Decimals and Volume Summary

KEY TERMS

- point
- line segment
- polygon
- geometric solid
- polyhedron
- face
- edge
- vertex
- right rectangular prism
- cube
- pyramid
- volume
- composite solid
- trailing zeros
- net
- surface area
- slant height

LESSON 1 Depth, Width, and Length

The mathematical definition of **point** is a location in space, often represented using a dot and named by a capital letter. A **line segment** is a portion of a line that includes two points and the points between those two points.

A **polygon** is a closed figure formed by three or more line segments. A **geometric solid** is a bounded three-dimensional geometric figure. A **polyhedron** is a three-dimensional solid figure that is made up of polygons that are called **faces**. An **edge** is the intersection of two faces and a **vertex** is the point where the edges meet.

For example, Figure A is a **right rectangular prism**, which is a polyhedron with three pairs of congruent and parallel faces.

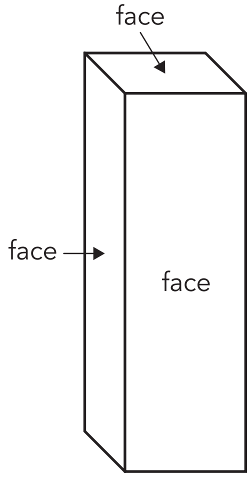


Figure A

Figure B is a **cube**, which is a polyhedron that has six congruent squares as faces.

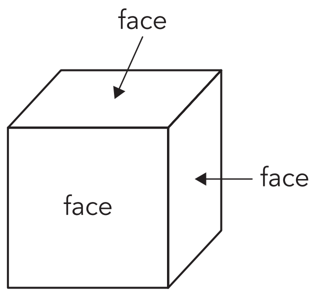


Figure B

Figure C is a rectangular pyramid. A **pyramid** is a polyhedron with one base and the same number of triangular faces as there are sides of the base.

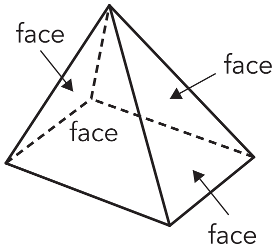
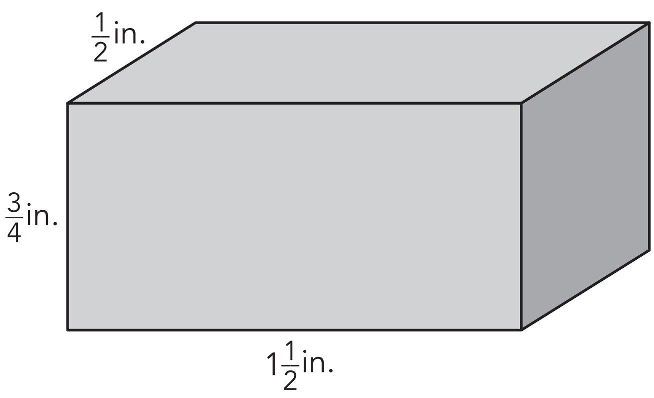


Figure C

Volume is the amount of space occupied by an object. The volume of an object is measured in cubic units. A unit cube is a cube whose sides are all 1 unit long.

The volume of a rectangular prism is a product of its length, width and height: $V = l \cdot w \cdot h$.

For example, to determine the volume of the right rectangular prism shown with the given dimensions, you can fill the prism with cubes, but smaller unit cubes with fractional side lengths are required.



Assign a unit fraction to the dimensions of each cube. Use the least common multiple (LCM) of the fraction denominators to determine the unit fraction.	LCM(2, 4) = 4 So, each cube will measure $\frac{1}{4}$ in. $\times$ $\frac{1}{4}$ in. $\times$ $\frac{1}{4}$ in. The volume of each unit cube is $\frac{1}{64}$ cubic inches.
Determine the number of cubes needed to pack the prism in each dimension.	length: $1\frac{1}{2} \div \frac{1}{4} = 6$ width: $\frac{1}{2} \div \frac{1}{4} = 2$ height: $\frac{3}{4} \div \frac{1}{4} = 3$
Determine the number of cubes that make up the right rectangular prism.	$6 \times 2 \times 3 = 36$
Multiply the number of cubes by the volume of each cube to determine the volume of the right rectangular prism.	$36 \times \frac{1}{64} = \frac{36}{64}$ $= \frac{9}{16}$

The volume of the right rectangular prism is $\frac{9}{16}$ cubic inches.

You can use the formula $V = Bh$ to calculate the volume of any prism. However, the formula for calculating the value of B will change depending on the shape of the base. In a rectangular prism, $B = l \cdot w$.

LESSON 2

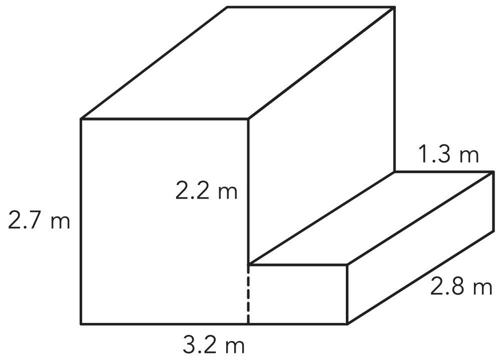
Which Warehouse?

A **composite solid** is made up of more than one geometric solid. You can decompose a composite solid into more than one polyhedron in order to determine its volume.

For example, to determine the volume of the composite solid shown, you can decompose the solid into two rectangular prisms and calculate the volume of each.

Volume of larger prism = $1.9 \times 2.8 \times 2.7 = 14.364 \text{ m}^3$

Volume of smaller prism = $1.3 \times 2.8 \times 0.5 = 1.82 \text{ m}^3$



To calculate the sum or difference of decimals, line up the decimals so that like place values are in the same column. Use the decimal point to help you correctly align.

$$\begin{array}{r} 14.364 \\ +1.820 \\ \hline 16.184 \end{array}$$

A *trailing zero* was added to 1.82. **Trailing zeros** are a sequence of 0s in a decimal representation of a number, after which no non-zero digits follow. Trailing zeros do not affect the value of a number.

The volume of the composite solid is 16.184 cubic meters.

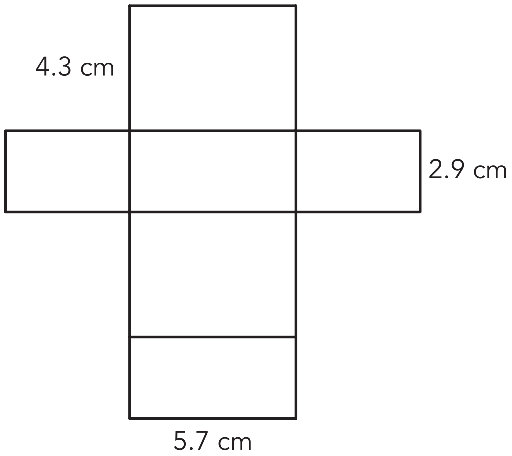
LESSON 3 Breaking the Fourth Wall

A **net** is a two-dimensional representation of a three-dimensional geometric figure.
A net has all of these properties:

- The net is cut out as a single piece.
- All of the faces of the geometric solid are represented in the net.
- The faces of the geometric solid are drawn so that they share common edges.

The **surface area** of a polyhedron is the total area of all its two-dimensional faces.

For example, you can use the net to calculate the surface area of the right rectangular prism.



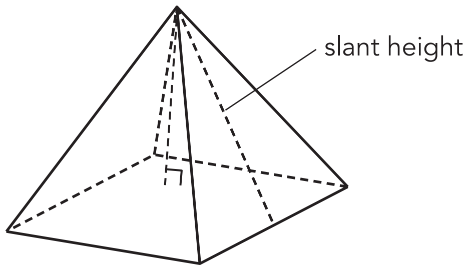
Determine the area of each unique face.

$4.3 \text{ cm} \times 5.7 \text{ cm} = 24.51 \text{ cm}^2$
$2.9 \text{ cm} \times 5.7 \text{ cm} = 16.53 \text{ cm}^2$
$4.3 \text{ cm} \times 2.9 \text{ cm} = 12.47 \text{ cm}^2$

Determine the sum of all faces of the right rectangular prism.

$2(24.51) + 2(16.53) + 2(12.47)$
$= 49.02 + 33.06 + 24.94$
$= 107.02$

The surface area of the right rectangular prism is 107.02 cm^2.



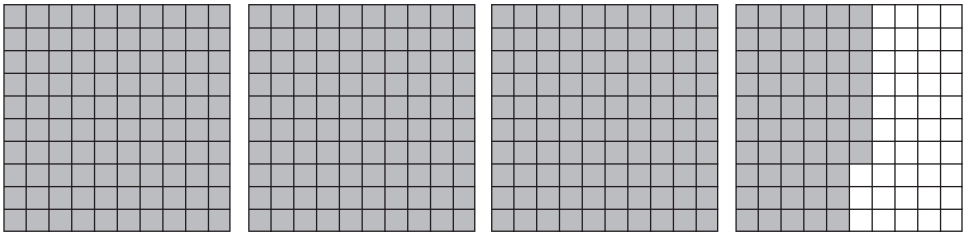
Remember, the vertex of a pyramid is the point at which all the triangular faces of the pyramid intersect. A **slant height** of a pyramid is the distance measured along a triangular face from the vertex of the pyramid to the midpoint, or center, of the base.

LESSON 4

Dividend in the House

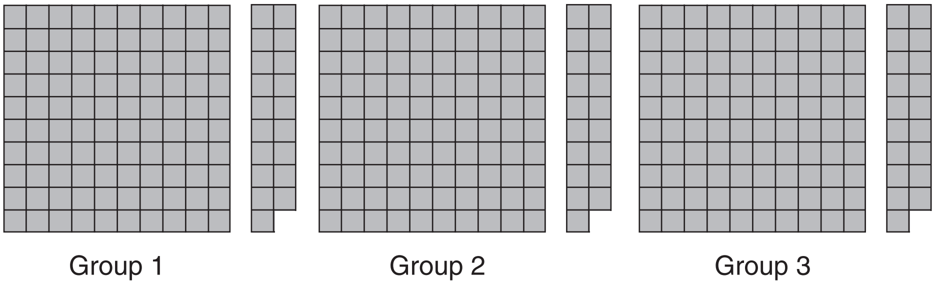
You can use a hundredths grid to model dividing decimals, such as 3.57 ÷ 3.

First, shade hundredths grids to represent 3.57.

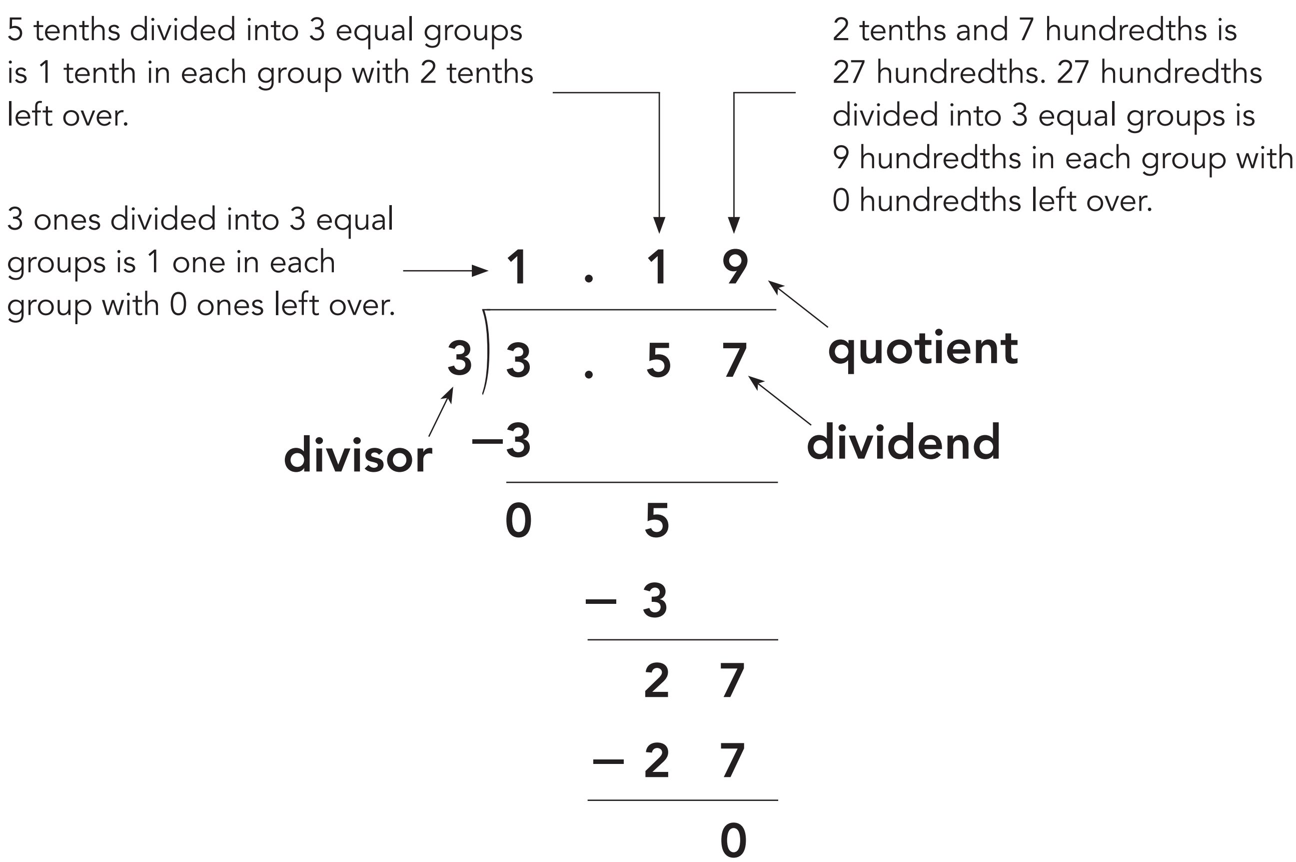


Next, divide the shaded model into 3 equal groups.

One whole grid and 19 small squares are in each group. So, 3.57 ÷ 3 = 1.19.



You can also use a standard algorithm to divide 3.57 ÷ 3.



If you multiply or divide both the dividend and divisor by the same number, the quotient remains the same.

$$7.7 \div 3.5 = 77 \div 35$$

$$\frac{7.7}{3.5} = \frac{77}{35}$$

You can use what you know about dividing with decimals to solve problems about volume and surface area. For example, suppose the surface area of a cube is 48.24 square inches. Calculate the area of each face of the cube.

$$\begin{array}{r} 8.04 \\ 6\overline{)48.24} \end{array}$$

Since a cube has six congruent square faces, each face has an area of 8.04 square inches.