only helps to solidify the students' mathematical thinking, but also allows ELL students an opportunity to speak using proper mathematical terms.

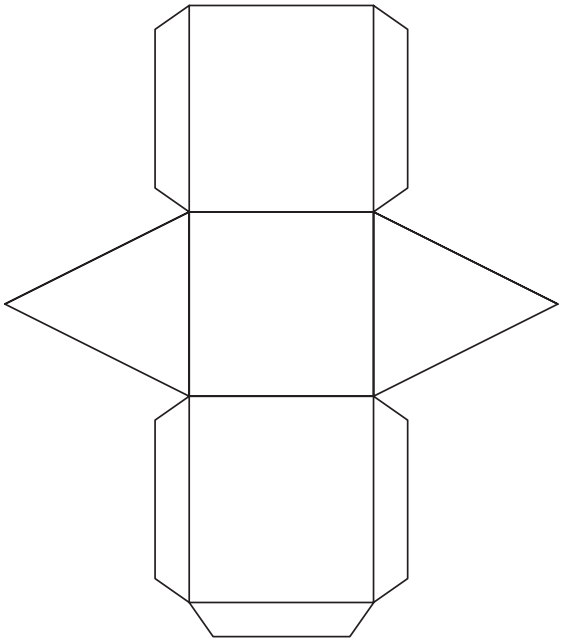
## Cube Net



## Why is this page blank?

So you can cut out the net on the other side.

## Triangular Prism Net



## Why is this page blank?

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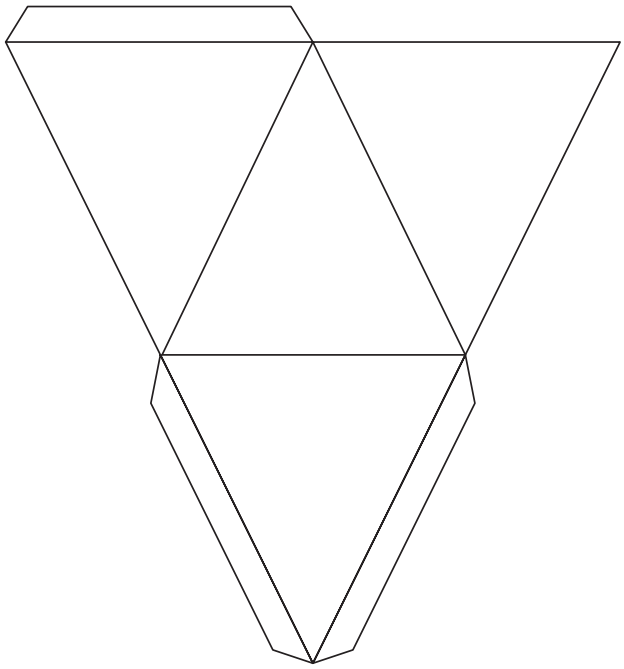
## Triangular Pyramid Net



## Why is this page blank?

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