How to support your student as they learn about **Composing and Decomposing**

Mathematics is a connected set of ideas, and your student knows a lot. Encourage them to use the mathematics they already know when seeing new concepts in this module.

Module Introduction

In this module your student will deepen their understanding of numbers and shapes. There are four topics in this module: *Factors and Multiples*, *Positive Rational Numbers*, *Angles and Shapes*, and *Decimals and Volume*. Your student will use what they already know about fraction and decimal operations in this module.

Academic Glossary

Each module will highlight an important term. Knowing and using these terms will help your student think, reason, and communicate their math ideas.

Term	Analyze
Definition	• To study or look closely for patterns • To break a concept down into smaller parts to gain a better understanding of it
Questions to Ask Your Student	• Do you see any patterns? • Have you seen something like this before? • What happens if the shape, model, or numbers change?
Related Phrases	• Examine • Evaluate • Determine • Observe • Consider • Investigate • What do you notice?

TABLE OF CONTENTS

Draw any triangle on a piece of paper. Tear off the triangle's three angles. Arrange the angles so that they are adjacent angles.

What do you notice about these angles?

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Math Process Standards

Each module will focus on a process (or a pair of processes) that will help your student become a mathematical thinker. The "I can" statements listed below help your student to develop their mathematical learning and understanding.

Communicate mathematical ideas, reasoning, and their implications using multiple representations including symbols, diagrams, graphs, and language as appropriate.

I can:

- explain what a problem "means" in my own words.
- create a plan and change it if necessary.
- ask useful questions when trying to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

Look for examples of these processes in the Topic Summaries.

The Carnegie Learning Way

Our Instructional Approach

Carnegie Learning's instructional approach is based on how people learn and real-world understandings. It is based on three key components:

ENGAGE	DEVELOP	DEMONSTRATE
Purpose: Provide an introduction that creates curiosity and uses what students already know and have experienced.	**Purpose:** Build a deep understanding of mathematics through different activities.	**Purpose:** Reflect on and evaluate what was learned.
Questions to Ask: How does this problem look like something you did in class?	**Questions to Ask:** Do you know another way to solve this problem? Does your answer make sense?	**Questions to Ask:** Is there anything you do not understand?

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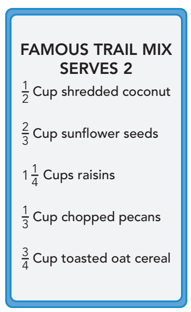
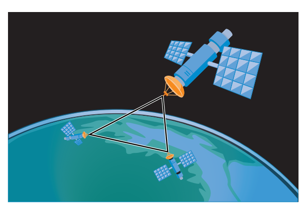
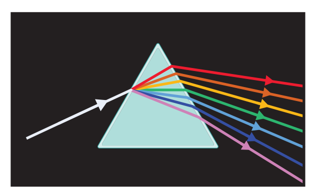
Module Overview

TOPIC 1	TOPIC 2	TOPIC 3	TOPIC 4
Factors and Multiples	**Positive Rational Numbers**	**Angles and Shapes**	**Decimals and Volume**
5 Days	7 Days	8 Days	9 Days
Your student will study the relationship between numbers and area.	**Your student will review fraction multiplication and use the inverse relationship between multiplication and division to understand fraction by fraction division.**	**Your student will establish important triangle relationships through reasoning, logic, investigating, and testing conjectures.**	**Your student will be introduced to prisms and pyramids as they discuss volume and surface area.**
Did you know that? You can use an area model to represent the product of the two numbers 15 and 42. 40 2 10 400 20 5 200 10 Can you write the expression? *[400 + 20 + 200 + 10]*	What in the world?  Dividing fractions is commonly used when cooking and baking. How many cups of chopped pecans would you need for half the serving size? *[$\frac{1}{6}$ cup of chopped pecans]*	Where in the world? The Global Positioning System, or GPS, uses triangle relationships to help us navigate and find our location anywhere on Earth. 	Did you know that? Sir Isaac Newton shined light through glass prisms to discover the seven colors of the visible light spectrum: red, orange, yellow, green, blue, indigo, and violet. 

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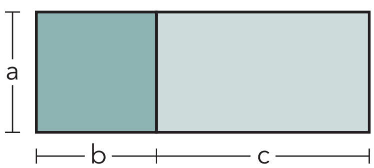
Topic 1: Factors and Multiples

Key Terms		
• numeric expression • equation • Distributive Property • base • power	• exponent • common factor • relatively prime • greatest common factor (GCF)	• multiple • Commutative Property • least common multiple (LCM)
The **Distributive Property**, when applied for multiplication, states that for any numbers *a*, *b*, and *c*, $a(b + c) = ab + ac$.  	The **exponent** of a power is the number of times the base is used as a factor. $8^4 = 8 \cdot 8 \cdot 8 \cdot 8$ ↑ exponent	A **multiple** is the product of a given whole number and another whole number. multiples of 10: 10, 20, 30, 40, 50 … ↑ ↑ ↑ ↑ ↑ $10 \cdot 1$, $10 \cdot 2$, $10 \cdot 3$, $10 \cdot 4$, $10 \cdot 5$ …

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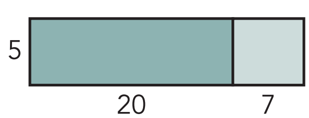
In this topic, students learn about factors and multiples. They use area models and factor trees to show the factors of a given number and organize the factors. Students also use tables to determine common factors, the greatest common factor (GCF), and the least common multiple (LCM) of two or more numbers.

Area Models

The equation $5 \cdot 27 = 135$ shows that the expression $5 \cdot 27$ is equal to the expression 135. An **equation** is a mathematical sentence that uses an equals sign to show that two or more quantities are the same as one another.



The area model shows the side length of 27 split into two parts.

$$5 \cdot 27 = 5(20 + 7)$$

The factors for each region are $(5 \cdot 20) + (5 \cdot 7)$.

The area of each smaller region is $100 + 35$.

The total area is 135 units^2.

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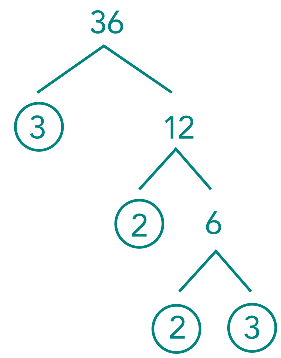
Factor Trees to Determine the Prime Factors of a Number

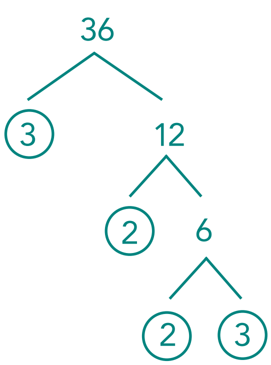
A factor tree is a way to organize the prime factors of a number. A prime factor is a number that is only divisible by itself or 1. Choose any factor pair to get started.

- Begin with the number 36.
- Pick any whole number factor pair of 36, other than 1 and 36.
- Draw a branch from 36 to each factor, 3 and 12.
- Since one of the factors is not prime, you are not finished.
- Use branches to write a factor pair for 12.
- Since one of the factors of 12 is not prime, you are not finished.
- Because 2 and 3 are prime, this factor tree is complete.



$$\mathbf{36 = 2 \cdot 2 \cdot 3 \cdot 3}$$

Repeated Multiplication as a Power

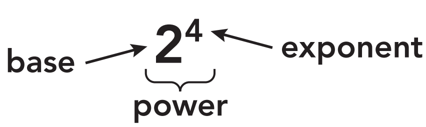


$$\mathbf{36 = 2 \cdot 2 \cdot 3 \cdot 3}$$
$$\mathbf{= 2^2 \cdot 3^2}$$

The prime factorization shown has repeated factors. You can represent repeated multiplication as a power. A **power** has two parts: the base and the exponent.

The **base** of a power is the factor multiplied by itself repeatedly, and the **exponent** of the power is the number of times you use the base as a factor.

$$2 \cdot 2 \cdot 2 \cdot 2 = 2^4$$



Using a Table to Determine the GCF and LCM

You can organize the prime factors of two or more numbers into a table. Only list shared factors that are in both rows in the same column.

Number	Prime Factors				
56	2	2	2		7
42	2			3	7

In the table shown, the common prime factors of 56 and 42 are 2 and 7.

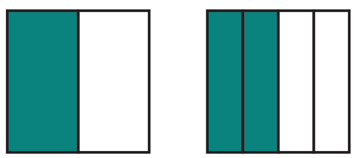
The **greatest common factor (GCF)** is the product of the shared prime factors. $2 \cdot 7 = 14$, so the GCF of 56 and 42 is 14. The **least common multiple (LCM)** is the product of the shared and non-shared prime factors. Both 2 and 7 are shared factors. They are only used once in the product. $2 \cdot 2 \cdot 2 \cdot 3 \cdot 7 = 168$, so the LCM of 56 and 42 is 168.

Topic 2: Positive Rational Numbers

Key Terms

• unit fraction • equivalent fraction • benchmark fractions • algorithm	• rational number • reciprocal • multiplicative inverse • complex fraction

Fractions that represent the same part-to-whole relationship are **equivalent fractions**.



$\frac{1}{2}$ $\frac{2}{4}$

The **multiplicative inverse** of a number $\frac{a}{b}$ is the number $\frac{b}{a}$, where *a* and *b* are nonzero numbers. The product of any nonzero number and its multiplicative inverse is 1. The multiplicative inverse of a number is also known as the **reciprocal** of a number.

The multiplicative inverse of $\frac{3}{7}$ is $\frac{7}{3}$.

$\frac{3}{7} \cdot \frac{7}{3} = \frac{21}{21} = 1$

The multiplicative inverse of 5 is $\frac{1}{5}$.

$\frac{5}{1} \cdot \frac{1}{5} = \frac{5}{5} = 1$

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In this topic, students focus on fraction multiplication and division. They will create physical models to represent and compare fractions, and determine equivalent fractions. They use an area model for multiplication with fractions before using an **algorithm**, or step-by-step method.

Using Area Models to Multiply Fractions

MATH PROCESS STANDARDS

How do the activities in *Positive Rational Numbers* promote student expertise in the math process standards?

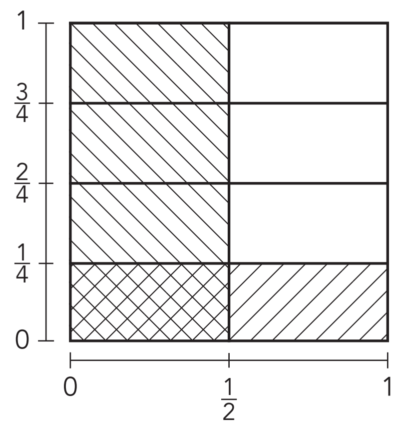
NOTE: This is an example of the math process standard:

Communicate mathematical ideas, reasoning, and their implications using multiple representations including symbols, diagrams, graphs, and language as appropriate.

- I can explain what this area model means in my own words.

Refer to page 2 for more "I can" statements.

Consider the expression $\frac{1}{4} \cdot \frac{1}{2}$ represented in the area model shown.

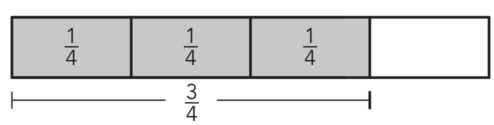


How are the factors $\frac{1}{4} \cdot \frac{1}{2}$ represented in the model?

Also in this topic, students use models of fractions with division, and then use a dividing across strategy.

Using Strip Diagrams to Represent Quotients with Fractions

A strip diagram can show the quotient of two fractions, such as $\frac{3}{4} \div \frac{1}{4}$. The division expressions asks, "How many $\frac{1}{4}$s are in $\frac{3}{4}$?"



There are 3 one-fourths in $\frac{3}{4}$, so $\frac{3}{4} \div \frac{1}{4} = 3$.

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Finally, students learn the standard algorithm, or steps, for dividing fractions. They rewrite division expressions as multiplication by the reciprocal, or multiplicative inverse.

Dividing Fractions

$$\frac{5}{8} \div \frac{3}{4} = \frac{\frac{5}{8}}{\frac{3}{4}}$$

Rewrite the division expression as a **complex fraction**.

$$= \frac{\frac{5}{8}}{\frac{3}{4}} \cdot \frac{\frac{4}{3}}{\frac{4}{3}}$$

Multiply the numerator and denominator by the multiplicative inverse of $\frac{3}{4}$.

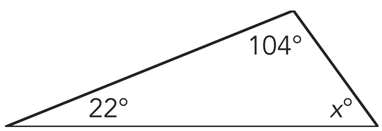
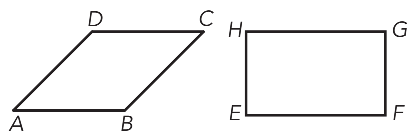
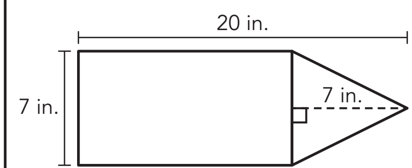
$$= \frac{\frac{5}{8} \cdot \frac{4}{3}}{\frac{3}{4} \cdot \frac{4}{3}} = \frac{\frac{5}{8} \cdot \frac{4}{3}}{1}$$

Perform multiplication and rewrite the denominator as 1.

$$\frac{5}{8} \div \frac{3}{4} = \frac{5}{\not{8}_{2}} \cdot \frac{\not{4}^{1}}{3} = \frac{5}{6}$$

TEXAS MATH SOLUTION

Topic 3: Angles and Shapes

Key Terms		
• Triangle Inequality Theorem • Triangle Sum Theorem	• parallelogram • variable	• trapezoid • composite figure
The **Triangle Sum Theorem** states that the sum of the measures of the interior angles of a triangle is 180°.  	A **parallelogram** is a four-sided figure with two pairs of parallel sides and opposite sides that are equal in length.  	A **composite figure** is a figure that is made up of more than one geometric figure. 

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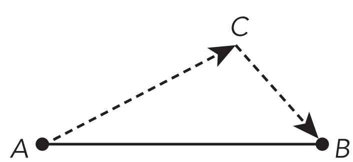
In this topic, students determine if three given line segments will create a triangle or not.

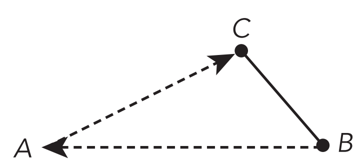
Triangle Inequality Theorem

The Triangle Inequality Theorem states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$AC + CB > AB$

$BA + AC > BC$





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Also in this topic, students use hands-on tools to learn about the sum of the interior angles of a triangle and the relationship between triangle side and angle measures.

Triangle Sum Theorem

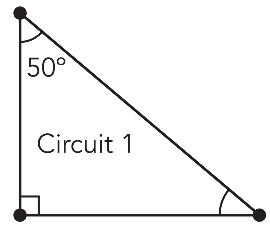
The **Triangle Sum Theorem** states the relationship between the three angles in a triangle.

THEOREM

TRIANGLE SUM THEOREM

The sum of the measures of the interior angles of a triangle is 180°.

Trevor organizes a bike race called the Tri-Cities Criterium. Criteriums consist of several laps around a closed circuit. He designs a triangular circuit.



Use the Triangle Sum Theorem to determine the measure of the third angle in the triangular circuit.

$$x + 90^\circ + 50^\circ = 180^\circ$$

$$x + 140^\circ = 180^\circ$$

$$x = 40^\circ$$

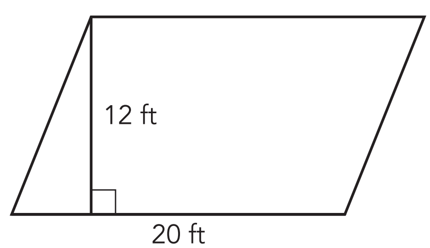
From their knowledge of rectangles and area, students develop the formula, or rule, for determining the area of parallelograms, triangles, and **trapezoids**. Students calculate the volume of right rectangular prisms.

Area of a Parallelogram

A **parallelogram** is a four-sided figure with two pairs of parallel sides with each pair equal in length. In a parallelogram, the height is the distance from the base to the opposite side at a right angle. The area of a parallelogram is equal to $b \cdot h$, where the variable b represents the base and h represents the height. A **variable** is a letter used to represent a number.

For example, in this parallelogram, the base, b, is 20 feet and the height, h, is 12 feet.

$$\begin{aligned}\text{Area of a parallelogram} &= bh \\ &= (20)(12) \\ &= 240 \text{ ft}^2\end{aligned}$$



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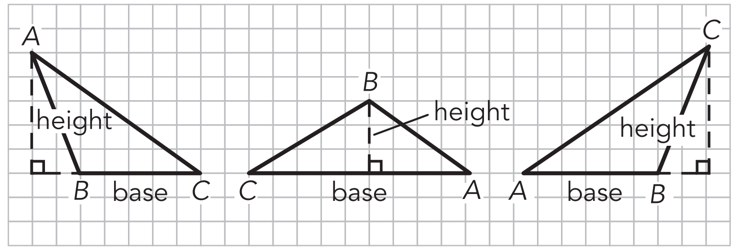
Area of a Triangle

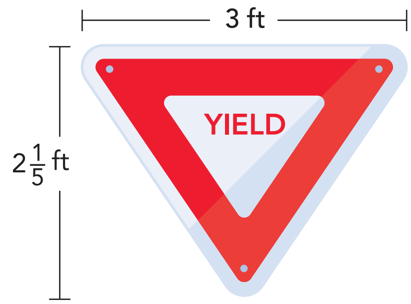
The area of a triangle is equal to $\frac{1}{2}bh$. The base of a triangle can be any of its sides.

Draw a line straight down from the top corner of the triangle to the bottom, or base at a right angle. This is called the height of the triangle.



For example, in this triangle, the base, b, is equal to 3 feet and the height, h, is equal to $2\frac{1}{5}$ feet.

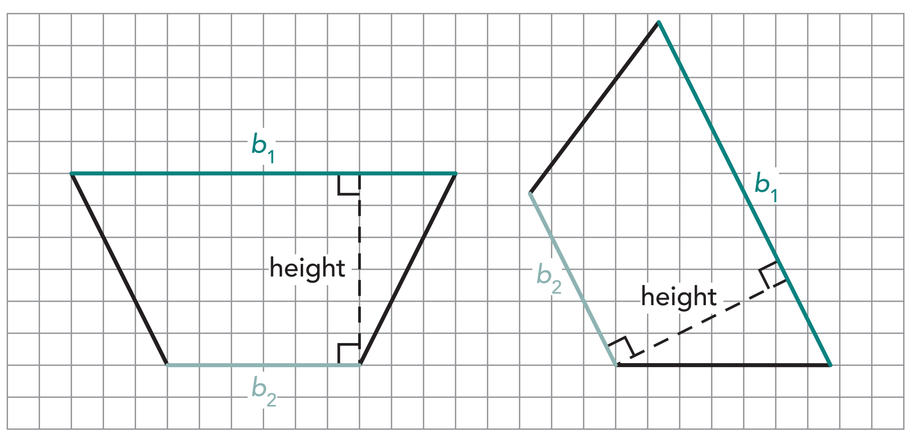
$$\text{Area of a triangle} = \frac{1}{2}bh$$
$$= \frac{1}{2}(3)\left(2\frac{1}{5}\right)$$
$$= 3\frac{3}{10}\ \text{ft}^2$$



Area of a Trapezoid

A trapezoid has two bases that are parallel to each other, often labeled b_1 and b_2. The other two sides of a trapezoid are called the legs of the trapezoid. A height of a trapezoid is the length of the shortest line drawn from one base to the other at a right angle.

The area of a trapezoid is equal to $\frac{1}{2}(b_1 + b_2)h$.



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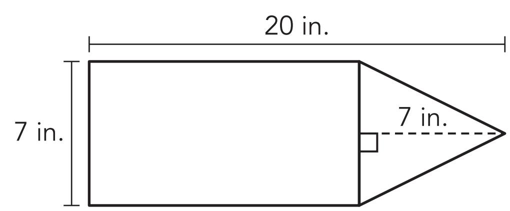
Composite Figures

A composite figure is a figure that is made up of more than one geometric figure. Area is additive. The area of a composite figure can be determined by decomposing it into familiar shapes and then adding together the areas of those shapes. The composite figure shown is composed of a rectangle and a triangle.

Area of composite figure = Area of Rectangle + Area of Triangle

$= (7)(13) + \frac{1}{2}(7)(7)$

$= 91 + 24\frac{1}{2} = 115\frac{1}{2}$



The area of the composite figure is $115\frac{1}{2}$ in.^{2}.

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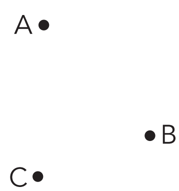
Topic 4: Decimals and Volume

Key Terms

- point
- line segment
- polygon
- geometric solid
- polyhedron
- face
- edge
- vertex
- right rectangular prism
- cube
- pyramid
- volume
- composite solid
- trailing zeros
- net
- surface area
- slant height

A **point** is a location in space, often represented using a dot and named by a capital letter.

Points A, B, and C are shown.



A **line segment** is a portion of a line that includes two points and the points between those two points.

Line segment $\overline{AB}$ is shown.

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In this topic students calculate volumes of right rectangular prisms. Next, students determine the surface area of three-dimensional solids by determining the area of their two-dimensional nets. Through the problem-solving activities with area and volume, students review operating with decimals.

TEXAS MATH SOLUTION

Polygons and Polyhedrons

A polygon is a closed figure formed by three or more line segments. A geometric solid is a bounded three-dimensional geometric figure. A polyhedron is a three-dimensional solid figure that is made up of polygons that are called faces. An edge is the intersection of two faces and a vertex is the point where the edges meet.

For example, Figure A is a right rectangular prism, which is a polyhedron with three pairs of congruent and parallel faces.

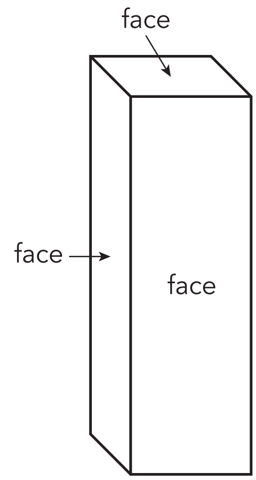

Figure A

Figure B is a cube, which is a polyhedron that has six congruent squares as faces.

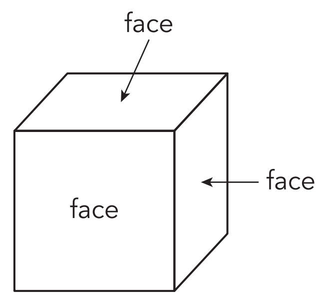

Figure B

Figure C is a rectangular pyramid. A pyramid is a polyhedron with one base and the same number of triangular faces as there are sides of the base.

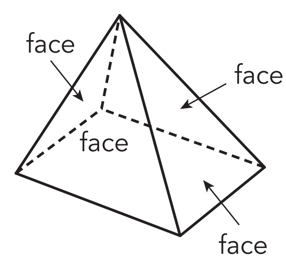

Figure C

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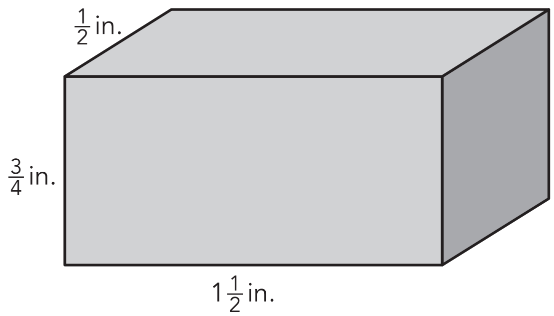
Volume

Volume is the amount of space occupied by an object. The volume of an object is measured in cubic units. A unit cube is a cube whose sides are all 1 unit long. The volume of a rectangular prism is a product of its length, width and height: $V = l \cdot w \cdot h$.

For example, to determine the volume of the right rectangular prism shown with the given dimensions, you can fill the prism with cubes, but smaller unit cubes with fractional side lengths are required.



Assign a unit fraction to the dimensions of each cube. Use the least common multiple (LCM) of the fraction denominators to determine the unit fraction.	LCM(2, 4) = 4 So, each cube will measure $\frac{1}{4}$ in. $\cdot$ $\frac{1}{4}$ in. $\cdot$ $\frac{1}{4}$ in. The volume of each unit cube is $\frac{1}{64}$ cubic inch.
Determine the number of cubes needed to pack the prism in each dimension.	length: $1\frac{1}{2} \div \frac{1}{4} = 6$ width: $\frac{1}{2} \div \frac{1}{4} = 2$ height: $\frac{3}{4} \div \frac{1}{4} = 3$
Determine the number of cubes that make up the right rectangular prism.	$6 \cdot 2 \cdot 3 = 36$
Multiply the number of cubes by the volume of each cube to determine the volume of the right rectangular prism.	$36 \cdot \frac{1}{64} = \frac{36}{64}$ $= \frac{9}{16}$

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The volume of the right rectangular prism is $\frac{9}{16}$ in^3.

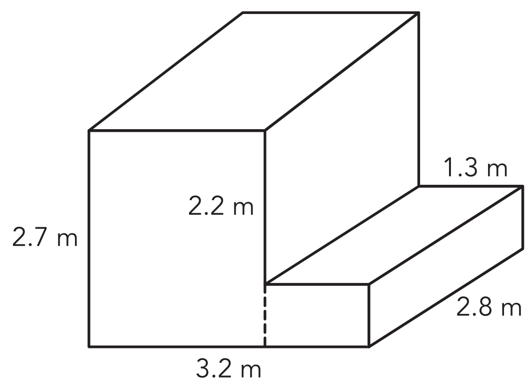
You can use the formula $V = B \cdot h$ to calculate the volume of any prism. However, the formula for calculating the value of B will change depending on the shape of the base. In a rectangular prism, $B = lw$.

Composite Solids

A composite solid is made up of more than one geometric solid. You can decompose a composite solid into more than one polyhedron in order to determine its volume. For example, to determine the volume of the composite solid shown, you can decompose the solid into two rectangular prisms and calculate the volume of each.

Volume of larger prism = $1.9 \cdot 2.8 \cdot 2.7 = 14.364$ m^3

Volume of smaller prism = $1.3 \cdot 2.8 \cdot 0.5 = 1.82$ m^3



To calculate the sum or difference of decimals, line up the decimals so that like place values are in the same column. Use the decimal point to help you correctly align.

A trailing zero was added to 1.82. Trailing zeros do not affect the value of a number.

$$\begin{array}{r} 14.364 \\ +1.820 \\ \hline 16.184 \end{array}$$

The volume of the composite solid is 16.184 m^3.

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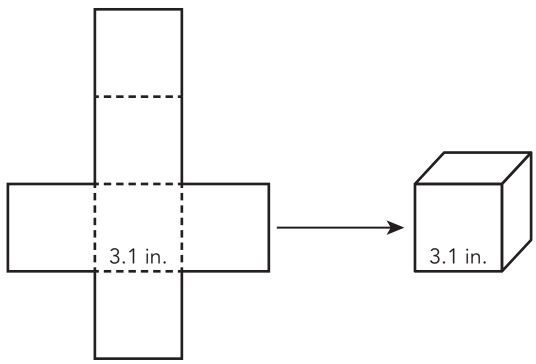
Nets

A net is a two-dimensional representation of a three-dimensional geometric figure. It has the following properties:

- The net is cut out as a single piece.
- All of the faces of the geometric solid are represented in the net.
- The faces of the geometric solid are drawn so that they share common edges.

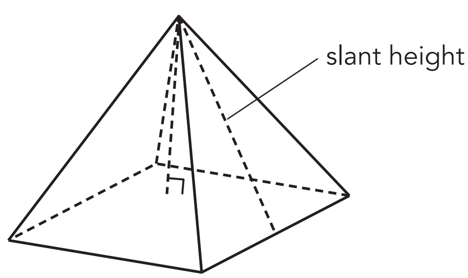
The surface area of a three-dimensional geometric figure is the total area of all of its two-dimensional faces.



Pyramids

A pyramid is a polyhedron with one base and the same number of triangular faces as there are sides of the base.

The vertex of a pyramid is the point at which all the triangular faces intersect. A slant height of a pyramid is the distance measured along a triangular face from the vertex of the pyramid to the midpoint, or center, of the base.



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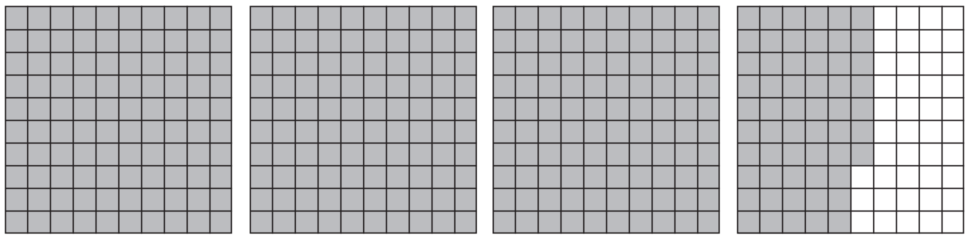
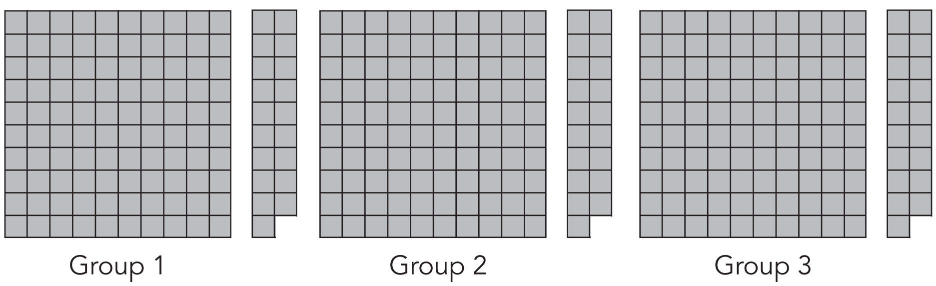
Using a Hundredths Grid to Divide Decimals

You can use a hundredths grid to model dividing decimals, such as 3.57 ÷ 3. First, shade hundredths grids to represent 3.57.



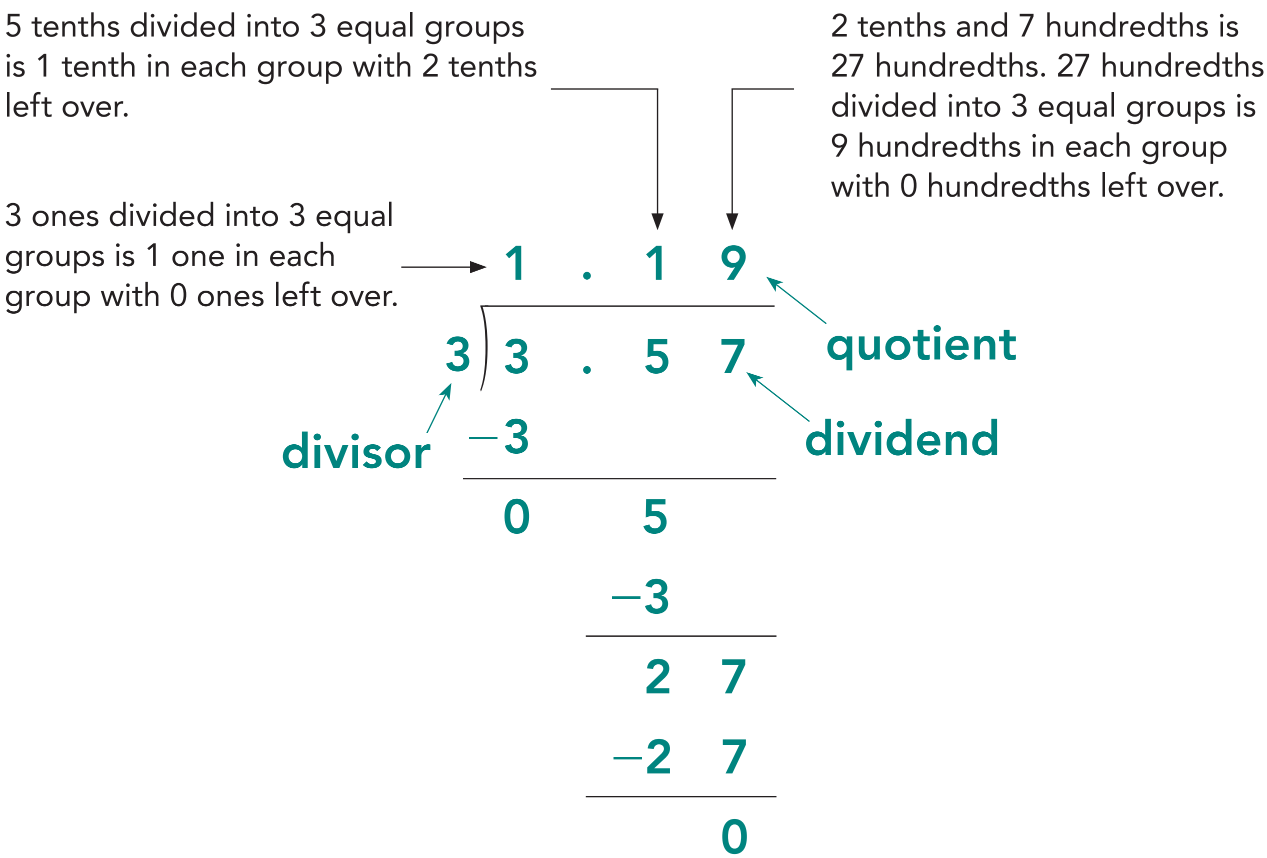
Next, divide the shaded model into 3 equal groups. One whole grid and 19 small squares are in each group. So, 3.57 ÷ 3 = 1.19.



Using a Standard Algorithm to Divide Decimals

The long division algorithm uses organized estimation and place value to determine a quotient, or the number of times the divisor is contained in the dividend.

Let's use the standard algorithm to divide 3.57 ÷ 3. The dividend is 3.57 and the divisor is 3.



The quotient is 1.19; therefore, 3.57 ÷ 3 = 1.19.

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Discuss important dates throughout this module such as assessments, assignments, or class events with your student. Use the table to record these dates and reference them as your student progresses through the module.

Important Dates	
Date	Reason

Using the link below, visit the Texas Math Solution Support Center for students and caregivers to access additional resources such as:

- Mathematics Glossaries
- Videos
- Topic Materials
- A Letter to Families and Caregivers

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